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Restrained Edges Effect on the Dynamics of Thermoelastic Plates under Different End Conditions

Wael R. Abdul-Majeed* Muhsin J. Jweeg Adnan N. Jameel*****

**Department of Mechatronics Engineering/Al-Khawarizmi College of Engineering/ University of Baghdad*
*** College of Engineering/University of Al-Nahrain*
**** Department of Mechanical Engineering/College of Engineering/University of Baghdad*

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Abstract

Frequency equations for rectangular plate model with and without the thermoelastic effect for the cases are: all edges are simply supported, all edges are clamped and two opposite edges are clamped others are simply supported. These were obtained through direct method for simply supported ends using Hamilton's principle with minimizing Ritz method to total energy (strain and kinetic) for the rest of the boundary conditions. The effect of restraining edges on the frequency and mode shape has been considered. Distributions temperatures have been considered as a uniform temperature the effect of developed thermal stresses due to restrictions of ends conditions on vibration characteristics of a plate with different will be investigated. it is noticed that the thermal stress will increase with increasing the heatnig temperature and that will cause the natural frequency to be decreased for all types of end conditions and for all modes of frequency.

Keywords: *Thermoelasticity, thin plate, ends condition, mode shape, natural frequency.*

1. Introduction

Thermoelasticity is concerned with questions of equilibrium of bodies treated as thermodynamic systems whose interaction with the environment is confined to mechanical work, external forces, and heat exchange. Because of constraints, a non -uniform temperature distribution in a component having a complex shape usually gives rise to thermal stresses. It is essential to know the magnitude and effect of these thermal stresses when carrying out on rigorous design of such components. The thermal stresses alone and in combination with the mechanical stresses produced by the external forces will be effect on dynamics properties of apart such as natural frequency and mode shape . Naji, et al. [1] studied the thermal stresses generated within a rapidly heated thin metal plate when a parabolic two-step heat conduction equation is used.

The effect of different design parameters on the thermal and stress behavior of the plate is investigated. Al-Huniti, et al. [2] investigated the thermally induced vibration in a thin plate under a thermal excitation .The excitation is in the form of a suddenly applied laser pulse (thermal shock). The resulting transient variations of temperature are predicted using the wave heat conduction model (hyperbolic model), which accounts for the phase lag between the heat flux and the temperature gradient. The resulting heat conduction equation is solved semi analytically using the Laplace transformation and the Riemann sum approximation to calculate the temperature distribution within the plate. The equation of motion of the plate is solved numerically using the finite difference technique to calculate the transient variations in deflections. Norris and Photiadis [3] enabled direct calculation of thermoelastic damping in vibrating elastic solids.

The mechanism for energy loss is thermal diffusion caused by inhomogeneous deformation, flexure in thin plates. The general result is combined with the Kirchhoff assumption to obtain a new equation for the flexural vibration of thin plates incorporating thermoelastic loss as a damping term. The thermal relaxation loss is inhomogeneous and depends upon the local state of vibrating flexure, specifically, the principal curvatures at a given point on the plate. The influence of modal curvature on the thermoelastic damping is described through a modal participation factor. The effect of transverse thermal diffusion on plane wave propagation is also examined. It is shown that transverse diffusion effects are always small provided the plate thickness. Tran a, et al. [4] studied the thermally induced vibration and its control for thin isotropic and laminated composite plates. The structural intensity (SI) pattern of the plates which have different material orientations and boundary conditions was analyzed. The thermoelasticity simulation is performed using the finite element method. It shows that the structural energy flows are dependent on the material structures as well as the boundary conditions for a prescribed thermal source. The position to attach a damper for controlling the thermally induced vibration is investigated based on the virtual sources and sinks of the SI patterns.

2. Analytical Study

The plate analyzed has usually been assumed to be composed of a single homogeneous and isotropic material with shape and dimensions as in Fig. (1) [5].

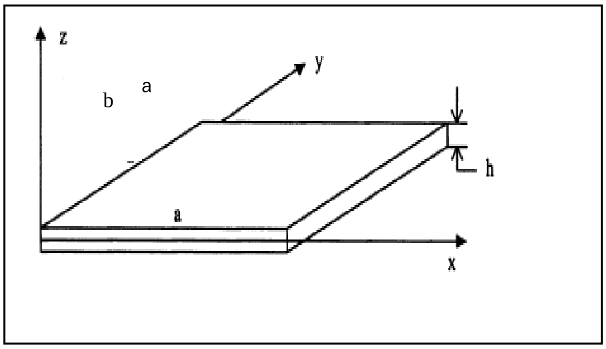

Fig. 1.Schematic Diagram of Thin Plate.

