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Intelligent H_2/H_∞ Robust Control of an Active Magnetic Bearings System

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Abstract

Robust controller design requires a proper definition of uncertainty bounds. These uncertainty bounds are commonly selected randomly and conservatively for certain stability, without regard for controller performance. This issue becomes critically important for multivariable systems with high nonlinearities, as in Active Magnetic Bearings (AMB) System. Flexibility and advanced learning abilities of intelligent techniques make them appealing for uncertainty estimation. The aim of this paper is to describe the development of robust H_2/H_∞ controller for AMB based on intelligent estimation of uncertainty bounds using Adaptive Neuro Fuzzy Inference System (ANFIS). Simulation results reveal that the robust controller design objectives of wide bandwidth and improved performance are satisfied for a wide range of frequency variations. It can be concluded that the intelligent uncertainty weighting functions can precisely compensate for the effects of modelling errors and nonlinearities in the system.

Keywords: *Active Magnetic Bearings (AMB) , Adaptive Neuro Fuzzy Inference System (ANFIS), H_2/H_∞ robust controller, modelling errors, uncertainty bounds.*

1. Introduction

Recently research involving the combination of hard and soft computing has developed with the aim of complementing these sets [1]. Real-world problems can be solved in more innovative ways by combining the attractive features of both computing methods. The goal of this paper is to show such combination applied to an industrial process. This research will show that the soft computing aspects of adaptive neuro fuzzy inference system can be used together with the hard computing features of robust H_∞ control to accomplish valuable results in an easier, more efficient fashion.

Robust control is a hard computing design methodology dedicated to provide assured stability and performance for uncertain dynamic systems. Robust control synthesis needs a precise mathematical model of the plant dynamics and bounds on the uncertainty related to that model. Such uncertainties may result from parameter variations, under-modeled dynamics, or process disturbances [2]. By specifying a nominal model and a “hard bound” on the uncertainty related to that model, robust control aims to guarantee robust stability for the actual system, which must lie within the set defined by the model plus the uncertainty bound.

For robust control synthesis it is accustomed to choose uncertainty bounds (uncertainty weighting functions) that are somewhat random and exceedingly conservative to guarantee stability, usually at the expense of performance. In order to overcome this drawback, intelligent methods are developed to design the unstructured uncertainty (or the modelling error) weighting function for H_∞ robust control synthesize. Reliable and efficient tool are obtained as in [3],[4].

As a combination of neural networks (NNs) and fuzzy logic, neural-fuzzy systems can make good use of both sensory numerical data and expert linguistic information; system performance tuning is flexible as the number of membership functions and training epoch numbers can be altered easily [5]. Therefore, the ANFIS technique is used in [6] to estimate the uncertainty bounds for robust motion controlled system. Then the applied ANFIS estimation is further improved in [7] to estimate uncertainties in more difficult situations. The frequency ranges of the uncertainties in the model are precisely located, and the synthesized controlled system becomes insensitive to them while guaranteeing a specified performance and larger stability margin of robust controllers, as measured by the *v*-gap metric. The design application to an uncertain MIMO system model of an Active Magnetic Bearing (AMB) system [8] has been extended in this paper to include the theoretical formulation of the control problem and the estimation of uncertainties using different ranges of frequencies. Precise quantification of uncertainty due to 'mainly' the modelling errors has been achieved. Robustness against changes in operating frequencies has been observed.

This paper is organized as follows: Section 2 describes the dynamics of the AMB system, Section 3 introduces a brief description of robust identification, Section 4 presents Adaptive Neuro Fuzzy Inference System (ANFIS) for Uncertainty Estimation, Section 5 defines the applied H_2/H_∞ robust control, Section 6 presents some simulation results and discussion and finally, in Section 7 conclusions will be drawn.

2. Dynamic AMB System Model

AMB is a collection of electromagnets producing a magnetic field to support a rotating iron shaft without any physical contact. AMBs are open loop unstable and the stabilization of the system can only be done by feedback control. In addition, their complex structure requires powerful control system design approaches for robust stability and robust performance. The rigid body diagram for an AMB is shown in Figure. 1. The steel rotor has a mass of 1.549 Kg and a length of 0.457 *m*, the two steel disks are positioned to modify the modal characteristics at high speeds. Two radial AMBs are located at the ends of the rotor, orthogonally aligned in the *x* and *y* directions, together with two orthogonal pairs of sensors to measure rotor displacements from the bearing line of centre. These radial AMBs comprise a four input (bearing currents) and four output (displacements) dynamic system. A linearized system dynamics obtained from a Lagrangian analysis of an AMB can be expressed using a state vector composed of the rotor displacement and their time derivatives [4]:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad \text{...(1)}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad \text{...(2)}$$

