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Experimental and Numerical Study of Crack Effect on Frequency of Simple Supported Beam

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Abstract

In this research the natural frequency of a cracked simple supported beam (the crack is in many places and in different depths) is investigated analytically, experimentally and numerically by ANSYS program, and the results are compared. The beam is made of iron with dimensions of L*W*H= (0.84*0.02* 0.02m), and density = 7680kg/m^3, E=200Gpa. A comparison made between analytical results from ANSYS with experimental results, where the biggest error percentage is about (7.2 %) in crack position (42 cm) and (6 mm) depth. Between Rayleigh method with experimental results the biggest error percentage is about (6.4 %) for the same crack position and depth. From the error percentages it could be concluded that the Rayleigh method gives close results to experimental than ANSYS. Also it is found that the frequency of beam when the crack is in the middle position is less than the frequency with crack near the end position.

Keywords: *Vibration beam, crack beam, health monitoring, experimental vibration beam, cracks effect beam frequency.*

1. Introduction

The importance of the beam and its engineering applications is obvious, and it undergoes many different of loading. Many types of loading may cause cracks in the beam. These cracks and their locations effect on the shapes and values of the beam frequency. Recently these topics are so prevailing in the industry of spacecraft, airplanes, wind turbines, turbines, robot arm and many other applications.

Many studies were performed to examine the vibration and dynamic of cracked beams; one of them was Shen and Pierre[1],who present a finite element approach which make it possible to predict in the first few eigen frequencies due to cracks (pairs or single open cracks).The change in the first natural frequency with crack depth is matched closely by the present finite element approach and also with the experimental results.

Also, Shen and Pierre [2] derived the equation of motion with associated boundary conditions for uniform Bernoulli-Euler beam. The resulting equation is solved for simple supported and cantilevered beams with single edge cracks by Galerkin and Ritz procedure; they matched the theoretical results with the experimental and finite element results and they match closely.

Chati et . al. [3] modeled the non-linearity as a piecewise-linear system. In an attempt to define effective natural frequencies for this piecewise linear system, the idea of a 'bilinear frequency' is utilized. The finite element method is used to obtain the natural frequencies in each linear region.

Choondros et. al [4]studied the dynamics of a cracked fixed-free bar with a breathing crack in longitudinal vibration. They showed in results the eigenfrequency changes due to a single open-edge breathing crack in order to depend on the bilinear character of the system. They tested their method fordifferent bar configurations corresponding to crack location, crack depths, cross-section dimensions, andPoisson's ratio. The natural frequencies obtained from this model agree well with experimental results.

Choondros et. al [5]used a continuous cracked beam vibration theory for predication of changes in transverse vibration of simply supported beam with a breathing crack. They found that the changes in vibration frequencies for fatigue breathing crack are smaller than the ones caused by open cracks. Utilizing aluminum beams with fatigue cracks for experimental setup they compared the results with the analytical.

Chondros[6]used a continuous cracked beam vibration theory for predication of changes in dynamic characteristic due to loading conditions and vibration amplitude. He used the numerical results to correlate the analytical results for lumped crack beam vibration analysis for aluminum and steel beams with open cracks. He supported the theoretical result by experimental results for the same cases.

Cam and et. al. [7] studied ,experimentally and theoretically, the effect of the crack on vibration of cracked beam. They used echo method for predication the size and location of the crack in cracked beam. They found that the theoretical results (ANSYS) agreed with experimental results.

In this paper, three approaches are employed, an analytical approach is compared with experimental result and with that gained numerically by ANSYS program to verify the results.

The objective of this paper is to study the effect of crack depth and position on the natural frequency of the simple supported beam and to find the best method that gives good results to be compared with experimental results.

2. Theoretical Approach

2.1. Analytical Approach (Rayleigh method)

Rayleigh method is a good method and simpler than the other analytical methods for finding the natural frequencies. It includes calculating the kinetic energy and potential energy of the system. where the kinetic energy can be calculated by integration the mass through length of the beam and the potential energy by integration the stiffness through the length of the beam. So one can get from the above, S. S. Rao [12]:

$$\omega^2 = \frac{\int_0^l EI\left(\frac{d^2y(x)}{dx^2}\right)^2 dx}{\int_0^l \rho A(y(x))^2 dx} = \frac{g\sum_{i=1}^{n+1} m_i y_i}{\sum_{i=1}^{n+1} m_i y_i^2} \qquad \dots (1)$$

Where,

ω is the natural frequency of beam.

E is the modulus of elasticity of beam (N/m2).

I is the second moment area od cross section area of beam (m4).

ρ is the mass density of beam (kg/m3).

A is the cross section area of beam (m2).

g is the gravity acceleration (9.81 m/s2).

m_i is the mass in each Rayleigh divided point of beam.

y_i is the deflection in each Rayleigh divided point of beam.

By calculating the deflection of the beam(y(x)) using the following steps:

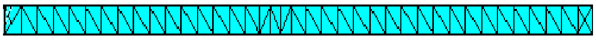
1. Dividing the beam into (n) parts (i.e. (n+1) nodes).
2. Calculating the delta matrix $[\delta]_{((n+1)*(n+1))}$.
3. Calculating the mass matrix $[m]_{((n+1)}$.
4. Calculating the deflection at each node by multiplying delta matrix and mass matrix ($[y]_{(n+1)}= [\delta]_{((n+1)*(n+1))} [m]_{((n+1)}$) after applying the boundary conditions.

The analytical results are solved using MATLAB. Where a MATLAB program that simulated the Rayleigh method is written in order to calculate the first natural frequency of any beam (Different materials, different dimensions and different shape).

