Al-Khwarizmi Engineering Journal

Al-Khwarizmi Engineering Journal, Vol. 9, No. 1, P.P. 29-38 (2013)

Aircraft Lateral-Directional Stability in Critical Cases via Lyapunov Exponent Criterion

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(Received 13 December 2011; accepted 20 December 2012)

Abstract

Based on Lyapunov exponent criterion, the aircraft lateral-directional stability during critical flight cases is presented. A periodic motion or limit cycle oscillation isdisplayed. A candidate mechanism for the wing rock limit cycle is the inertia coupling between an unstable lateral-directional (Dutch roll) mode with stable longitudinal (short period) mode. The coupling mechanism is provided by the nonlinear interaction of motion related terms in the complete set equations of motion. To analyze the state variables of the system, the complete set of nonlinear equations of motion at different high angles of attack are solved. A novel analysis including the variation of roll angle as a function of angle of attack is proposed. Furthermore the variation of Lyapunov exponent parameter as function of time is introduced. The numerical result indicated that the system became lightly damped at high angle of attack with increasing the amplitude of aircraft state variables limit cycle. A good agreement between the numerical result and published work is obtained for the onset of limit cycle oscillation, almost at($\alpha = 20^{\circ} - 23^{\circ}$).

Keywords: *wing rock; nonlinear dynamic system; limit cycle oscillation.*

1. Introduction

The wing rock phenomenon is a self-sustained oscillatory mode, typical of several modern high performance aircraft which exhibit constant amplitude rolling oscillations at moderate and high angle of attack. Such dynamic systems typically possess a limit cycle which becomes stable after a buildup phase. These oscillations are sustained around a state at which the energy generation at lower amplitudes and the dissipation at larger amplitudes are balanced [1]. While maneuvering at angle of attack, many high performance aircraft can experience a self-induced limit cycle roll oscillation called a wing rock. This unwanted motion may result in limitations being placed on the aircraft's flight envelop [2]. The wing rock may diminish the flight effectiveness or even present a serious danger due to potential instability of aircraft [3]. The wing rock is undesirable because the oscillatory motion has an adverse effect on maneuverability and reduces the tracking accuracy [4]. From dynamics view, aerodynamic stability derivatives show that the wing rock is related to instability in Dutch roll mode of motion of aircraft [4]. Studies of the candidate wing rock mechanism showed that the unstable lateral-directional (Dutch roll) mode, when inertial coupled with a stable longitudinal (short period) mode, can establish a stable response (limit cycle) in sense of Lyapunov[4].Brad S.Liebst studied the trigger parameter and a simple procedure developed to predict the onset of a wing rock [5].Chung-Hao Hsu studied the aerodynamic mathematical model to calculate the wing rock characteristics. The resulting nonlinear flight dynamics equations of both one and three degree of freedom are solved with Beecham-Titchener asymptotic method for limit cycle amplitude and wing rock frequency[6]. Emad N. studied the behavior of a limit cycle wing rock motion resulting from new couple effect including dihedral effect derivative and directional stability derivative[7].

The present work focused on the variation of the aircraft state variables limit cycle amplitude at different values of the angles of attack. The numerical results indicated that the system became more lightly damped at an angle of attack $(\alpha = 20^{\circ} - 23^{\circ})$. The variation of Lyapunov exponent parameter as a function of time to predict the stability of aircraft is presented.

2. Aerodynamic Model

The high performance baseline aircraft, F-15 with Mach number=0.6 and height=10.6 Km, was taken into consideration in the present work. The aircraft fixed-coordinate systems, geometrical configuration, inertia properties and the aerodynamic model are a hybrid from references [8], [9] and [10].These values are listed in table[A].

Summaries of estimated lift coefficient C_L , drag coefficient C_D are shown in Figs. 1 to 2, respectively.

Table A,
Geometrical Configuration, Inertia Properties and the Aerodynamic Model for F-15 Aircraft:

Wing area (m^2)	56.485
Wing span (m)	13.045
Mean chord (m)	4.861
Vehicle weight (kg)	18522.44
I_{xx} (kg.m^2)	38968.9
I_{yy} (kg.m^2)	224172.8
I_{zz}(kg.m^2)	255130.7
Aspect ratio	3.0
$C_{L\alpha}$	3.4385
C_D	0.0315