where, $\mathbf{x} = \begin{bmatrix} \mathbf{z}_M \\ \dot{\mathbf{z}}_M \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & I \\ \mathbf{M}_B^{-1}\mathbf{K}_s & -\mathbf{M}_B^{-1}\mathbf{G}_B \end{bmatrix}$,

$$\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}_B^{-1}\mathbf{K}_i \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}^T,$$

$$\mathbf{M}_B = \mathbf{T}_F^{-1}\mathbf{M}\mathbf{T}_z, \mathbf{G}_B = \mathbf{T}_F^{-1}\mathbf{G}\mathbf{T}_z, \mathbf{z}_M = \begin{bmatrix} x_a \\ x_b \\ y_a \\ y_b \end{bmatrix},$$

$$\mathbf{K}_s = \begin{bmatrix} k_s & 0 & 0 & 0 \\ 0 & k_s - k_c & 0 & 0 \\ 0 & 0 & k_s & 0 \\ 0 & 0 & 0 & k_s - k_c \end{bmatrix},$$

$$\mathbf{K}_i = \begin{bmatrix} k_i & 0 & 0 & 0 \\ 0 & k_i & 0 & 0 \\ 0 & 0 & k_i & 0 \\ 0 & 0 & 0 & k_i \end{bmatrix},$$

$$\mathbf{T}_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_a & -l_b & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -l_a & l_b \end{bmatrix},$$

$$\mathbf{T}_z = \frac{1}{l_a + l_b}\begin{bmatrix} l_b & l_a & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & l_b & l_a \\ 0 & 0 & -l_a & l_b \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_y & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & I_y \end{bmatrix},$$

$$\mathbf{G} = \Omega\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_z \\ 0 & 0 & 0 & 0 \\ 0 & I_z & 0 & 0 \end{bmatrix},$$

with system parameters m=1.549kg, $I_x = I_y = 2.39\times10^{-2}\,Kg.m^2$, $I_z = 10^{-4}\,Kg.m^2$, $l_a = 0.153m$, $l_b = 0.170m$, Ω = 627.0*rad/sec.*, $k_s = -96.5\times10^3\,N/m$, $k_i = 29.9N/A$, and $k_c = 2.6\times10^3\,N/m$. The resulting continuous-time model is unstable, with eigenvalues [13]

$$\lambda = -471 \pm 3.6i, 471 \pm 3.6i, \\ -351 \pm 0.006i, 351 \pm 0.006i. \quad \text{...(3)}$$

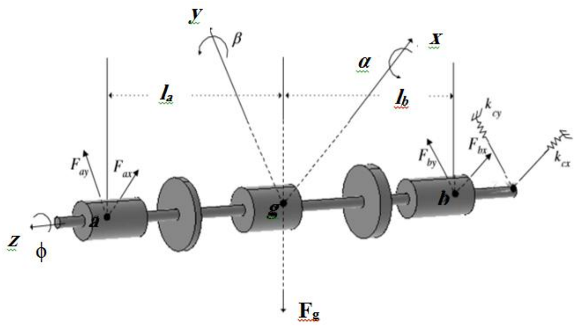


Fig. 1. Generalized rigid rotor supported by two radial bearings [4].

3. Robust Identification

In standard identification problems, the perturbations potentially arise from two different sources as illustrated in Table 1. In most practical situations, the model error is considered, especially when the order of the nominal model must be small, as usually required in robust design techniques. Moreover, while prior information on measurement noise can be obtained, similar hypothesis on unmodeled dynamics is less realistic.

Three main approaches for robust identification have been addressed in literature:

1. Stochastic Embedding (SE), which can be described as: "A frequency domain method which assumes that unmodeled dynamics can be represented adequately by a non- stationary stochastic process whose variance increases with frequency" [9].
2. Set Membership Identification (SMI), provides efficient algorithms for estimating the set of feasible models, compatible with the available data and the UBB error assumption [10].

Table 1,
Sources of perturbations in a standard identification problem.

	Sources	Characteristics
Variance error	Measurement noises	Generally uncorrelated with the input signal (when the data is collected in open loop).
Bias	The effect of un-modeled dynamics	Strongly depends on the estimated nominal model structure and on the input signal used in the identification experiment.

3. Model Error Modelling (MEM), is an effective robust parametric identification approach to estimate the model uncertainty bound. MEM technique employs standard prediction error methods to identify an error model from input–output time domain data. The un-modelled dynamics can be estimated by looking at that part of identification of residuals that originates from the input, [11], and [12]. In addition, the uncertainty can be estimated regardless of the order of the nominal model. For these appealing features, this method is implemented in this work.