3. Boundary Conditions

General closed – form solutions are given of a thermoelastic rectangular plate with various elementary boundary conditions on each of the four edges. Appendix A collect some important combinations of end boundary conditions. [Let the plate be placed in a coordinate system with the origin at it center and the edge width (a) be parallel to x – axis and and the edge width (b) be parallel to y as in Fig. (1)

4. Natural Frequency and Mode Shape of dynamic Thermoelastic plates

Free, transverse vibrations of the thermoelastic structural with neglecting the effect of in plane vibrations are studied with different end boundary conditions under uniform temperature distribuation.

4.1. All Edges are Simply Supported

The general governing differential equation of free vibration of thermoelastic plate is represented by [6]:

$$D\nabla^4 w = \rho h\ddot{w} - \frac{\nabla^2 M_t}{1-\upsilon} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} \quad \ldots(1)$$

Where $D = \dfrac{Eh^3}{12(1-\upsilon^2)}$, and the quantities

$$N_t = \alpha E\int_{-h/2}^{h/2}(\Delta T)dz$$

$$M_t = \alpha E\int_{-h/2}^{h/2}(\Delta T)zdz \quad \ldots(2)$$

Which represents the thermal stress resultants .

Then the boundary conditions for the deflection w are represented in Appendix C

$$w_{x=0} = w_{x=a} = 0 \quad , \quad w_{y=0} = w_{y=b} = 0$$

$$\frac{\partial^2 w_{x=0}}{\partial x^2} = \frac{\partial^2 w_{x=a}}{\partial x^2} = 0 \quad , \quad \frac{\partial^2 w_{y=0}}{\partial y^2} = \frac{\partial^2 w_{y=b}}{\partial y^2} = 0$$

The initial conditions assuming the plate initially at rest in the refrence position ,are given by

$$w(x,y,0) = \frac{\partial w}{\partial t}(x,y,0) = 0 \qquad 0 \le x \le a\ , \quad 0 \le y \le b \quad \ldots(3)$$

The displacement function $w(x,y,t)$ is approximated by means of the expansion [7].

$$w(x,y,t) \approx w(x,y)\sin\omega_{mn}t = \sin\omega t\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad \ldots(4)$$

And the displacement function $w(x,y)$ is assumed from functions, that satisfies identically the boundary conditions; these functions are different due to the types of end conditions at x and y axis and this will be studied .

The plate will have uniform temperature

$$\Delta T = T_c \qquad \ldots(5)$$

substitution of Eq.(5) in Eq. (2) we have

$$N_t = \alpha EhT_c \qquad M_t = 0 \qquad \ldots(6)$$

So that for all edges are restrained

$$N_x = N_y = -\frac{N_t}{1-\upsilon} \qquad N_{xy} = 0 \qquad \ldots(7)$$

with all edges are ***restrained*** ,substituting the thermal forces in Eq. (7) and the deflection from Eq. (4) into the governing differential equation of free vibration of thermoelastic plate in Eq.(1) noting that $Mt = 0$, one obtains the following frequency equation.

$$D\pi^4\left[(\frac{m}{a})^2 + (\frac{n}{b})^2 \right]^2 - \frac{N_t\pi^2}{1-v}\left[(\frac{m}{a})^2 + (\frac{n}{b})^2\right] = \rho h\omega_{mn}^2 \qquad \ldots(8)$$

for natuaral frequancy of plate without thermal load $N_t = 0$

$$\omega_{mnf}^2 = \frac{D\pi^4}{\rho ha^4}\left[m^2 + r^2n^2\right]^2 \qquad \ldots(9)$$

Then

$$\omega_{mn}^2 = \omega_{mnf}^2 - \frac{N_t\pi^2}{\rho h(1-v)a^2}(m^2 + r^2n^2) \qquad \ldots(10)$$

Substituting Eq. (7) into Eq. (10) , the natuaral frequancy as a function of uniform temperature T_c can be presented as

$$\omega_{mn}^2 = \omega_{mnf}^2 - \frac{\alpha ET_c\pi^2}{\rho h(1-v)a^2}(m^2 + r^2n^2) \qquad \ldots(11)$$