2.2. Numerical Approach (Finite Elements Method)

In this method, the finite elements method was applied by using the ANSYS program(ver.13). The three dimensional model were built and the element (Solid Tet 10 node 187) were used.

Generally the number of nodes was approximately (1250-1300) and the number of elements was (550-600). A sample of meshed beam is shown in Fig. 1.



a) Meshed Beam without Crack.

b) Meshed Beam with Crack at 12 cm from Left End.

c) Meshed Beam with Crack at 22 cm from Left End.

d) Meshed Beam with Crack at 32 cm from Left End.

e) Meshed Beam with Crack at 32 cm from left End.

Fig. 1. A sample of a Meshed Beam.

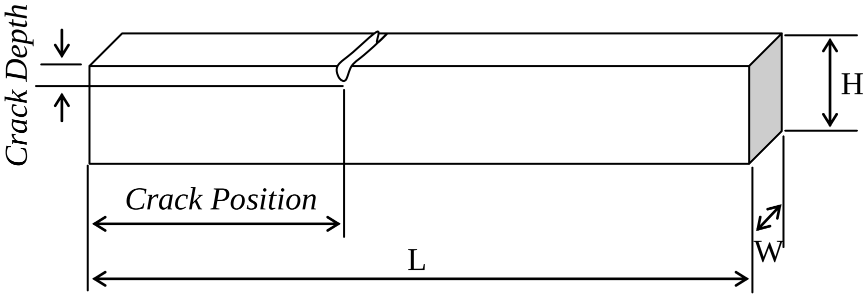
3. Experimental Approach

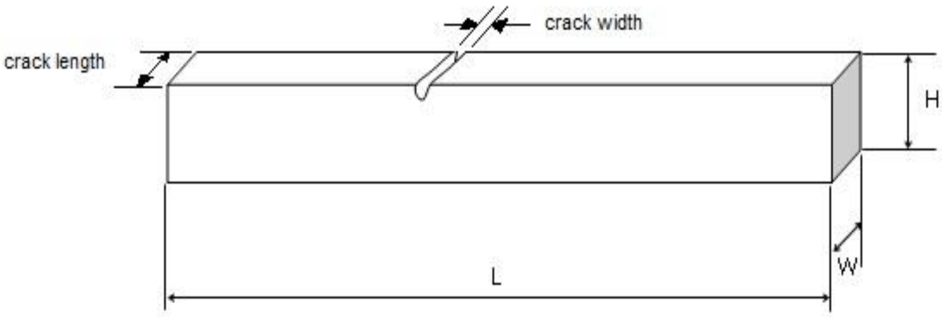
The (TM16 universal vibration apparatus) from TQ company is employed in this study and is shown in Fig. 3. The dimensions of the specimen used are (L*W*H=0.84*0.02 *0.02 m) as shown in Fig. 2. The material of the specimen was stainless steel (Code No.: 314, Robert L. Norton [16]) with density of (7680 kg/m^3), calculate by divided the weight of beam per volume of beam, Young modulus (200 GPa) and Poisson’s Ratio (0.3).

The crack was created in the specimens with certain dimensions of crack ;(see Table (1)).

Table 1,
Dimensions of the Cracks that Used Experimentally.

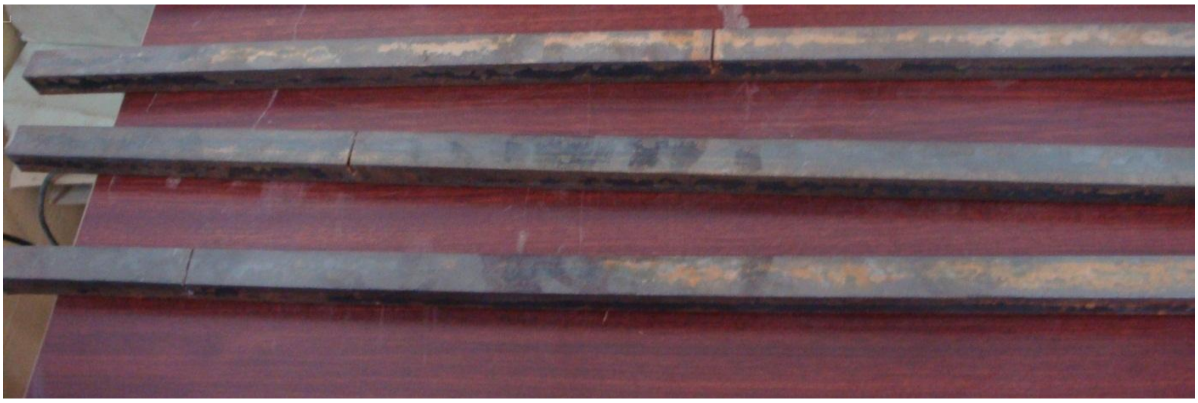
Specimen No.	Crack Location (m)	Crack Length (m)	Crack Width (m)	Crack Depth (mm)
1	0.12	0.2	0.0015	0 2 4 6 8 10
2	0.22	0.2	0.0015	0 2 4 6 8 10
3	0.32	0.2	0.0015	0 2 4 6 8 10
4	0.42	0.2	0.0015	0 2 4 6 8 10
5	0.54	0.2	0.0015	0 2 4 6 8 10
6	0.64	0.2	0.0015	0 2 4 6 8 10
7	0.74	0.2	0.0015	0 2 4 6 8 10