MEM can be briefly described as follows: If assuming that (u, y_o) is the measurement data set, and G_N is the system nominal model estimated with (u, y_o), then the model error modelling method can be summarized as follows:

i. First, compute the residual:

$$\varepsilon = y_o - G_N u \qquad \dots(4)$$

ii. Consider the "error" system with input u and output ε, and identify the model-error model Ge for this system. This model provides the estimation of the under- modelling error.
iii. From the nominal model and the model error, the uncertainty region can be constructed by adding the model error to the nominal model in the frequency domain. This gives a region where true system is supposed to be found.
iv. Model validation: The nominal model is not falsified if and only if it lies inside its own uncertainty region (as delivered in step iii).

Nevertheless, the drawback of this technique is that it leads to conservative uncertainty sets because it is based on the worst case assumptions [2], [13], and [14]. MEM has been modified in [4] to a non-parametric regression problem in the frequency domain using feed forward neural networks. The identification reduces the conservativeness of the estimated uncertainty bounds with a suitable confidence region. In this work, it will be further modified using ANFIS to simplify the computational efforts and reduce the calculating time, making real time implementation more suitable.

4. Adaptive Neuro Fuzzy Inference System (ANFIS) for Uncertainty Estimation

ANFIS as created by [15] may be attributed to have generated a new paradigm in fuzzy-neural computation which strongly supports Zadeh's soft-computing ideas [16]. The use of the ANFIS is characterized by [14]:

- The fuzzy logic component gives the algorithm a degree of robustness and combines it with the learning abilities of the neural network.
- The neural network aspects of the ANFIS allows for multiple attempts of generating the rule base and membership functions. This provides a detection scheme that has a low error rate in predicting the output, with the fewest number of rules and membership functions to keep computation times to a minimum, the consequent parameters thus identified are optimal under the condition that the premise parameters are fixed.
- The hybrid approach that combines neural network and fuzzy logic is much faster than the strict gradient descent.

One of the most important and effective areas of applications of ANFIS is modelling, estimation and prediction of systems with uncertainties. Some of these applications can be found in [17]-[24]. The common advantages between these works can be summarized as follows:

- There is the ability to converge much faster that the back-propagation ANN; the number of epochs is several orders of magnitude less than the one needed for training of the corresponding back-propagation ANN.
- The ability to adapt to environmental changes.
- The ANFIS is also repeatable over time, with minor recalibration.

4.1. ANFIS Structure

Figure 2 shows the equivalent type-3 Takagi and Sugeno (T-S) fuzzy if-then rules used for ANFIS architecture [15], where the system has two inputs *x* and *y*. The output of each rule is a linear combination of input variables plus a constant term, and the final output is the weighted average of each rule's output.

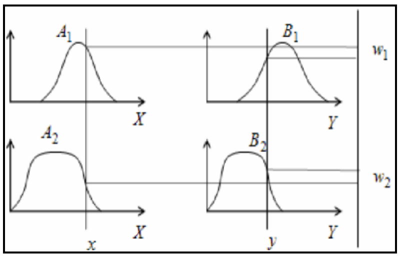
The fuzzy IF-THEN rule set, in which the outputs are linear combinations of their inputs, is:

- **Rule 1**: If x is A_1 and y is B_1 Then $f_1=p_1x+q_1y+r_1$
- **Rule 2:** If x is A2 and y is B2 Then f2=p2x+q2y+r2

where, *A*'s and *B*'s are particular fuzzy subsets defined by nonlinear coefficient, namely premise parameters, while *p*'s, *q*'s and *r*'s are linear coefficients determining the output of each applied Fuzzy rule, usually known as consequent parameters.

Figure 3 shows the basic architecture of Adaptive Neuro-fuzzy Inference System (ANFIS). In general, ANFIS has input and output layers, and three hidden layers that represent the membership functions and the fuzzy rules [15]. Layer 1 is the input layer. It consists of adaptive nodes, which generate membership grades of linguistic labels based upon premise signal that use the generalized bell membership function. Layer 2 is the input membership or fuzzification layer with fixed nodes designated Π that represents the firing strength of each rule. The output of each node is the fuzzy AND of all the input signals. Layer 3 is the fuzzy rule layer. The outputs of Layer 3 are the normalized firing strengths. Each node is a fixed rule labelled *N*. The adaptive nodes in Layer 4 calculate the rule outputs based upon consequent parameters. The single node in Layer 5, labelled *Σ*, calculates the overall ANFIS output from the sum of the node inputs.