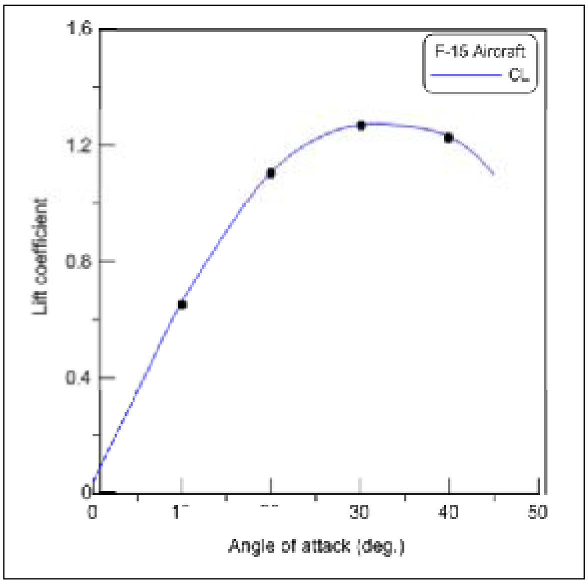


Fig. 1. Lift Coefficient

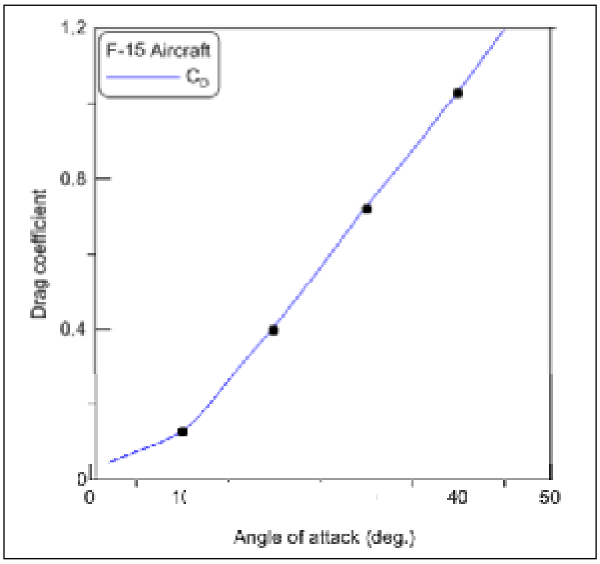


Fig. 2. Drag Coefficient

The aerodynamic model is based on the representative experimental and flight test data [8].The stability derivatives are estimated according to the mathematical relations:

Stability derivatives of equations:

$$M_\alpha = \frac{Q\,S\,C}{I_Y} C_{m\alpha} \quad \ldots(1)$$

$$M_q = \frac{Q\,S\,C}{I_Y} C_{mq} \quad \ldots(2)$$

$$Z_\alpha = -\frac{Q\,S}{m}(C_{L\alpha} + C_D) \quad \ldots(3)$$

$$L_r = \frac{Q\,S\,b}{I_X}\left(\frac{b}{2U}\right) C_{Lr} \quad \ldots(4)$$

$$N_r = \frac{Q\,S\,b}{I_Z}\left(\frac{b}{2U}\right) C_{nr} \quad \ldots(5)$$

$$L_p = \frac{Q\,S\,b}{I_X}\left(\frac{b}{2U}\right) C_{Lp} \quad \ldots(6)$$

$$N_p = \frac{Q\,S\,b}{I_Z}\left(\frac{b}{2U}\right) C_{nr} \quad \ldots(7)$$

$$Y_\beta = \frac{Q\,S}{m} C_{Y\beta} \quad \ldots(8)$$

$$L_\beta = \frac{Q\,S\,b}{I_X} C_{L\beta} \quad \ldots(9)$$

$$N_\beta = \frac{Q\,S\,b}{I_Z} C_{n\beta} \quad \ldots(10)$$

3. Nonlinear Equations of Motion Model

The aircraft simulation model represents F-15 fighter aircraft which is considereda nonlinear modal in flight dynamics system. Therefore, the behavior of the flight regime model aircraft may consider the limit cycle or another periodic motion. The wing rock is one of the limit cycle motions.

The aircraft model (F-15 fighter aircraft) is a nonlinear state space system, considering small perturbations around the trim conditions. The lateral-directional system in general form can be defined as [11]:

$$\{\dot{X}_{Lat-Dir}\} = [A_{Lat-Dir}]\{X_{Lat-dir}\} + [B_{Lat-Dir}]\{U_{Lat-Dir}\} \quad \ldots(11)$$