And for *restrained* edges at x=0,a and *unrestrained* at y=0,b thermal forces will be

$$N_x = -\frac{N_t}{1-\upsilon} \qquad N_{xy} = N_Y = 0 \qquad \ldots(12)$$

and the natuaral frequancy will be

$$\omega_{mn}^2 = \omega_{mnf}^2 - \frac{N_t m^2}{\rho h(1-v)a^2} \qquad \ldots(13)$$

and the function of the uniform temperature T_c will be

$$\omega_{mn}^2 = \omega_{mnf}^2 - \frac{\alpha ET_c\pi^2 m^2}{\rho(1-v)a^2} \qquad \ldots(14)$$

4.2. All Edges are Clamped

To derive the differential equation for lateral vibration of rectangular thermoelastic plate a kinetic energy of the plate in edition to the total strain energy of the plate and apply the Hamilton's principle to derive the equation of motion. The kinetic energy due to the velocity $\dot{w}$ only is represented as

$$T = \frac{1}{2}\iint_A \rho h\dot{w}^2 dxdy \qquad \ldots(15)$$

the Hamilton's principle for the plate undergoing small deflection can be set as [8]:

$$\delta\int_{t1}^{t2}(T - \Pi_{strain})dt = 0 \qquad \ldots(16)$$

Then the lagrangian of the plate from the above equation can be written as

$$L = \frac{1}{2}\iint_A D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 dxdy - \frac{1}{2}\iint_A \Big\{ N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) \Big\}dxdy + \iint_A \frac{M_t}{1-v}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)dxdy - \frac{1}{2}\iint_R \rho h\dot{w}^2 dxdy \qquad \ldots(17)$$

For free vibration the solution is assumed

$$w(x,y,t) = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}X_i(x)Y_j(y)\sin\omega t \qquad \ldots(18)$$

Substituting Eq. (18) by Eq. (19) and minimizing the resulting lagrangian with respect to A_{ij} ,we get

$$\sum_{k=1}^{m}\sum_{j=1}^{n}\left[\begin{array}{l} D\int_0^a\int_0^b \left((X'')^2Y^2+2X''XY''Y+X^2(Y'')^2\right)dxdy \\ -\int_0^a\int_0^b \left(N_x(X')^2Y^2+N_yX^2(Y')^2+2N_{xy}XX'YY'\right)\,dxdy \end{array}\right]A_{ij}+\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{M_t}{(1-\nu)}\int_0^a\int_0^b(X''Y+XY'')dxdy=\sum\sum\left[\rho h\omega^2\int_0^a\int_0^b X^2Y^2dxdy\right]A_{ij} \quad \ldots(20)$$

This is the general frequency equation**.**

With uniform temperature T_c and all edges are *restrained* with the aid of Eq. (2) for thermal forces and thermal moments into general frequency equation we have:

$$\omega^2=\frac{D\int_0^a\int_0^b\left((X'')^2Y^2+2X''XY''Y+X^2(Y'')^2\right)dxdy+\frac{N_t}{(1-\nu)}\int_0^a\int_0^b\left((X')^2Y^2+X^2(Y')^2\right)dxdy}{\rho h\int_0^a\int_0^b X^2Y^2dxdy} \quad \ldots(21)$$

The frequency of plate without thermal effect has the form

$$\omega_{ijf}^{\ 2}=\frac{D\int_0^a\int_0^b\left((X'')^2Y^2+2X''XY''Y+X^2(Y'')^2\right)dxdy}{\rho h\int_0^a\int_0^b X^2Y^2dxdy} \quad \ldots(22)$$

Then with substituting the mode shape of clamped ends X_i and Y_j from Appendix C

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{N_t\left(\alpha_1^2+r^2\alpha_3^2\right)}{\rho ha^2(1-\nu)} \quad \ldots(23)$$

With ω_{ijf} for free vibration of clamped plate

$$\omega_{ijf}^{\ 2}=\frac{D\left(\alpha_1^4+2r^2\alpha_2+r^4\alpha_3^4\right)}{\rho ha^4} \quad \ldots(24)$$

Then ω_{ij} terms of uniform temperature will be as:

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{\alpha ET_c\left(\alpha_1^2+r^2\alpha_3^2\right)}{\rho a^2(1-\nu)} \quad \ldots(25)$$

Where α_1,α_2 and α_3 are calculated from Appendix C

For clamped edges *restraind* at x=0,a and *unrestrained* at y=0,b

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{N_t\alpha_1^2}{\rho ha^2(1-\nu)} \quad \ldots(26)$$