Starting from initial model and following modelling optimization procedure, as described in [24], the optimal number of fuzzy rules is determined. Accordingly, ANFIS of four rules is found to be most suitable for this application.



$f_1=p_1x+q_1y+r_1$

$f_2=p_2x+q_2y+r_2$

Fig. 2. T-S fuzzy reasoning [15].

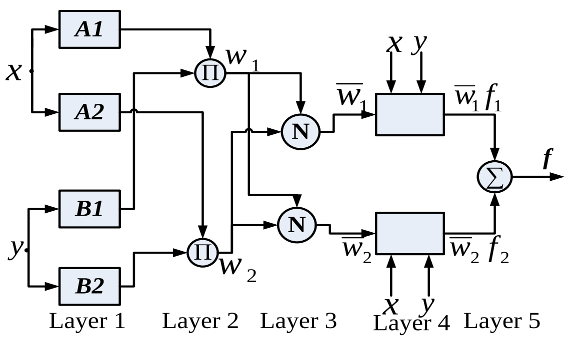


Fig. 3. Architecture of an ANFIS equivalent to a first-order sugeno fuzzy model with two inputs and two rules [15].

4.2. Hybrid Learning Algorithm

From the ANFIS system shown in Figure 3 with fixed premise parameters values, the overall output can be expressed as a linear consequent of the consequent parameters. The output *f* in Figure 3 can be rewritten as:

$$f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} = \frac{w_1}{w_1 + w_2} f_1 + \frac{w_2}{w_1 + w_2} f_2$$
$$f = \overline{w}_1(p_1 x + q_1 y + r_1) + \overline{w}_2(p_2 x + q_2 y + r_2) \quad \ldots(5)$$

which is linear in the consequent parameters p_1, q_1, r_1, p_2, q_2, r_2.

The training algorithm of ANFIS takes the initial fuzzy model and tunes it by means of a hybrid technique combining gradient descent back-propagation in the backward pass and mean least-squares optimization algorithms in the forward pass, as shown in Figure 4. At each epoch, an error measure defined as the sum of the squared difference between actual and desired output is reduced. Training stops when either the predefined epoch number or error rate is obtained. The gradient descent algorithm is implemented to tune the nonlinear premise parameters, while the basic function of the mean least-squares is to optimize or adjust the linear consequent parameters.

4.3. Implementation of ANFIS for Identification of Uncertainty Bounds

The main purpose of the intelligent uncertainty identification is to estimate the upper magnitude bound of the model error frequency response function $G_e(j\omega)$[4],[6]:

$$|G_e(j\omega)| = |G_r(j\omega) - G_N(j\omega)| = \frac{|E(j\omega)|}{|U(j\omega)|} \quad \ldots(6)$$

where $G_r(j\omega)$ is the measured frequency response function of the actual system, $G_N(j\omega)$ is the frequency response function of the nominal linear model of the system, $E(j\omega)$ is the Fast Fourier Transform (FFT) of prediction error $e(t)$ and $U(j\omega)$ is the FFT of $u(t)$. Note that plant uncertainties and non-deterministic effects give rise to frequency dependent intervals associated with $|G_e(j\omega)|$.

Figure 5 shows non parametric estimation of the model error frequency response function $|G_e(j\omega)|$ using sampled input-output data and a simple ANFIS structure as described in Section 4.1 enhanced by feedback signal. The ANFIS provides an estimate of the model error magnitude $G_u(j\omega, f)$ that is conditioned on the input frequency ω and the ANFIS output function f. The feedback signal helps to further eliminate the error between the actual model error frequency response $|G_e(j\omega)|$ and the intelligently estimated uncertainty bound $|G_u(j\omega, f)|$. Using this approach, the corresponding intelligent estimation of uncertainty bound can be formulated as:

$$G_u(j\omega_k, f) = \overline{w}_1 f_1(G_e(j\omega_k), e_{ud}(j\omega_{k-1}), \omega_k) + \overline{w}_2 f_2(G_e(j\omega_k), e_{ud}(j\omega_{k-1}), \omega_k) \quad \ldots(7)$$

$$\omega_k(k = 1,...,nn)$$

where f_1 and f_2 are selected as second order nonlinear function models, nn is the number of data samples and e_{ud} is a prediction error:

$$e_{ud}(k) = [G_e(j\omega_k) - G_u(j\omega_k, f)] \quad \ldots(8)$$

$G_u(j\omega, f)$ (12) is trained to minimize a cost function of prediction errors by automatically adjusting the ANFIS function f. The goal is to enhance the search for less conservative uncertainty bound through iterative minimization procedure until the stopping criteria is met

$$J(e_{ud}) = |e_{ud}| < \delta \quad \ldots(9)$$

where $|e_{ud}|$ is the absolute value of the prediction error and δ is a pre-specified very small numerical value, e.g. less than 10^{-3}.