Where $[A_{Lat-dir}]$ = lateral-directional plant matrix,
$[B_{Lat-Dir}]$ = Control effectiveness matrix,
$\{X_{Lat-Dir}\}$ =lateral-directional
State vector,
$[\beta p\ \emptyset\ r]^T$, $\{U_{Lat-Dir}\}$ = control input, $[\delta_R \delta_A]^T$;
$\beta$is the sideslip angle, p is the roll rate, ϕ is the roll angle,
ris the yaw rate and δ_R,δ_A are the rudders, aileron control angles,
respectively.
If only rudder or aileron control were under consideration, then the control input would simplify to a single (scalar) value. The uncoupled set of nonlinear equations of motion was [12]:

$$\{\dot{X}_{Lat-Dir}\} = [A_{Lat-Dir}]\{X_{Lat-Dir}\} \quad \ldots(12)$$

$$\{\dot{X}_{Long}\} = [A_{Long}][X_{Long}] \quad \ldots(13)$$

And $\{X_{Long}\}$ =longitudinal state vector,$[\alpha\ q]^T$.
The lateral-directional plant matrix was defined by:

$$[A_{Lat-Dir}] = \begin{bmatrix} Y_\beta/U & 0.0 & (g\cos\theta_0)/U & -1 \\ L_\beta & L_P & 0.0 & L_r \\ 0.0 & 1 & 0.0 & 0.0 \\ N_\beta & N_P & 0.0 & N_r \end{bmatrix}$$

And

$$[A_{Long}] = \begin{bmatrix} Z_\alpha/U & 1.0 & g/U \\ M_\alpha & M_q & M_{\dot{\alpha}} \end{bmatrix}$$

Where Y_β is the change in side force caused by a variation in thesideslip angle, $L_\beta$is the dihedral effect, $L_p$is the roll damping, $L_r$is the change in rolling moment caused by yawing, N_β is the directional stability, N_p is the change in yawing moment caused by rolling,$N_r$is the yaw damping, U is the free stream velocity, Z_α is the normal force due to angle of attack, $q$is the pitch rate,α is angle of attack, M_α is the longitudinal stability derivative and $M_q$is the pitch damping. In full set of equations, the small angle assumption for motion about an initial level of flight condition was removed and anEuler angle relation was introduced. Roll angle,ϕbecame the Euler angle Θwhile the pitch angle, θ, became the Euler angle Θ [4]. The nonlinear relations listed below were added to corresponding terms in Eqs. (2), (3) to obtain a full set of motion expressions. The nonlinear additions are denoted by *(NL)* with the subscript denoting the nonlinear expression to which it is applied. Thus,

$$(NL)_\beta = p\,\alpha + \frac{g(C_\Theta S_\Phi - C_{\Theta 0}\Phi)}{U} \quad \dots(14)$$

$$(NL)_P = q\,r\,(I_Y - I_Z)/I_X \quad \dots(15)$$

$$(NL)_\emptyset = (q\,S_\Phi + r\,C_\Phi)T_\Theta \quad \dots(16)$$

$$(NL)_r = q\,\frac{p(I_X - I_Y)}{I_Z} \quad \dots(17)$$

$$(NL)_\alpha = -p\,\beta + \frac{g(C_\Theta C_\Phi - C_{\Theta 0})}{U} \quad \dots(18)$$

$$(NL)_q = p\,r + (I_Z - I_X)/I_Y + M_\alpha (NL)_\alpha \quad \dots(19)$$

In Eqs. (4)– (9), trigonometric terms are shown as $C_\Phi = \cos\Phi$,$S_\Phi = \sin\Phi$,$C_\Theta = \cos\Theta$and $T_\Theta = \tan\Theta$

An Euler angle relationship for $\dot{\Theta}$was introduced by:

$$\dot{\Theta} = q\,C_\Phi - r\,S_\Phi \quad \dots(20)$$

The nonlinear set of motion relations were obtained by including Eqs. (4)–(9) as an additional set of terms added to Eqs. (2)– (3). In addition, a seventh relation was added, Eq. (10), to account for the dynamics of the Euler angle *Θ*. As a result, the state vector for the nonlinear system became:

$$\{X_{NL}\} = [\beta\,p\,\Phi\,r\,\alpha\,q\,\Theta]^T \quad \dots(21)$$

The time history of the complete set of nonlinear motion equations was solved.