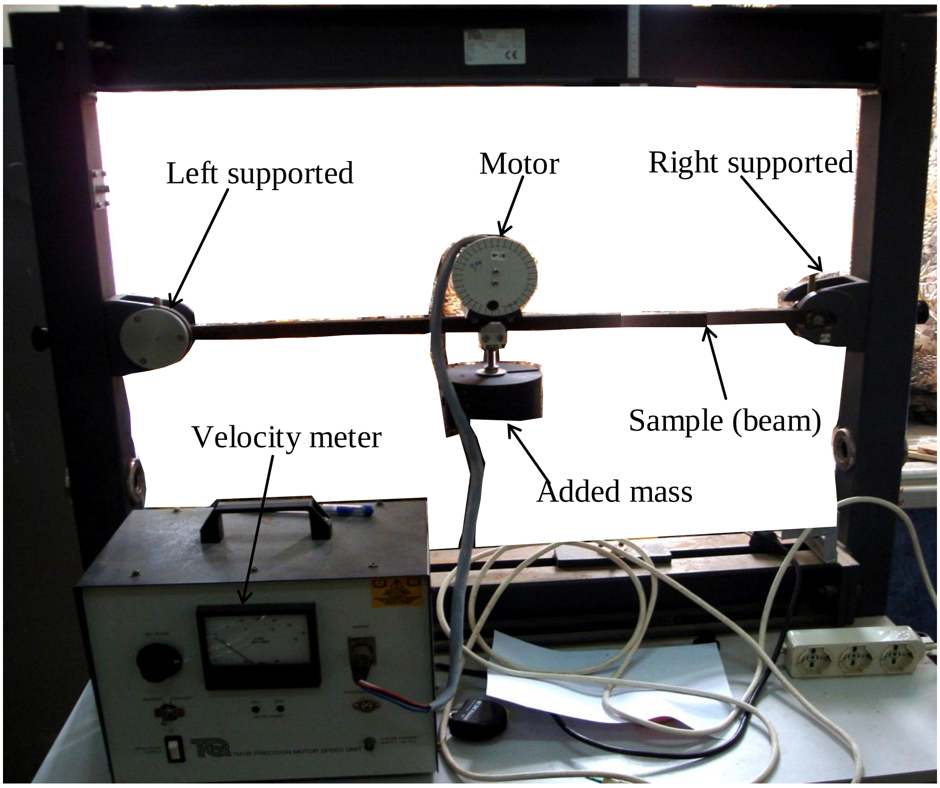
(a) Dimensions of sample

(b) Sample of beam



(c) Different Crack Position

Fig. 2. Dimension and Crack Depth and Crack Position of Samples Test.



(a) The Universal Vibration Apparatus.

(b) Motor. **(c) Added Mass.** **(d) Velocity Meter.**

(e) Left and Right Supported.

Fig. 3. The Universal Vibration Apparatus.

Table 2,
Natural Frequency with Different Crack Position and Crack Depth.

Crack Position (cm)	Crack Depth (mm)	Frequency (Hz)			Error Exp. and ANSYS (%)	Error Exp. and Rayleigh (%)
		Experimental	ANSYS	Rayleigh Method		
12	0	143.74	148.81	145.2758759	3.5	1.06
	2	141.42	148.7	145.2784708	5.15	2.73
	4	146.805	148.46	145.2810655	1.13	1.04
	6	141.99	148.01	145.2836602	4.2	2.3
	8	142.5665	147.39	145.2862548	3.4	1.9
	10	139.75	146.5	145.2888493	4.8	3.96
22	0	143.74	148.81	145.2758759	3.5	1.07
	2	153.57	148.8	145.2784708	3.1	5.4
	4	149.4	148.77	145.2873931	0.42	2.75
	6	152.85	148.69	145.2931524	2.7	4.9
	8	149.4	148.61	145.2989122	0.53	2.7
	10	141.42	148.43	145.3046725	4.95	2.7
32	0	143.74	148.81	145.2758759	3.5	1.07
	2	150	148.65	145.2784708	0.9	3.15
	4	139.2	148.17	145.2873931	6.4	4.4
	6	139.75	147.35	145.2931524	5.4	3.96
	8	142.56	146.03	145.2989122	2.4	1.9
	10	138.144	144.03	145.3046725	4.26	5.18
42	0	143.74	148.81	145.2758759	3.5	1.06
	2	140.3	148.54	145.2886394	5.87	3.6
	4	141.3	147.77	145.3014064	4.6	2.8
	6	136.59	146.44	145.3141768	7.2	6.4
	8	140.3	144.43	145.3269508	2.9	3.6
	10	141.99	141.44	145.3397282	0.39	2.4

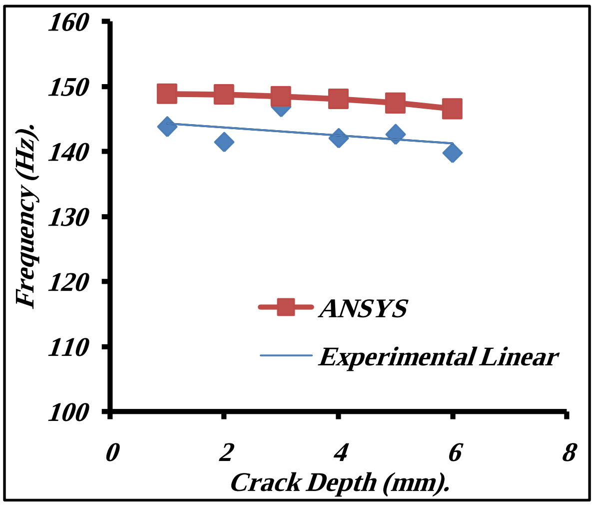


Fig. 4. The Comparison between the Experimental and ANSYS Results at Different Crack Depths When the Crack lies at (12 cm) .