In terms of temperature

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{\alpha ET_c\alpha_1^2}{\rho a^2(1-\nu)} \quad \ldots(27)$$

4.3. Edges are Clamped at x=0,a and Simply Supported at y=0,b

The general frequency equation of clamped edges Eq. (20) are suitable for edges clamped at x=0,a and simply supported at y=0,b. With uniform temperature T_c and all edges *restrained* with the aid of Eq. (2) for thermal forces and thermal moments into general frequency equation and arranged with substituting the mode shape of two clamped ends and two simply supported ends X_i and Y_j from Appendix C into above equations the result will be

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{N_t\left(\beta_1^2+r^2\beta_3^2\right)}{\rho ha^2(1-\nu)} \quad \ldots(28)$$

With ω_{ijf} for free vibration suitable for edges clamped at x=0,a and simply supported at y=0,b .

$$\omega_{ijf}^{\ 2}=\frac{D\left(\beta_1^4+2r^2\beta_2+r^4\beta_3^4\right)}{\rho ha^4} \quad \ldots(29)$$

Then ω_{ij} in terms of uniform temperature will be:

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{\alpha ET_c\left(\beta_1^2+r^2\beta_3^2\right)}{\rho a^2(1-\nu)} \quad \ldots(30)$$

Where β_1,β_2 and β_3 calculated from Appendix C

For clamped edges *restraind* at x=0,a and simply supported *unrestrained* at y=0,b

$$\omega_{ij}^{\ 2}=\omega_{ijf}^{\ 2}-\frac{N_t\beta_1^2}{\rho ha^2(1-\nu)} \quad \ldots(31)$$

In terms of temperature

$$\omega_{ij}^{\ 2} = \omega_{ijf}^{\ 2} - \frac{\alpha E T_c \beta_1^2}{\rho a^2 (1-v)} \qquad \dots(32)$$

5. Results and Discussions

The sample of calculations was made on Aluminum 1060-H18 rectangular plate which has the mechanical and thermal properties given in appendix A respectively. Rectangular plate with three aspect ratio a/b (r = 1.2). and a/h (φ =120) and owing constant magnitude of a=0.12 m has been considered. The effects of the uniform increase of temperature of plates (thermoelastic behavior) on the natural frequency and mode shapes with different three types of ends conditions have been studied.

Figures (2), (3) and (4) show the effect of temperature rising on natural frequencies analytical magnitudes till it reaches the thermal buckling temperature for plates with all edges restrained. The types are SSSS, CCCC and CSCS respectively

It is observed that the lowest natural frequencies of all types reach zero when the temperatures get to the thermal buckling temperature; also the first five natural frequencies of plates decreas with increasing the temperature. Second and third natural frequencies of CSCS plate have the same magnitudes almost.

Figures (5), (6) and (7) show the effect of temperature rising on natural frequencies analytical magnitudes till it reaches the thermal buckling temperature for plates with edges at x=0,a restrained the types are SSSS, CCCC and CSCS respectively

The lowest natural frequencies of all types reach zero when the temperatures has the thermal buckling temperature. The first five natural frequencies of plates decrease with increasing the temperature.

The fifth natural frequency of SSSS plate will become the fourth natural frequency and vice versa when the temperature has magnitude close to 6 C^0 . Also CCCC natural frequencies have the same behavior of SSSS type but they are switching at magnitude close to 3 C^0.

CSCS natural frequencies have the switching behavior between second and third natural frequencies at magnitude close to 1 C^0.