4. Lyapunov Exponent Criterion

In order for a system to exhibit a chaotic behavior, it must be non-linear, sensitive to varying parameters and it initial conditions. Predictive criteriasuch as periodic doubling criteria, Chirikov's overlap criteria [12], or Lyapunov exponent criteria [13] were used to describe the chaotic behavior of the non-linear dynamic

System In the present paper, the Lyapunov exponent criteria will be considered to describe the chaos behavior of non-linear state equations. Consider an attractor point $x_0$and a neighboring attractor point $x_0 + \varepsilon$. Then, the iterated map function is applied n times to each value and the absolute value of the difference between those results is considered:

$$d_n = \left|f^{(n)}(x_0 + \varepsilon) - f^{(n)}(x_0)\right| \quad \dots(22)$$

If the behavior is chaotic, this distance is expected to grow exponentially with *n*, so

$$\frac{d_n}{\varepsilon} = \frac{\left|f^{(n)}(x_0+\varepsilon) + f^{(n)}(x_0)\right|}{\varepsilon} \equiv e^{\lambda n} \quad \dots(23)$$

or

$$\lambda = \frac{1}{n}\ln\left[\frac{\left|f^{n}(x_0+\varepsilon) + f^{n}(x_0)\right|}{\varepsilon}\right] \quad \dots(24)$$

Whereλ,is Lyapunov exponent for the trajectory. Applying the chain rule for differentiation, the derivative of *f (n)* can be written as a product of nderivatives of*f (x)* evaluated at successive trajectory points x_0,x_1,x_2, and so on. Thus, the definition of Lyapunov exponent in a more intuitive form is

$$\lambda = \frac{1}{n}\ln\left(\left|f'(x_0)\right|\left|f'(x_1)\right| \dots \left|f'(x_{n-1})\right|\right) \quad \dots(25)$$

Where $f'(x) = df/dx$ …(16)

Also, one can rewrite equation (16) as

$$\lambda = \frac{1}{n}\left(\ln\left|f'(x_0)\right| + \ln\left|f'(x_1) + \dots + \left|f'(x_{n-1})\right|\right|\right) \quad \dots(27)$$

When

$\lambda < 0$: the orbit attracts to a stable periodic orbit.

$\lambda = 0$: the orbit is a neutral fixed point (a bifurcation occurs).

$\lambda > 0$: the orbit is unstable and chaotic.

5. Result and Discussion

A baseline aircraft F-15 is taken into consideration in this analysis with Mach no. =0.6 at H=10.6 Km.To demonstrate the state variables in a wing rock phenomenon, the nonlinear equations of motions at high angles of attack were solved.In this study, the wing rock limit cycle F-15 is characterized by modifying the directional stability derivatives in a plant matrix. A buildup of roll angle to limit cycle oscillation can be found in Fig. 3. with $\zeta_{D-R} = -0.4$ and angle of attack =20 deg.

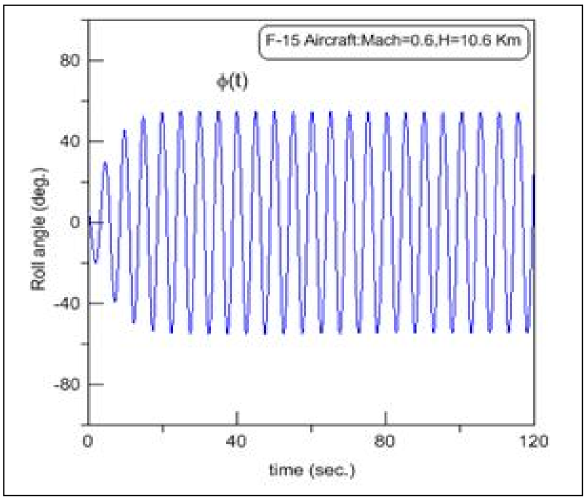


Fig. 3. Roll Limit Cycle Buildup, ζ=-0.4at α=20 deg.

A representative buildup roll and sideslip angles are indicated in Fig.5. It can be noted that the amplitude ratio of $|\Phi|:|\beta| = 7.45$ with a phase lag of sideslip to roll angle Fig.4.

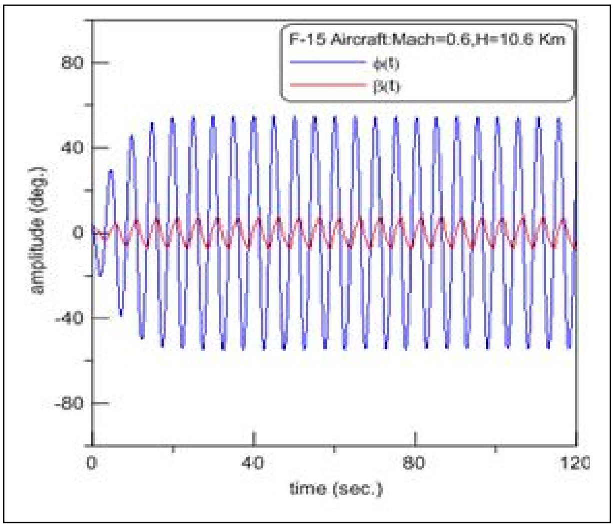


Fig. 4. Limit Cycle Histories of Φ and β, ζ=-0.4 and α=20 deg.