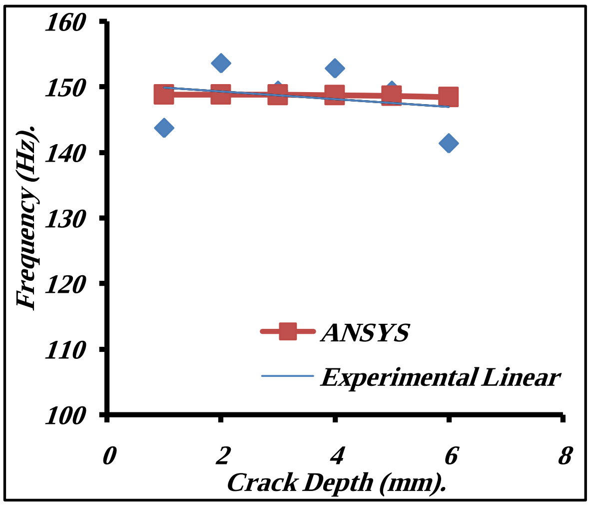


Fig. 5. The Comparison between the Experimental and ANSYS Results at Different Crack Depths When the Crack lies at (22 cm) .

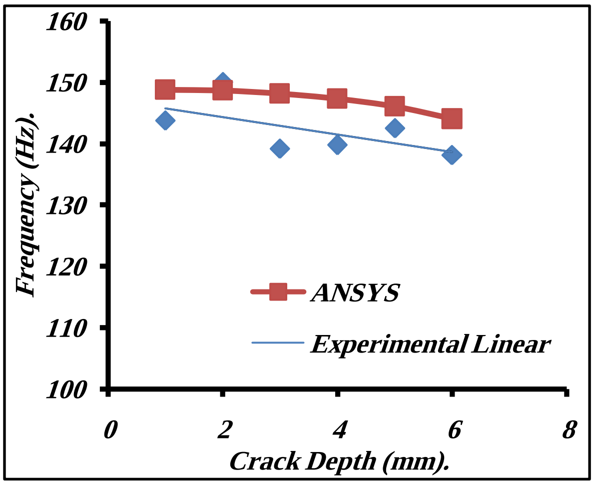


Fig. 6. The Comparison between the Experimental and ANSYS Results at Different Crack Depths When the Crack lies at (32 cm) .

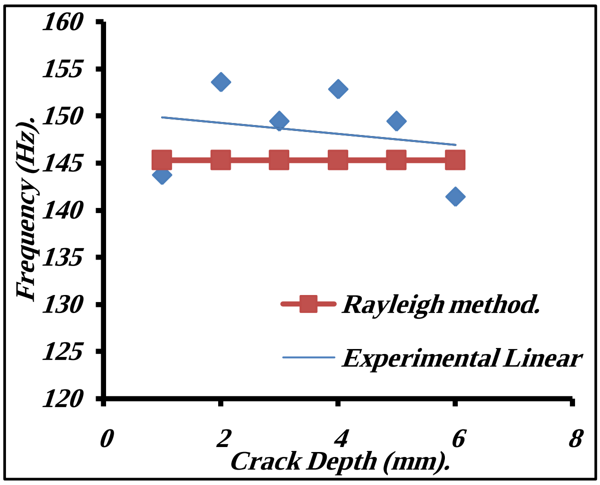


Fig. 9. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Depths When the Crack lies at (22 cm) .

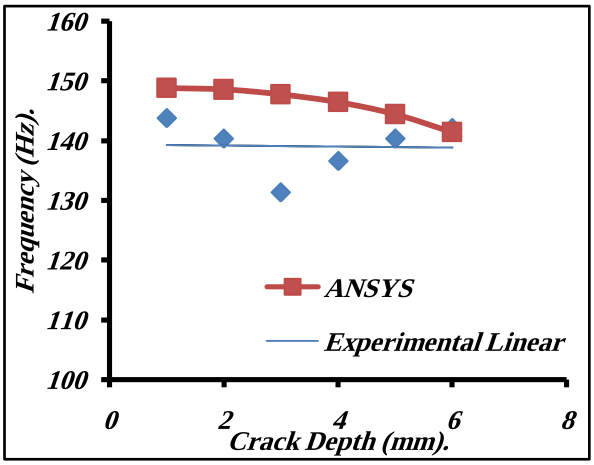


Fig. 7. The Comparison between the Experimental and ANSYS Results at Different Crack Depths When the Crack lies at (42 cm) .

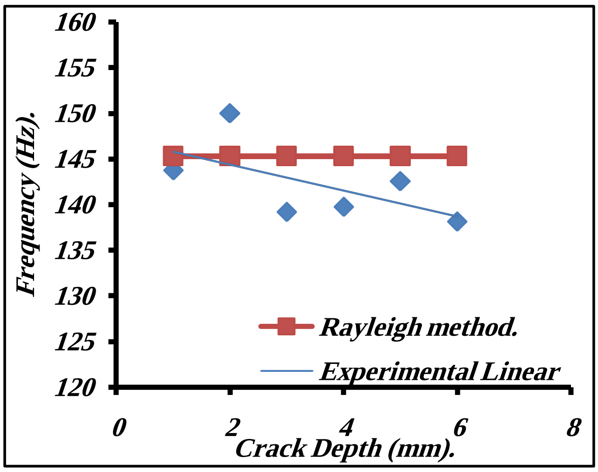


Fig. 10. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Depths When the Crack lies at (32 cm) .