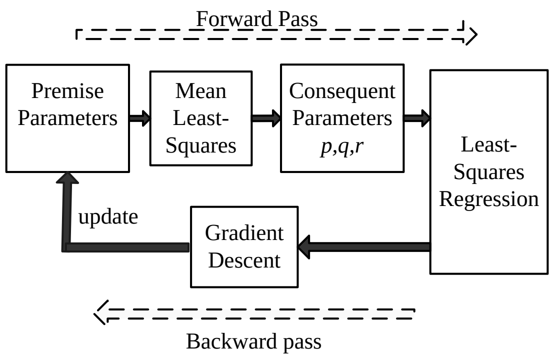


Fig. 4. ANFIS learning using hybrid technique.

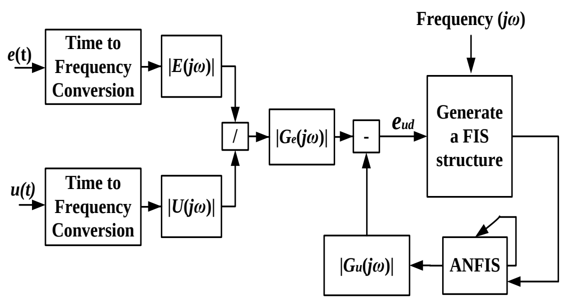


Fig. 5. Intelligent estimation of the uncertainty weighting function, using ANFIS.

Theorem 1: Let $e_{ud_{\min}} = \min_{\omega} e_{ud}(\omega_k)$; $\omega_k (k = 1,...,nn)$ and $G_{u_{\min}} = G_u(e_{ud_{\min}})$ for a given model error estimation. $G_{u_{\min}}$ satisfies certain stopping criteria. Hence, the intelligent uncertainty weighting function W_a obtained from the intelligent estimated uncertainty bounds G_u is:

$$W_a = | G_{u_{\min}} | \qquad \text{...(10)}$$

Proof: Given the measured controlled signal, *u*, and estimated error model, *ε,* required to develop the upper magnitude bound of the model-error function as defined in Equation (6), and considering ANFIS based intelligent estimation of uncertainty as given in Equation (7). If Equation (8) is minimized within a given number of iterations while satisfying Equation (9), then the statement of Equation (10) is true for ω_k (k=1,…,nn).

The training of ANFIS should cover the frequency response function magnitude of G_e. In order to illustrate this issue, the ANFIS should be trained to identify the model error magnitude associated with the linear model of the system over a suitable frequency range, e.g. within the system's Nyquist frequency. The ANFIS within this scheme can efficiently estimate non-conservative uncertainty bound using large amplitude signals over different range of frequencies within short time of calculations and considerably simplified computation [7].

For a MIMO system, it is necessary to estimate as many uncertainty weighting functions as the number of measured variables. Consequently, the resulted weighting functions will be combined in the following uncertainty weighting matrix and used to synthesize the robust controller, as will be described next.

$$\overline{W_a} = \begin{bmatrix} W_{a1} & 0 & 0 & 0 \\ 0 & W_{a2} & 0 & 0 \\ 0 & 0 & W_{a3} & 0 \\ 0 & 0 & 0 & W_{a4} \end{bmatrix} \qquad \text{...(11)}$$

5. Simplified H_2/H_∞ Robust Control

The generalized plant considered for a system is:

$$\dot{x} = Ax + B_1\omega_i + B_2 u \qquad \text{...(12)}$$

$$z = C_1 x + D_{12} u \qquad \text{...(13)}$$

$$y = C_2 x + D_{21}\omega_i \qquad \text{...(14)}$$

where ω_i is the exogenous inputs external to the closed-loop system such as measurement noise, *u* is the control input vector, *y* is the measured output vector, and *z* is the regulated output vector.

Then the plant's transfer matrix can be partitioned as follows:

$$P(s) = \left[\begin{array}{cc|c} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right] \qquad \text{...(15)}$$

such that

$$z(s) = P_{11}(s)\omega_i(s) + P_{12}(s)u(s) \qquad \text{...(16)}$$

and

$$y(s) = P_{21}(s)\omega_i(s) + P_{22}(s)u(s) \qquad \text{...(17)}$$

with a proper rational controller *K(s)*, the control is given by

$$u = K(s)y \qquad \text{...(18)}$$

Substituting equation (18) into equation (16) and equation (17), the following relationship is obtained:

$$z(s) = \left[P_{11}(s) + P_{12}(s)K(s)z_i P_{21}(s)\right]\omega_i(s) \qquad \text{...(19)}$$