A comparative time history of roll angle and angle of attack perturbations during the limit cycle is shown in Fig.5. It should be noted that the angle of attack perturbations show a doubling relative the roll limit cycle frequency. This feature, described as kinematic coupling, can be attributed to the terms of Eqs. (8) and (9) respectively. An alternate perspective of wing rock limit cycle may be obtained from a cross plot of sideslip angle with roll angle.

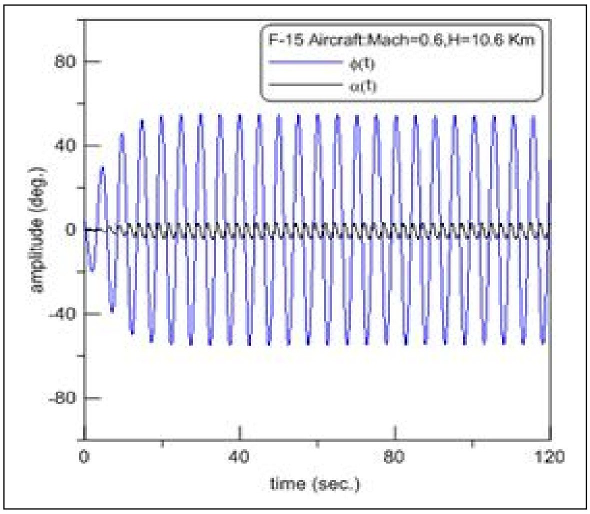


Fig. 5. Limit Cycle Hostories Φ and α, ζ=-0.4 and α=20 deg.

The trajectory corresponds to amplitude ratio of $|\Phi|$ to $|\beta|$ that equals to 7.45 and 54.8 phase lag of sideslip relative to the roll angle ,as illustrated in Fig.6.It explains that the oscillatory behavior of the map is at a constant amplitude.

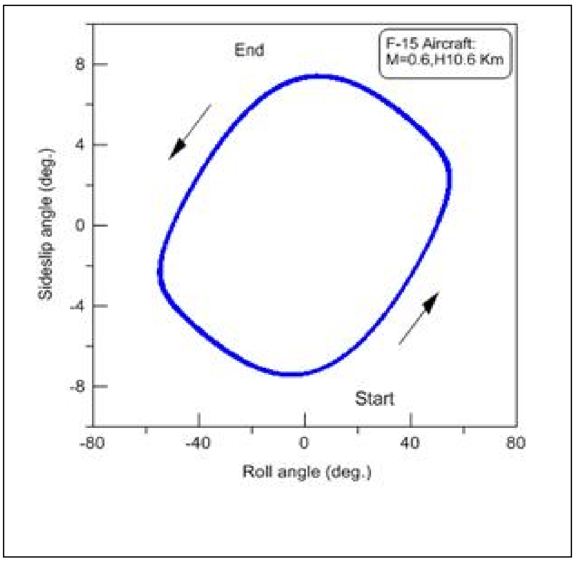


Fig. 6. Limit Cycle Phase Trajectory, ζ=-0.4 and α=20deg.

Fig.7. shows the variation of roll angle amplitude with damping ratio. It is apparent form this figure that the limit cycles, when Dutch roll damping less than zero, correspond to a pitchfork type of hopf bifurcation, because the static equilibrium does not exist.