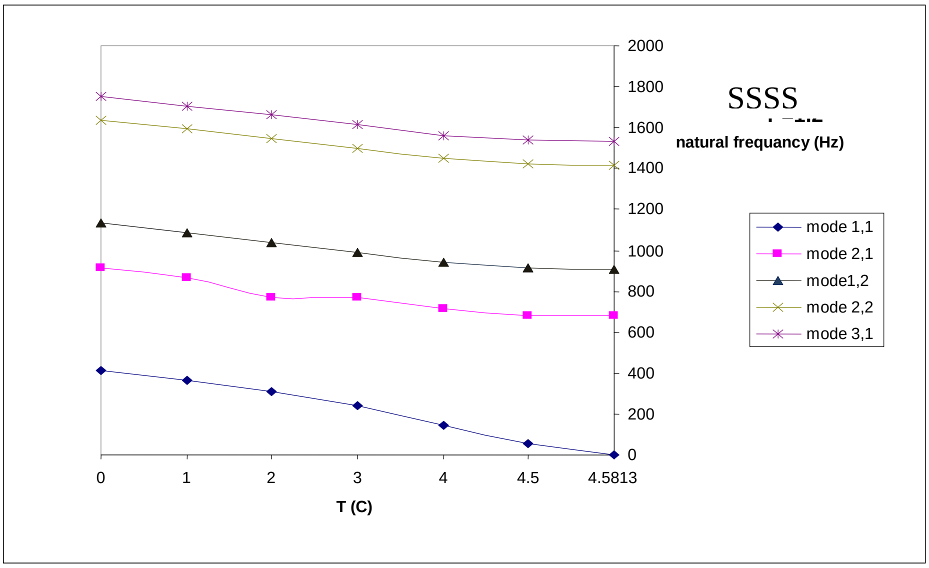

Fig. 2. Effect of Temperature on First Five Natural Frequencies Magnitude on SSSS Plate, All Edges are Restrained.

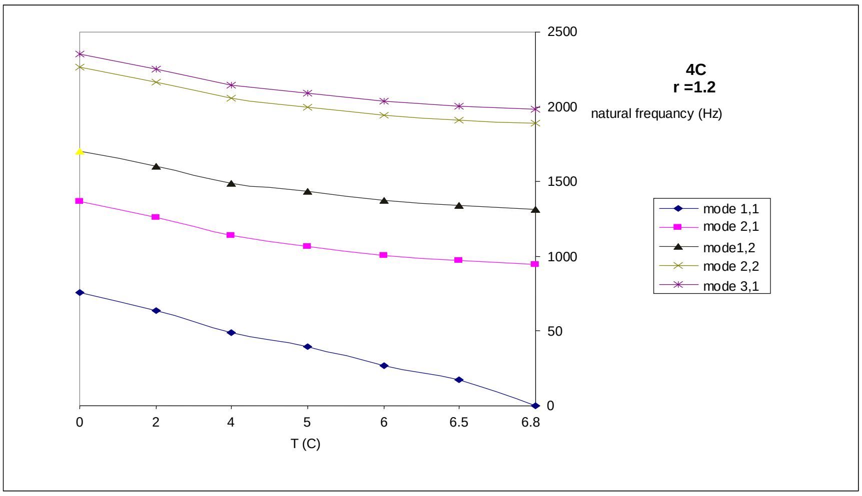

Fig. 3. Effect of Temperature on First Five Natural Frequencies Magnitude on CCCC Plate with All Edges are Restrained.

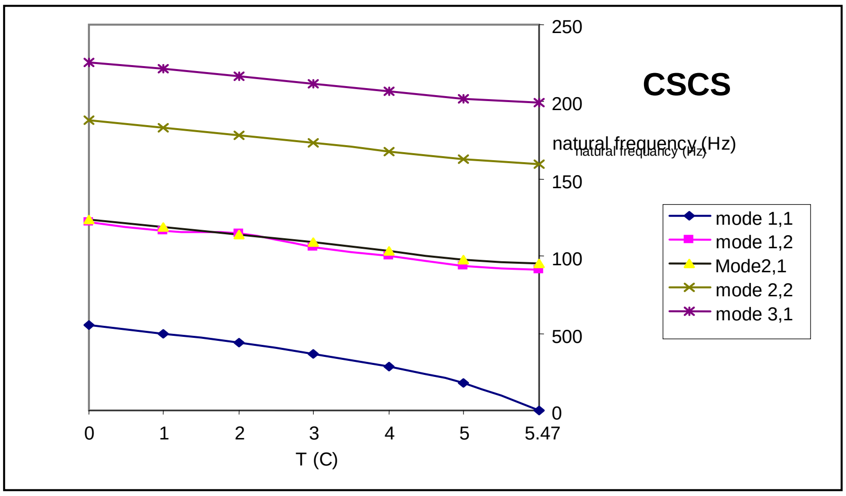

Fig. 4.Effect of Temperature on First Five Natural Frequencies Magnitude on CSCS Plate, All Edges is Restrained.