where $z_i = \left[I - P_{22}(s)K(s)\right]^{-1}$

The transfer matrix - which is the transfer function from the exogenous input $\omega_i(s)$ to the regulated output *z(s)* -can be denoted *F(s)* for simplicity of notation and can be written as:

$$z(s) = F(s)\omega_i(s) \qquad \text{...(20)}$$

where

$$F(s) = \left[P_{11}(s) + P_{12}(s)K(s)z_i P_{21}(s)\right]$$

The generalized plant model is shown in Figure 6, and the Linear Fractional Transformation (LFT) of model, is:

$$P(s) = \begin{bmatrix} 0 & 0 & W_a \\ -W_e & W_e & -G_N W_e \\ 0 & 0 & W_u \\ -I & I & -G_N \end{bmatrix} \qquad \text{...(21)}$$

where I is an identity matrix.

Since there are multi objectives that need to be satisfied; robust stability, robust performance, faster tracking and minimized the effects of uncertainties and measurements errors, the robust controller is formulated in this case as a mixed H_2/H_∞ problem. The prescribed specifications are translated into the following criterion:

$$\text{Minimize } \alpha_m \|T_\infty\|_\infty + \beta_m \|T_2\|_2 \quad \text{…(22)}$$

while maintaining the H_∞ of the closed loop transfer function T_∞ from ω_i to $z_\infty < \gamma_0$ and maintaining the H_2 of the closed loop transfer function T_2 from ω_i to $z_2 < v_0$, where $\alpha_m > 0$, $\beta_m > 0$, $\gamma_0 > 0$ and $v_0 > 0$ are some prescribed value. Moreover, places the closed loop poles in the LMI open left hand plan region.

W_e is selected to adjust the performance and eliminate the steady state error. W_u is selected to attenuate the high frequency noises as much as possible [2]. W_a is used to shape the closed-loop transfer function at frequencies where uncertainties expected to appear. The intelligent uncertainty weighting function derived directly from the identified ANFIS uncertainty bound can be used for robust control synthesis of a system. The intelligent weighting function accurately reflects additive uncertainty associated with the nominal model, as illustrated in Section 4.3. Since the order of the H_∞ controller is directly related to the order of this weighting function, it is recommended to substitute a low order transfer function for the ANFIS uncertainty bound.

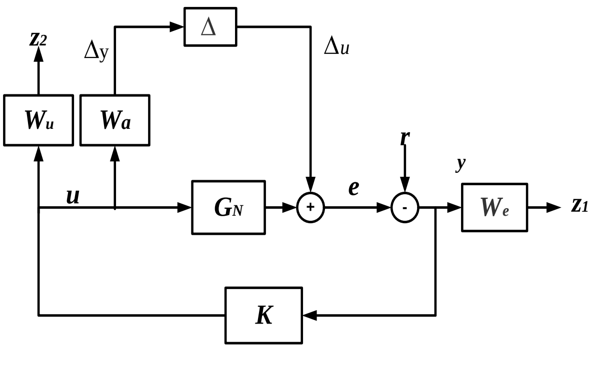


Fig. 6. The entire-connection of the robustly-controlled system.

6. Results and Discussion

The intelligent approach to estimate uncertainty bound was implemented to adaptively bound modelling uncertainty for robust controller design. As a result, an ANFIS estimation of uncertainty bounds for an AMB system model was developed and implemented to generate uncertainty weighting functions required for robust controller design. Then, in order to validate these weighting functions a corresponding robust controller $K(s)$ is designed. The entire connection of Figure 6 is used, with suitable selection of W_e and W_u.

Table 2 summarizes the numerical results of the intelligent estimation and robust controller design of the uncertainty bounds for the AMB system. Comparison was conducted with intelligent estimation method of intelligent Confidence Interval Network Neural Network (CIN NN) [8] for the purpose of validation; less complicated procedure, and reduced number of calculations are achieved. It is also obvious that the learning time required for ANFIS is almost ten times faster than the time required for CIN, and the number of iteration of ANFIS is five times less than the number of required iterations for CIN, for the same number of training data pairs.

Since the order of the robust H_2/H_∞ controller is directly related to the order of the uncertainty weighting function, it is desirable to substitute a low-order transfer function for the ANFIS bound. MATLAB’s *fitsys* command is applied to construct such a low-order weighting function. Figure 7 shows the four ANFIS estimated uncertainty weighting functions at 83.375 *Hz*. According to the applied method of MEM to estimate the upper magnitude bound, these transfer functions can accurately reflect additive uncertainties.