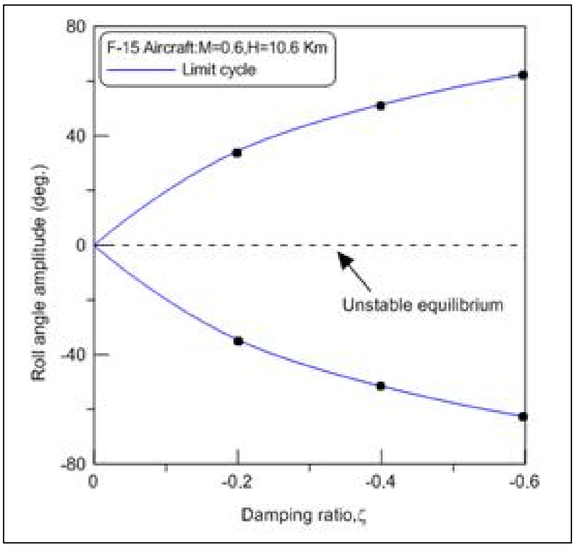
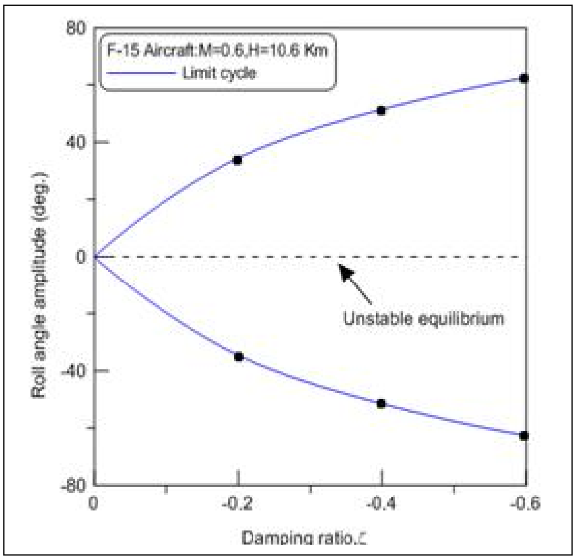


Fig.7. Stability Traits of Nonlinear Oscillator.

The effect of Dutch roll damping ratio ζ_{D-R} on the roll Φ,sideslip β and angle of attack α limit cycle amplitudes is shown in Fig.8 subject to constraint that the frequency ratio $(\omega_{sp}/\omega_{DR})$ was 2 .



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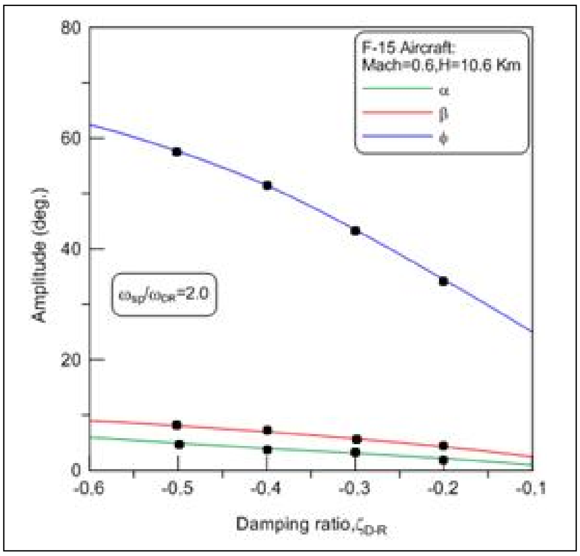

Fig. 8. Effect of Damping Ratio on Limit Cycle Amplitude, ζ=-0.4 and α=20 deg.

The trend is one of increasing amplitude with increasing levels Dutch-roll model damping instability. The effectof frequency ratio $(\omega_{sp}/\omega_{DR})$ on the amplitude of roll, sideslip and angle of attack is displayed in Fig.9.

The minimum value of the amplitude is obtained at an optimal value of $(\omega_{sp}/\omega_{DR})$ = 2.0 and that explains that ability of the longitudinal motion to couple and transfer energy for creation of a stable limit cycle.

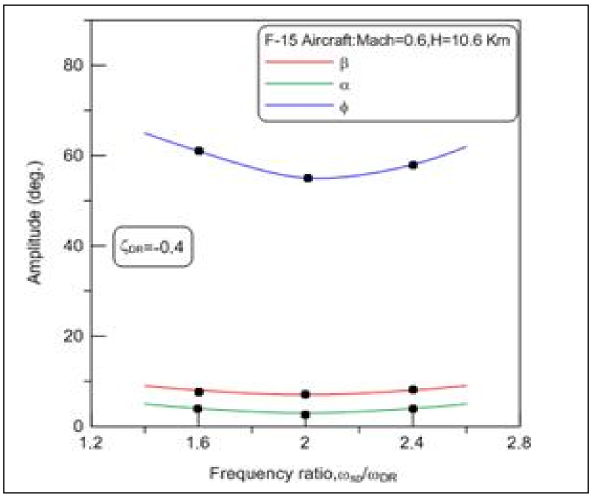

Fig. 9. Effect of Frequency Ratio on Limit Cycle Amplitude, ζ=-0.4 and α=20 deg.

The variation of the roll limit cycle amplitude at different angles of attack is presented in Fig.10. This figure shows the upper and lower values of the roll limit cycle as function of angle of attack.