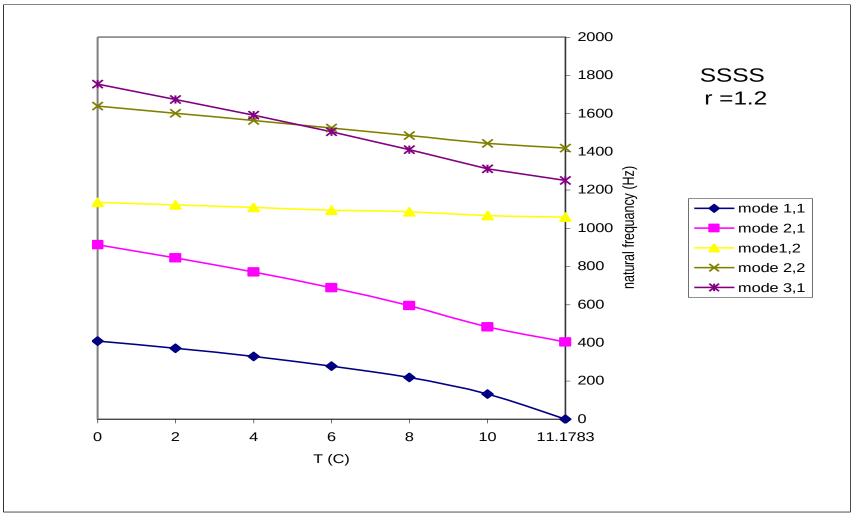

Fig. 5.Effect of Temperature on First Five Natural Frequencies Magnitude on SSSS Plate, Edges at y=0, b are Unrestrained.

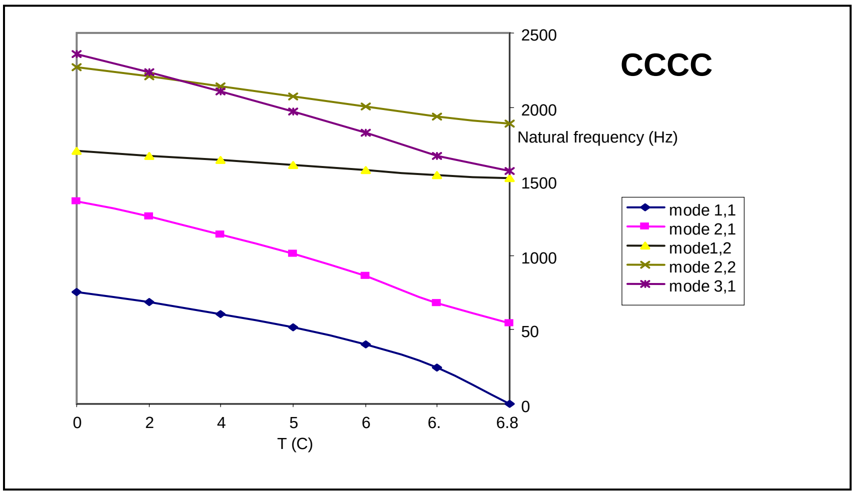

Fig. 6.Effect of temperature on First Five Natural Frequencies Magnitude on CCCC Plate, Edges at y=0, b are Unrestrained.

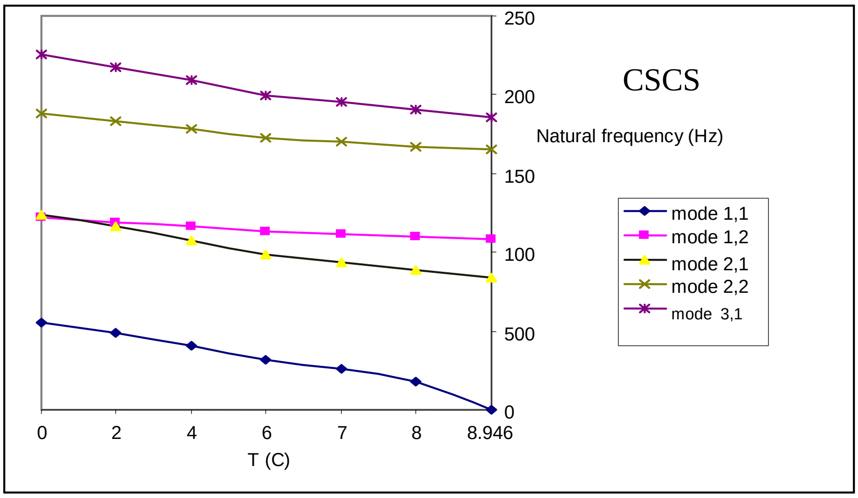

Fig. 7. Effect of Temperature on First Five Natural Frequencies Magnitude on CSCS Plate, Edges at y=0, b are Unrestrained.