Table 2,
Comparison between two intelligently identified uncertainty weighting functions. No. of training data pairs= 1900.

	Learning time (sec.)	No. of iteration of training	Order of W_{ak} (k=1,..,4)
CIN NNs	106.2426	100	4
ANFIS	16.2154	20	3
	Best objective LMI	K- $_{LMI}$ Dimension	No. of iteration LMI
CIN NNs	1.998	[4,32]	47
ANFIS	1.997	[4,28]	44

Intelligent robust controller is designed using the dynamic model described by Equation (1) based on a nominal rotation speed of 6.0 *krpm*. The controller integrated the ANFIS weighting function $\overline{W_a}$ of Figure 7, the performance weighting function W_e and input weighting function W_u . Table 2 indicates that the lower order of ANFIS- W_a results in a lower order controller matrix than that developed using CIN, indicating a relatively simplified controller.

Then, another set of experiments is conducted. The purpose is to illustrate the ability of the developed intelligent estimation of uncertainties to follow variations over a wide range of operating speeds. Figure (8-a) compares the ANFIS estimated uncertainty weighting functions W_{a1} at 83.375 Hz (the nominal value), 100 Hz and 200 Hz. The magnitudes of these uncertainty weighting functions show that as the rotational speed of the system moves away from the nominal speed that is used in the modelling, and clearly, the amount of uncertainty increases. It is clear that these intelligent weighting functions can accurately reflect model uncertainty variations in spite of changes in operating speeds. And according to Theorem 1, the intelligently estimated weighting functions are optimized weighting functions. Similarly, the weighting functions W_{a2} , W_{a3} , and W_{a4} are changed in accordance with variations of uncertainty bounds as frequency changed. These effects can be observed in Figure 8-b,c,d). As a result, the synthesized robust controller using these weighting functions of intelligent $\overline{W}_a$ will be less conservative with high performance quality. Meanwhile, the optimization results of the H_∞ performance under LMI constraints have been slightly affected by variations as illustrated in Table 3.

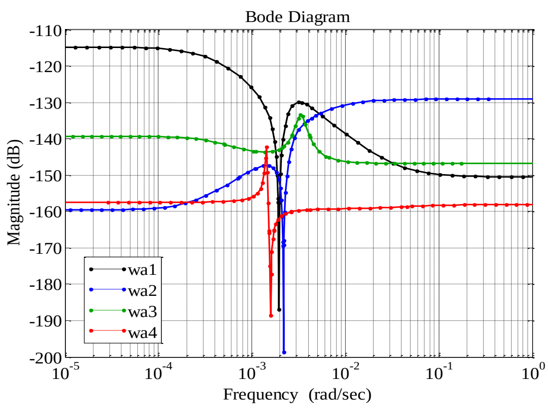


Fig. 7. The ANFIS identified weighting function W_a for AMB model, at 83.375 Hz [8].