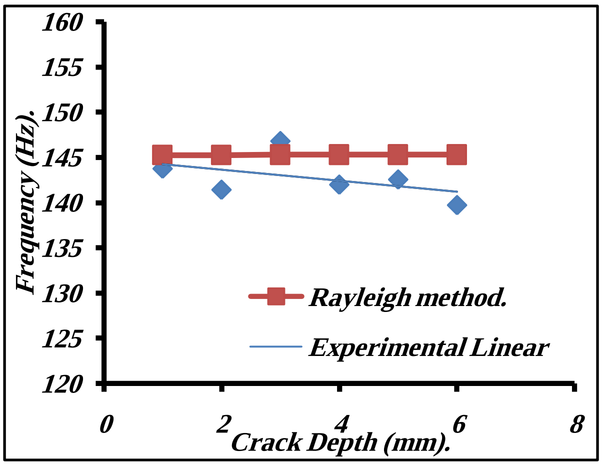


Fig. 8. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Depths When the Crack lies at (12 cm) .

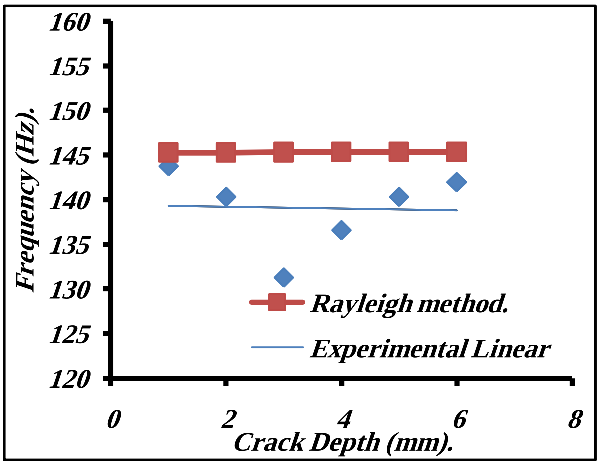


Fig. 11. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Depths When the Crack lies at (42 cm) .

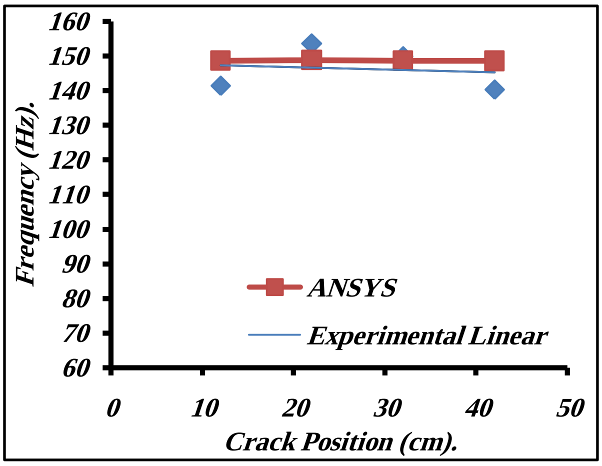


Fig. 12. The Comparison between the Experimental and ANSYS Results at Different Crack Position When the Crack Depth is (2mm).

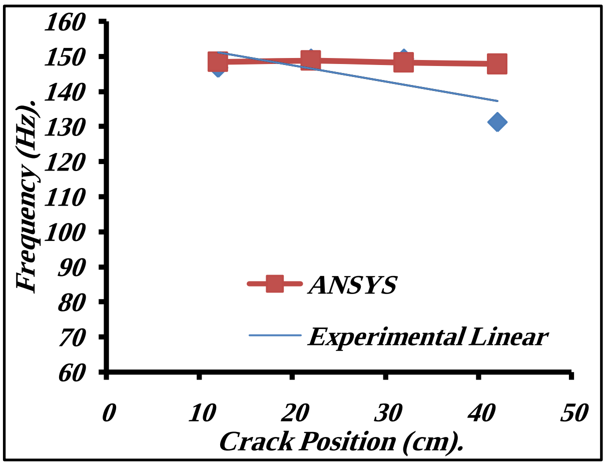


Fig. 13. The Comparison between the Experimental and ANSYS Results at Different Crack Position When the Crack Depth is (4mm) .

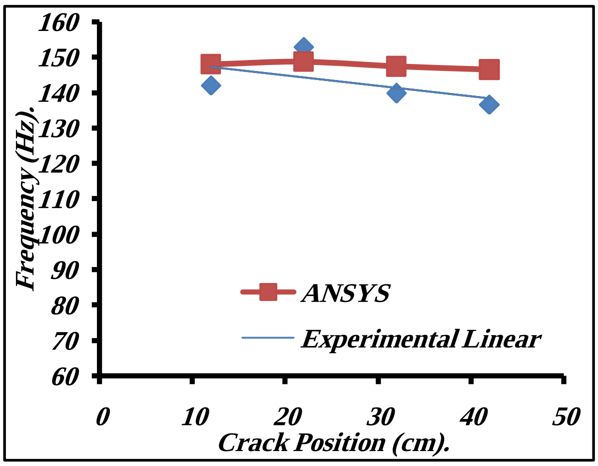


Fig. 14. The Comparison between the Experimental and ANSYS Results Different at Crack Position When the Crack Depth is (6mm) .