6. Conclusions

The following are the main summarized conclusions of this paper:

1. Thermal stresses have a significant influence on the natural frequency for the free boundary conditions compared with clamped boundaries, so that the boundary condition is one of the important factors that influence the vibration and mode shapes.
2. The lowest natural frequencies of all types reach zero when the temperatures has the thermal buckling temperature
3. The first five natural frequencies of plates decreasing with increasing of the uniform temperature of the plates for all types of ends conditions
4. In the case of the two opposite edges which are unrestrained, there is a switching between the modes of natural frequency when the temperature increases for each type of ends conditions.

Nomenclature

Latin Symbols

A	Area (mm^2)
a, b	Plate side length (mm)
D	Flexural rigidity of an isotropic plate (N.mm)
E	Modulus of elasticity of isotropic material (N/mm^2)

h	Plate thickness (mm)
i ,j	Integer
Mt	Thermal bending moment (N.m)
m,n	Integer
Nx, Ny	Edge forces per unit length (N/m)
Nxy	Shearing forces per unit length (N/m)
Nt	Thermal forces per unit length (N/m)
r	Dimensional aspect ratio a/b (m/m)
T	Temperature (C^0), Kinetic energy of the element (J)
t	Time (sec)
x, y, z	Cartesian coordinates

Greek Symbols

α_m, β_n	Coefficients
ν	Poisson's ratio
ρ	Mass density (Kg/mm^3)
Π_{strain}	Strain energy stored in complete plate (J)
$\omega_{ijf}, \omega_{ij}$	Angular frequency without and with thermal effect (rad/s)
φ	Dimensional aspect ratio side / thickness (m/m)
α	Coefficient of thermal expansion (1/C^0)
w	Deflection (mm)

Abbreviations Symbols

CCCC	Clamped-Clamped-Clamped-Clamped
CSCS	Clamped-Simply-Clamped-Simply
SSSS	Simply-Simply-Simply-Simply

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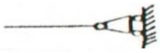
Appendices

Appendix A

Some Combinations of End Boundary Conditions

deflection	**Mid-plane deformation**	**symbol**
clamped	Restrained	
	unrestrained	
supported	restrained	
	unrestrained	
free	restrained	
	unrestrained	

Appendix B

Mechanical Properties of Aluminum 1060-H18

Density	2705 kg/m³
Hardness, Brinell	35
Ultimate Tensile Strength	27 MPa
Tensile Yield Strength	20 MPa
Elongation at Break	6 %
Modulus of Elasticity	69 GPa
Poisson's Ratio	0.3
Fatigue Strength	44.8 MPa
Machinability	30 %
Shear Modulus	26 GPa
Shear Strength	75.8 MPa

Thermal Properties of Aluminum 1060-H18

Heat Capacity	0.9 J/g °C
Thermal Conductivity	233 W/m °C
Coefficient of Thermal expansion	2.34e-5/°C
Convection Coefficient	2.5 W/m² °C

Appendix C

For SSSS ends condition

$$X_i = \sin \mu_i x \quad , \quad Y_j = \sin \mu_j y$$

$$w_{x=0} = w_{x=a} = 0 \quad , \quad w_{y=0} = w_{y=b} = 0 \quad , \quad \frac{\partial^2 w_{x=0}}{\partial x^2} = \frac{\partial^2 w_{x=a}}{\partial x^2} = 0 \, , \quad \frac{\partial^2 w_{y=0}}{\partial y^2} = \frac{\partial^2 w_{y=b}}{\partial y^2} = 0$$

For CCCC ends condition

$$X_i = \sin\mu_i x - \sinh\mu_i x - \eta_i(\cos\mu_i x - \cosh\mu_i x)$$
$$\eta_i = (\sin\mu_i a - \sinh\mu_i a)/(\cos\mu_i a - \cosh\mu_i a)$$
$$Y_j = \sin\mu_j y - \sinh\mu_j y - \eta_j(\cos\mu_j y - \cosh\mu_j y)$$
$$\eta_j = (\sin\mu_j b - \sinh\mu_j b)/(\cos\mu_j b - \cosh\mu_j b)$$
$$w_{x=0} = w_{x=a} = 0,\quad w_{y=0} = w_{y=b} = 0 \quad , \frac{\partial\, w_{x=0}}{\partial x} = \frac{\partial\, w_{x=a}}{\partial x} = 0, \quad \frac{\partial w_{y=0}}{\partial y} = \frac{\partial\, w_{y=b}}{\partial y} = 0$$