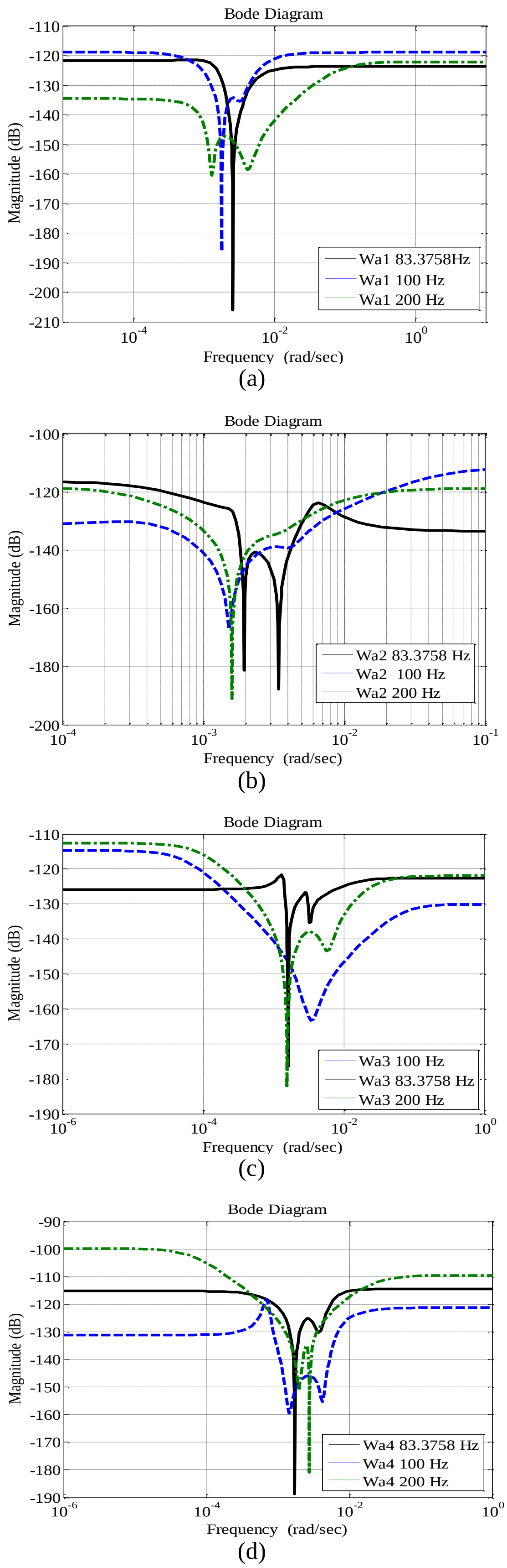


Fig. 8. Comparison of ANFIS W_a bounds for different operating speeds: (a) W_{a1}, (b) W_{a2} , (c) W_{a3} and (d) W_{a4}.

Table 3,
Comparison between three intelligently identified uncertainty weighting functions.

Frequency of estimated W_a (Hz)	Learning time (sec.)	Best objective LMI	K_{LMI} Dim-ension	LMI NOI*
83.3758	58.1862	1.998	[4*36]	41
100	57.2815	2.004	[4*36]	51
200	61.4213	1.997	[4*32]	43

*NOI :Number of Iterations.

7. Conclusions

The objective of this paper has been to develop an intelligent estimation of uncertainty bounds for robust LMI control of AMB systems. Robust control theory provides systematic representation of uncertainties. However, the selection of suitable weighting functions is a critical requirement for robust stability and performance. ANFIS in MEM framework is developed to estimate least conservative uncertainty weighting function. Successful application of the proposed ANFIS estimation algorithm for MIMO systems is accomplished; similar accuracy to neural network estimation is obtained in a shorter learning time and less number of iterations of training. Moreover the order of the estimated weighting function is reduced using the developed ANFIS method. The order of the evaluated LMI robust controller is reduced as well, which is preferred for practical applications. Different amount of uncertainties were identified using ANFIS technique for various frequency operating conditions, resulting in accurate uncertainty weighting functions.

The upcoming work is to extend the ANFIS method for Linear Parameter Varying (LPV) method to maintain the stability and robust performance over wider operating ranges. Moreover, online updating of the ANFIS bounds will be added to effectively adapt to parameter variations.

Notation

$K_{GN,K}$	A generalized stability margin of the stable loop $[G_N, _K]$
G_e	Model of Model Error
G_N	Model of Nominal model
G_u	Intelligent uncertainty bound
W_a	Additive Uncertainty weighting function
W_e	Performance weighting function
W_u	Control weighting function
e_{ud}	Prediction error
x,y	longitudinal displacement of radial active magnetic bearings
z	axial displacement of thrust active magnetic bearing.
m	rotor weight
I_x ,I_y	polar mass inertia of rotor
I_z	axial mass inertia of rotor
k_s	displacement stiffness
k_i	current stiffness
k_c	coupling stiffness
Ω	nominal rotor speed
λ	eigenvalues

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مسيطر H_2/H_∞ ذكي متين لمنظومة المحامل المغناطيسية النشطة

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الخلاصة

يتطلب تصميم المسيطر المتين تعريف مناسب لحدود عدم الوثوقية . و يتم عادة اختيار حدود عدم الوثوقية بشكل عشوائي و متحفظ لتحقيق الاستقرارية المطلوبة و بغض النظر عن جودة أداء المسيطر . هذه الحالة تصبح اشد صعوبة لمنظومات متعددة المتغيرات ذات لا خطية عالية كما في حالة منظومة المحامل المغناطيسية النشطة (AMB). ان المرونة التي تمتاز بها التقنيات الذكية و قابلياتها المتقدمة في التعلم يجعلها مناسبة لتطبيقات استنباط الحالة . الغرض من هذه المقالة هو بناء مسيطر H_2/H_∞ متين لمنظومة المحامل المغناطيسية النشطة بناءا على استنباط الحالة الذكي لحدود عدم الوثوقية باستخدام نظام الاستدلال العصبي الضبابي المتكيف (ANFIS). كشفت نتائج المحاكات ان اهداف المسيطر المتين بتحقيق حزمة واسعة للتردد الفعال و بأداء متين قد تحققت لمدى واسع من تغير الترددات . و لهذا نستنتج ان دوال عدم الوثوقية الذكية تمكن من تحديد تأثير أخطاء النمذجة و عدم الخطية بدقة عالية.