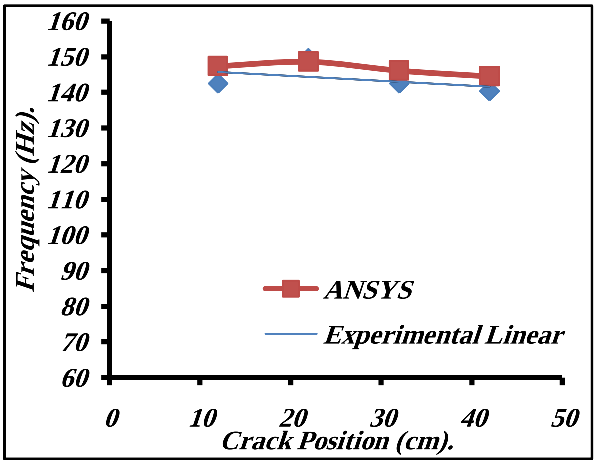


Fig. 15. The Comparison between the Experimental and ANSYS Results at Different Crack Position When the Crack Depth is (8mm).

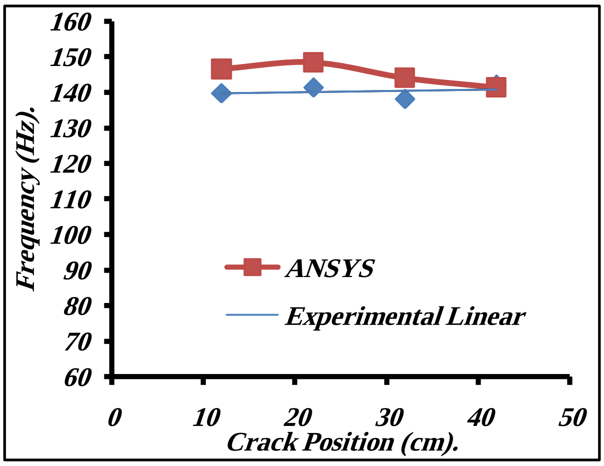


Fig. 16. The Comparison between the Experimental and ANSYS Results Different at Crack Position When the Crack Depth is (10mm) .

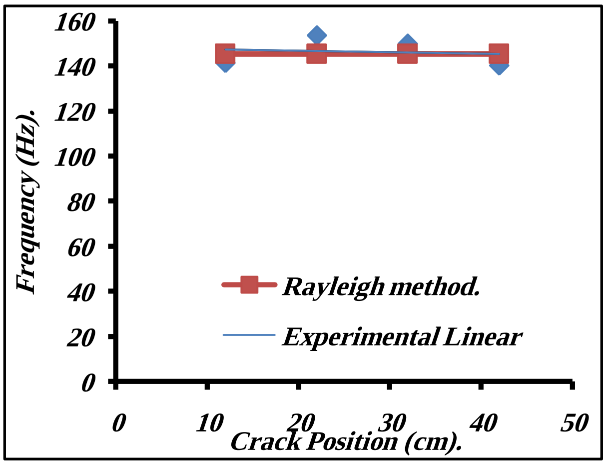


Fig. 17. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Position When the Crack Depth is (2mm) .

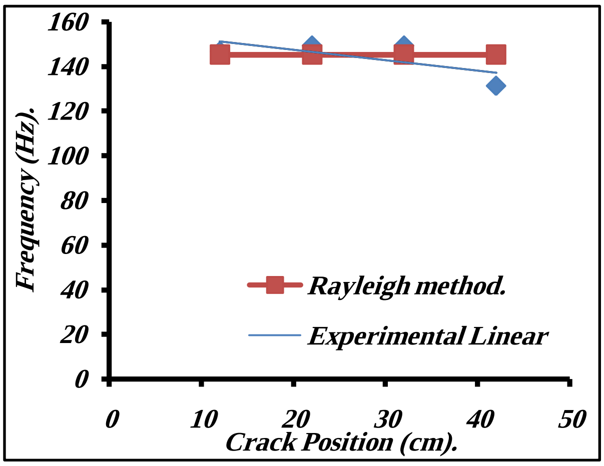


Fig. 18. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Position When the Crack Depth is (4mm).

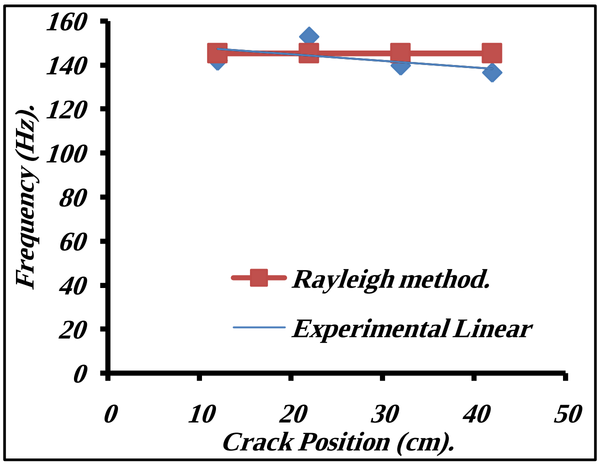


Fig. 19. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Position When the Crack Depth is (6mm).

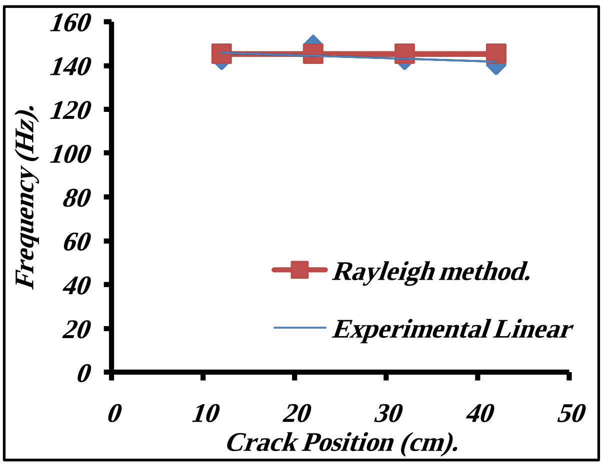


Fig. 20. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Position When the Crack Depth is (8mm).