For SCSC ends condition

$$X_i = \sin\mu_i x - \sinh\mu_i x - \eta_i(\cos\mu_i x - \cosh\mu_i x)$$
$$\eta_i = (\sin\mu_i a - \sinh\mu_i a)/(\cos\mu_i a - \cosh\mu_i a) \quad , \quad Y_j = \sin\mu_j y$$
$$w_{x=0} = w_{x=a} = 0 \quad , \quad w_{y=0} = w_{y=b} = 0 \quad , \quad \frac{\partial w_{x=0}}{\partial x} = \frac{\partial\, w_{x=a}}{\partial x} = 0, \quad \frac{\partial^2 w_{y=0}}{\partial y^2} = \frac{\partial^2 w_{y=b}}{\partial y^2} = 0$$

Where $\mu_i a$ and $\mu_j b$ are the roots of the above equations

The roots of SSSS ends condition are

$$\mu_i = \frac{m\pi}{a} \quad , \quad \mu_i = \frac{n\pi}{b}$$

The roots of CCCC ends condition are

$\alpha_1 = \alpha_3 = 4.73$, $\alpha_2 = 151.3$ For i=1 , j=1

$\alpha_1 = 4.73$, $\alpha_3 = (j+0.5)\pi$, $\alpha_2 = 12.3\alpha_3(\alpha_3 - 2)$ For i=1 , j=2,3,4,….

$\alpha_1 = (i+0.5)\pi$, $\alpha_3 = 4.37$, $\alpha_2 = 12.3\alpha_1(\alpha_1 - 2)$ For i=2,3,4,… j=1

$\alpha_1 = (i+0.5)\pi$, $\alpha_3 = (j+0.5)\pi$, $\alpha_2 = \alpha_1(\alpha_1 - 2)\alpha_3(\alpha_3 - 2)$ For i=2,3,4,. j=2,3,4,….\

$\alpha_1 = (i+0.5)\pi$, $\alpha_3 = (j+0.5)\pi$, $\alpha_2 = \alpha_1(\alpha_1 - 2)\alpha_3(\alpha_3 - 2)$ For i=2,3,4,. j=2,3,4,….

The roots of CSCS ends condition are

$\beta_1 = 4.73$, $\beta_3 = j\pi$, $\beta_2 = 12.3 j^2\pi^2$ For i=1 , j=1, 2, 3,..

$\beta_1 = (i+0.5)\pi$, $\beta_3 = j\pi$, $\beta_2 = \alpha_1(\alpha_1 - 2) j^2\pi^2$ For i=2,3,4,… j=1,2,3….

تاثير الحافات المحددة من الحركة على ديناميكية الصفائح المرنة حراريا تحت ظروف نهايات مختلفة

وائل رشيد عبد المجيد* محسن جبر جويج عدنان ناجي جميل*****

*قسم هندسة الميكاترونكس/ كلية الهندسة الخوارزمي/ جامعة بغداد
**كلية الهندسة / جامعة النهرين
***قسم الهندسة الميكانيكية/ كلية الهندسة/ جامعة بغداد

الخلاصة

صيغ معادلة التردد الطبيعي لصفائح مستطيلة الشكل مع وبدون تاثير المرونة الحرارية لحالات النهايات التالية: كل النهايات ذات اسناد بسيط ، كل النهايات مثبتة ، و نهايتين متقابلتين باسناد بسيط ونهايتين مثبتتين تم ايجادها من خلال الحل بالطريقة المباشرة للنهايات باسناد بسيط ، وباستخدام مباديء هاملتون والتخفيض بطريقة رتز للطاقة الكلية لباقي انواع النهايات . تاثير تثبيت النهايات افقيا بوجود درجة حرارة منظمة التوزيع على الترددات الطبيعية وشكل التردد تم دراستها كما تم التعرف على تاثير تولد الاجهادات الحرارية الناتجة من تثبيت النهايات افقيا على خواص الاهتزازات وتم ملاحظة ان الاجهادات الحرارية المتولدة تزداد مع ازدياد درجة حرارة التسخين وهذا يودي الى نقصان في الترددات الطبيعية لكل انواع النهايات ولكل اشكال الترددات الطبيعية.