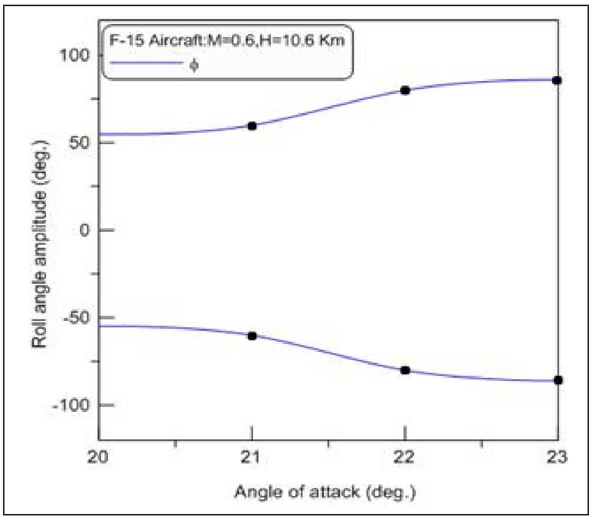

Fig. 10. Variation Limits Cycle Amplitude (Φ) at Different Angle of Attack.

The verification of the limit cycle existence $(\lambda = 0)$ at different values of angles of attack can be indicated in Fig. (11-14).

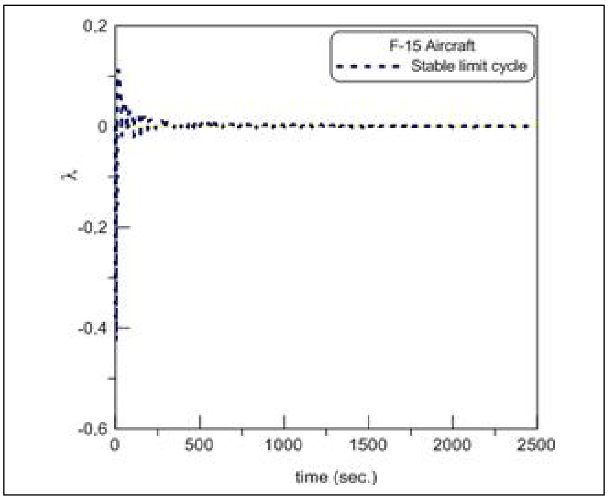

Fig. 11. Lyapunov Exponent Values (Verification L.C.O) at α=20 deg.

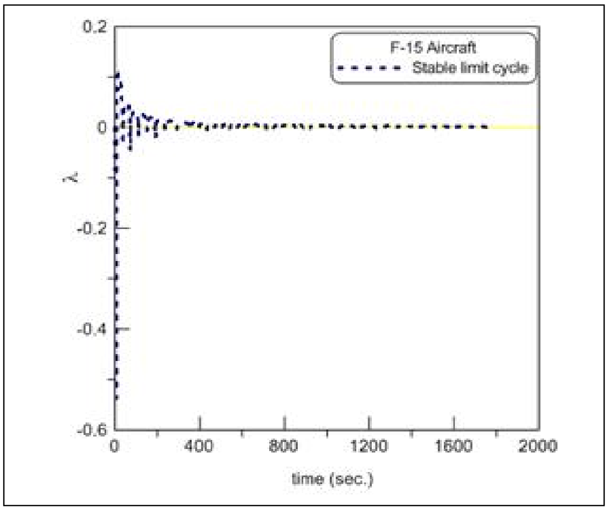

Fig. 12. Lyapunov Exponent Values (Verification L.C.O) at α=21 deg.

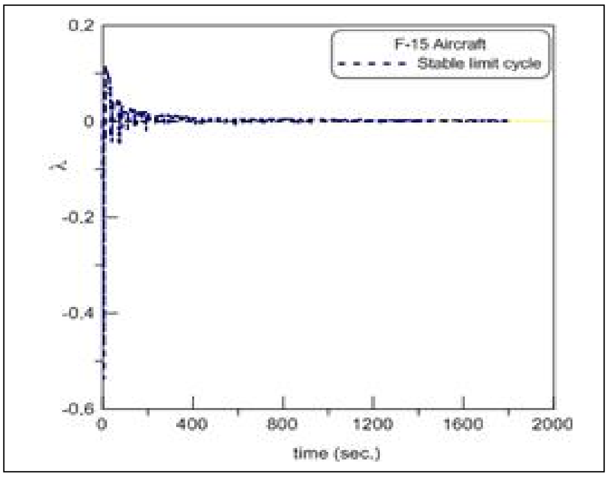

Fig. 13. Lyapunov Exponent Values (Verification L.C.O) at α=22 deg.