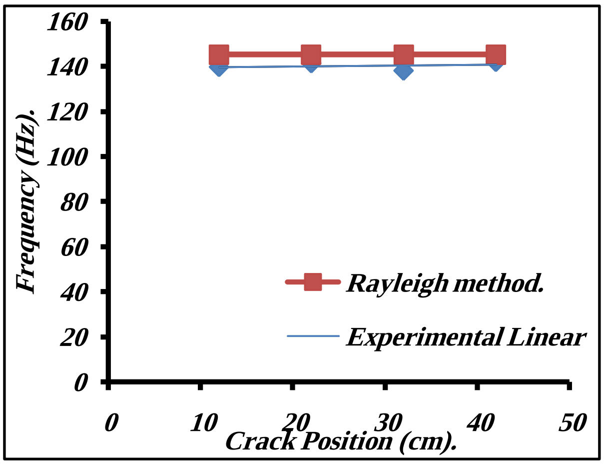


Fig. 21. The Comparison between the Experimental and Rayleigh Method Results at Different Crack Position When the Crack Depth is (10mm).

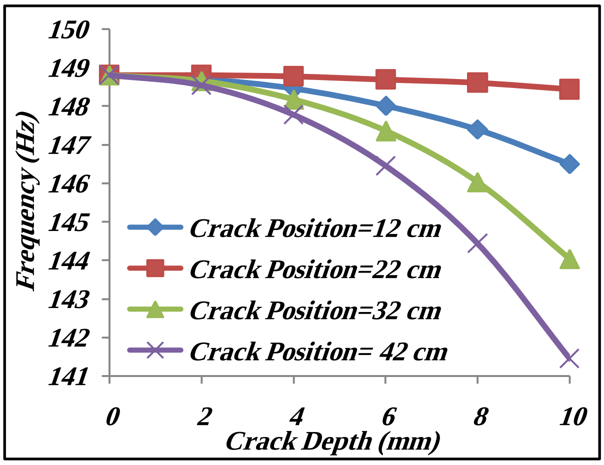


Fig. 22. Relationship between Natural Frequencies and Crack Depths for Different Crack position (12, 22, 32, 42 cm). ANSYS Results.

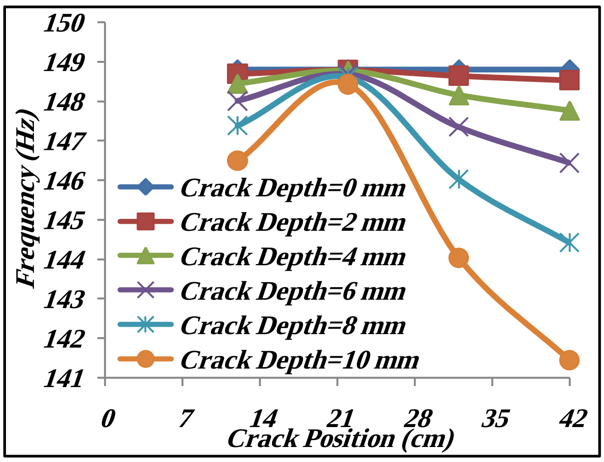


Fig. 23. Relationship between Natural Frequencies and Crack position for Different Crack Depths (0, 2, 4, 6, 8, 10 mm). ANSYS Results.

4. Results and Discussion

A comparison made between analytical results from ANSYS with experimental results shows a good approximation where the biggest error percentage is about (7.2 %) in crack position (42 cm) and (6 mm) depth, as illustrated in Fig. 7. and the comparison between Rayleigh method with experimental results shows a good approximation where the biggest error percentage is about (6.4 %) in crack position (42 cm) and (6 mm) depth, as illustrated in Fig. 11.

Figs. 4, 5, 6, and 7. explain the comparison between theoretical results (ANSYS) and experimental results for natural frequency and how it changes with crack depth for different crack position (12, 22, 32, and 42 cm) where the natural frequency decreased with increasing crack depth for the same crack position because of the decreasing of stiffness of the beam.

Figs. 8, 9, 10, and 11. show the comparison between Rayleigh method and experimental results for natural frequency and how it changes with crack depth for different crack position (12, 22, 32, and 42 cm) where the natural frequency decreased with increasing crack depth for the same crack position because of decreasing of stiffness of the beam.

Figs. 12, 13, 14, 15, and 16. The explain the comparison between theoretical results (ANSYS) and experimental results for natural frequency and how it changes with crack position for different crack depth (2, 4, 6, 8, and 10 mm) where the natural frequency decreased with increasing crack position for the same crack depth because of the decreasing of stiffness of the beam.

Figs. 17, 18, 19, 20, and 21. explain the comparison between Rayleigh method and experimental results for natural frequency and how it changes with crack position for different crack depth (2, 4, 6, 8, and 10 mm) where the natural frequency decreased with increasing crack position for the same crack depth because of the decreasing of stiffness of the beam.

Generally from all the figures one can see that the natural frequency decreases with increasing crack depth for different crack positions because of the changing in stiffness of the beam. And the rate of decreasing for experimental is close for different crack positions. For ANSYS the rate of decreasing of natural frequency is the biggest in crack position (42 cm). And, for Rayleigh method the rate of decreasing is close for different crack positions, as shown in Figs. 22. and 23. and Table (2).

5. Conclusion

From the results shown in the Figs. 4 to 23, the following concluding marks are observed:

1- A comparison made between analytical results from ANSYS with experimental results shows a good approximation where the biggest error percentage is about (7.2 %) in crack position (42 cm) and (6 mm) depth, as shown in Fig. 7.

2- The comparison between Rayleigh method with experimental results shows a good approximation where the biggest error percentage is about (6.4 %) in crack position (42 cm) and (6 mm) depth, as shown in Fig. 11.

3- From the error percentages in Table 2, the Rayleigh method gives close results to experimental than ANSYS.

4- The crack in the beam has an effect on the stiffness of the beam; this will affect the frequency of the beam. So, with the increasing of the crack depth, the stiffness of beam will decrease and this will cause a decreasing in the natural frequency of the beam.

5- The position of crack in the beam near the middle of the beam has more effect on the stiffness and natural frequency of beam from the other positions (near to the ends of the beam), i.e. frequency of the beam when the crack is in the middle position has a lower frequency of the beam with respect the cracks near to the end position.

6. References

[1] Shen, M. -H. H and Pierre C. . "Modes of free vibrations of cracked beams", paper (of 46 pages) presented to UM-MEAM, (1986).

[2] Shen, M. -H. H and Pierre C. . "Free Vibrations of Beams With a Single Edge Crack", Journal of sound and vibration, 170(2), pp. 237-259. (1994).

[3] Chati, M., Rand, R. and Mukherjee, S. "Modal analysis of a cracked beam", Journal of sound and vibration, 207(2), pp. 249-270. (1997).

[4] Chondros T. G., Dimarogonas A. D. and Yao J " Longitudinal vibration of a bar with a breathing crack", Engineering fracture mechanics, pp.503-518, 1998.

[5] Chondros T. G., Dimarogonas A. D. and Yao J. "Vibration of a beam with a breathing crack" journal of sound and vibration, 239(1), pp. 57-67. (2001).

[6] Chondros T. G. "The continuous crack flexibility model for crack identification" Blackwell Science Ltd., fatigue fact matter 24, pp. 643-650, 2001.
[7] Cam Ertugrul, SadettinOrhan and Murat Luy "An analysis of cracked structure using impact echo method", Elsevier journal, NDT&E International 38 (2005) 368–373.
[8] Alexander Tessler, Marco Di Sciuva and Marco Gherlone, "Refinement of Timoshenko Beam Theory for Composite and Sandwich Beams Using Zigzag Kinematics" paper of NASA, NASA/TP-2007-215086.
[9] Lin-Tai Tang, "Cracked Beam and Related Singularity problems", master thesis, National Sun Yat-sen University, Taiwan, July 1, 2001.
[10] H. R. Oz and M. T. Das, "In-Plane Vibrationsof Circular Curved Beamswitha Transverse Open Crack" Department of Mechanical Engineering, University of Gaziantep, 27310, Gaziantep,Turkey, Mathematical and Computational Applications, Vol. 11, No. 1, pp. 1-10, 2006.
[11] D.S rinivasarao, K.MallikarjunaRao and G.V.Raju, "Crack identification on a beam by vibration measurements and wavelet analysis ", International Journal of Engineering Science and Technology, Vol. 2(5), 2010, 907-912.
[12] S. S. Rao 'Mechanical Vibrations' Addison-Wesley Publishing Company, (1995).
[13] Krawczuk, M., Zak, A., and Ostachowicz, W. “Dynamics of Cracked Composite Material Structures,” Comput. Mech., 20, pp. 79–83, 1997.
[14] Birman, V., and Simitses, G. J. “Vibration of Sandwich Panels and Beams With Matrix Cracks in the Facings,” Compos. Sci. Technol., 61, pp. 1605–1613, 2001.
[15] Kisa, M. “Free Vibration Analysis of a Cantilever Composite Beam with Multiple Cracks”, Composites Science and Technology, Vol. 64, 1391–1402, 2004.
[16] Robert L. Norton “Machine Design” Pearson Education, Inc., 2006.

دراسة تحليلية وعددية لتأثير الشق على التردد الطبيعي للعتبات ذات التثبيط البسيط

لؤي صادق محمد مهند لفتة الوائلي علي محمد حسين
قسم الهندسة الميكانيكية/ كلية الهندسة/ جامعة الكوفة

الخلاصة

تم في هذا البحث دراسة تأثير الشقوق على التردد الطبيعي للعتبات الجاسئة بحرية حيث تم دراسة تأثير موقع وعمق الشق عل التردد الطبيعي للعتبة وذلك بطريقة عملية ،وتحليلية ،وعددية باستخدام برنامج (ANSYS). وتم مقارنة النتائج مع بعضها البعض لعتبة مصنوعة من الفولاذ بإبعاد(طول*عرض *سمك =0.84*0.02*0.02m) وخواص (E=200Gpa) وكثافة ($\rho=7680kg/m^3$).تم مقارنة النتائج المستحصلة من برنامجANSYS حيث وجد اكبر نسبة خطا هي 7.2 % لموقع شق (42سم) وعمق (6مم)، وتمت كذلك مقارنة النتائج المستحصلة من طريقة رايلي والعملي. و وجد اكبر نسبة خطا هي6.2 % لموقع شق (42سم) وعمق (6مم). من قيم نسب الخطأ يمكن الاستنتاج ان طريقة رايلي تعطي نتائج اقرب للعملي من طريقة ANSYS. وكذلك وجد أن تردد العتبة عندما يكون الشق في منتصفها اقل من تردد العتبة عندما يكون الشق بالقرب من أطرافها.