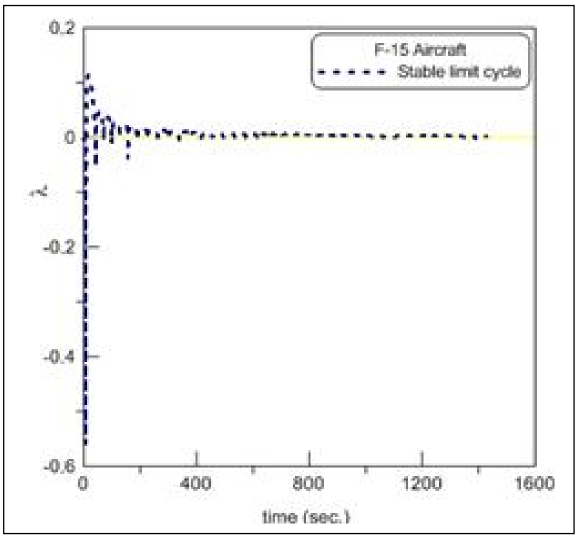

Fig. 14. Lyapunov Exponent Values (Verification L.C.O) at α=23 deg.

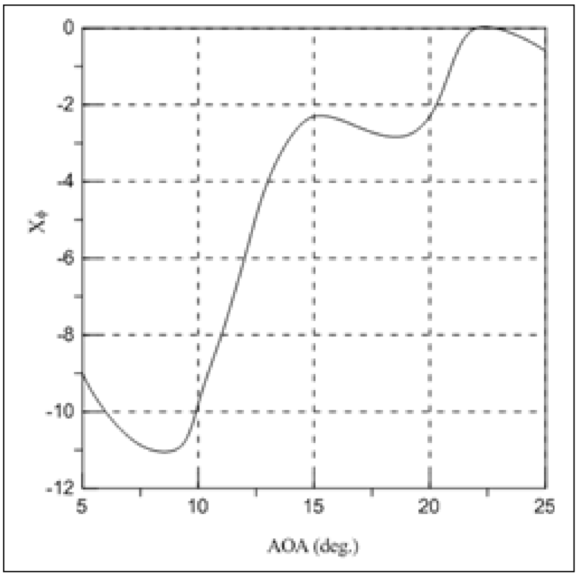

Fig. 15. Trigger Parameter VS AOA for F-15.

6. Conclusions

The characterization of the limit cycle oscillation in aircraft lateral dynamics is presented in this paper. Based on Lyapunov exponent criterion, the stability of aircraft in critical cases is displayed. The following conclusions wear obtained in the current work:

1. The system became more lightly damped at a high angle of attack and the influence of the aircraft roll damping on the behavior of the limit cycle was found to be weak.
2. The variation of Lyapunov exponent parameter as function of time is obtained. This will enable the tracking of aircraft stability.Once an indication of stability limit, the controller can intervene to bring it back to a steady state without having to consider the final value of Lyapunov parameter which is implemented in other methods and theories.
3. A good agreement between the numerical result and published work is obtained for the onset of limit cycle oscillation [14], as represented in Fig.16.

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الاستقرارية الجانبية العرضية للطائرة في الحالات الحرجة حسب نظرية ليبنوف الأسية

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الخلاصة

يتناول البحث الحالي تحليل الأستقرارية الجانبية- الأتجاهية أثناء حالات الطيران الحرجة بأعتماد نظرية ليبنوف الأسية. تم وصف الحركة الدورية أو التذبذب ذو الدورة المحددة. حيث كانت الألية المتبعة للدورة المحددة لتأرجح الجناح هي ربط القصور الذاتي بين طور الحركة الجانبية الأتجاهية غير المستقرة (الألتفاف الهولندي) وطور الحركة الطولية المستقرة (لفترة قصيرة). تم تحقيق ألية الربط هذه بواسطة التداخل اللاخطي بين الحدود ذات العلاقة في معادلات الحركة الكاملة. تم تحليل متغيرات الحالة للنظام عن طريق حل معادلات الحركة اللاخطية عند زوايا هجوم عالية وتم أقتراح أسلوب جديد للتحليل يضمن تغير زاوية الألتفاف كدالة لزاوية الهجوم أضافة لتغير معامل ليبنوف كدالة للزمن. أظهرت النتائج العددية بأن النظام (الطائرة) يصبح ذات تخميد ضعيف عند زوايا الهجوم العالية مع زيادة في سعة متغيرات الحالة ذات العلاقة. تم الحصول على توافق جيد بين نتائج البحث الحالي و نتائج لأبحاث لمنشورة عند نشوء التذبذب ذو الدورة المحددة.