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Design and Simulation of L_1-Adaptive Controller for Position Control of DC Servomotor

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Abstract

This paper presents $\mathcal{L}_1$-adaptive controller for controlling uncertain parameters and time-varying unknown parameters to control the position of a DC servomotor. For the purpose of comparison, the effectiveness of $\mathcal{L}_1$-adaptive controller for position control of studied servomotor has been examined and compared with another adaptive controller; Model Reference Adaptive Controller (MRAC). Robustness of both $\mathcal{L}_1$-adaptive controller and model reference adaptive controller to different input reference signals and different structures of uncertainty were studied. Three different types of input signals are taken into account; ramp, step and sinusoidal. The $\mathcal{L}_1$-adaptive controller ensured uniformly bounded transient and asymptotic tracking for both system's input and output signals, simultaneously with asymptotic tracking. Simulations of a DC servomotor with time-varying friction and disturbance are presented to verify the theoretical findings.

Keywords: *$\mathcal{L}_1$-adaptive controller, DC servomotor position control.*

1. Introduction

Research in adaptive control was motivated by the design of autopilots for highly agile aircraft that need to operate at wide range of speeds and altitudes, experiencing large parametric variations. In the early 1950s, adaptive control was conceived and proposed as a technology for automatically adjusting the controller parameters in the face of changing aircraft dynamics [1, 2]. The initial results in adaptive control were inspired by system identification, which led to an architecture consisting of an online parameter estimator combined with automatic control design [3]. Two architectures of adaptive control emerged: the direct method, where only controller parameters were estimated, and the indirect method, where process parameters were estimated and the controller parameters were obtained using some design procedure. To achieve identifiability, it was necessary to introduce a condition of persistency of excitation in order to guarantee that the parameter estimates converge. The progress in systems theory led to fundamental theory for development of stable adaptive control architectures [4-6]. The basic idea of all the modifications was to limit the gain of the adaptation loop and to eliminate its integral action. Examples of these modifications are the σ-modification and the e-modification. All these modifications attempted to provide a solution to the problem of *parameter drift* [6, 7]. However, despite the success of such techniques in many applications, they hold some drawbacks. For instance, adaptive controllers rely on the need of a persistency in parameter excitation before convergence which may lead to a bad transient behavior. A regressor is often required, involving with it a large parameter vector to be estimated. Moreover, a large adaptation gain may have undesirable effects, with the risk of parameter divergence. All of the arguments brought against adaptive schemes reveal that despite their numerous advantages, these controllers hold some drawbacks. For the sake of clarity, one can summarize some of them here [7, 8, 9]:

1. A wide range of such controllers exhibit undesirable frequency characteristics and are often used with restrictive assumptions. It has been shown that the sinusoidal reference inputs at certain frequencies and/or sinusoidal output disturbances at any frequency will significantly increase the adaptation gain which will destabilize the control system.
2. The need for the persistency in excitation can lead to a bad transient behavior.
3. An increase in the adaptation gain drives the closed-loop system closer to instability, while a small gain would slow down the convergence. Any parameter vector to be adapted must be adequately initialized, and this choice would depend on the specific configuration of the system.

Recently, a new variant of adaptive control has been developed named as $\mathcal{L}_1$-adaptive control. This version of adaptive control is utilizing fast and robust adaptation; it permits a transient analysis even for time varying uncertainties and is capable of handling constraints. The $\mathcal{L}_1$-adaptive control is presented as an adaptive approach for nonlinear time varying systems in the presence of state constraints [10].

2. Problem Formulation

The following class of systems will be considered [7, 11, 12];

$$\dot{x}(t) = A_m x(t) + b\,(\omega\, u(t) + \theta^{\mathrm{T}}(t)x(t)\sigma(t))$$
$$y(t) = c^{\mathrm{T}} x(t) \qquad \ldots(1)$$

where $x(t) \in \mathbb{R}^n$ is the system measured state vector; $u(t) \in \mathbb{R}$ is the control input, $y(t) \in \mathbb{R}$ is the regulated output; $b, c \in \mathbb{R}^n$ are known constant vectors; A_m is a known Hurwitz $n \times n$ matrix (all its eigenvalues have negative real values that specifying the desired closed-loop dynamics); $\omega \in \mathbb{R}$ is an unknown constant (with known sign); $\theta(t) \in \mathbb{R}^n$ is a vector of time-varying unknown parameters; and $\sigma(t) \in \mathbb{R}$ models input disturbances.

The objective of the desired controller is to ensure that $y(t)$ tracks a given bounded piecewise-continuous reference signal $r(t)$ with quantifiable performance bounds using full-state feedback adaptive controller based on the following assumptions [7, 11-14]

Assumption (I): Uniform boundedness of unknown parameters

Letting

$$\theta(t) \in \Theta, \qquad |\sigma(t)| \le \Delta_0\ , \forall t \ge 0. \qquad \ldots(2)$$

where Θ is a convex compact set, and $\Delta_0 \in \mathbb{R}^+$ is a conservative bound of $\sigma(t)$ [7, 11-14].

Assumption (II): Uniform boundedness of rate of parameters' variations

This assumes that $\theta(t)$ and $\sigma(t)$ have to be continuously differentiable with uniformly bounded derivatives [7, 12, 13]:

$$\|\dot{\theta}(t)\| \le d_\theta < \infty, \quad |\dot{\sigma}(t)| \le d_\sigma < \infty, \ \forall t \ge 0. \qquad \ldots(3)$$

Assumption (III): Uncertain system input gain is partially known

This assumes that

$$\omega \in \Omega_0 \triangleq [\omega_{l0}, \omega_{u0}], \qquad \ldots(4)$$

where $0 < \omega_{l0} < \omega_{u0}$ are given known lower and upper bounds on ω.

3. $\mathcal{L}_1$-Adaptive Control Architecture

In what follows, the elements of $\mathcal{L}_1$-adaptive controller will be considered. The controller comprises three main parts; state predictor, adaptation law and control law [7, 11, 14].

3.1. State Predictor

Let us consider the following state predictor:

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b\Big(\hat{\omega}(t)u(t) + \hat{\theta}^{\mathrm{T}}(t)x(t) +$$
$$\sigma t, \qquad x0{=}x0$$
$$\hat{y}(t) = c^{\mathrm{T}} \hat{x}(t) \qquad \ldots(5)$$

which has the same structure as the system in (1); the only difference is that the unknown parameters ω, $\theta(t)$, and $\sigma(t)$ are replaced by their adaptive estimates $\hat{\omega}(t)$, $\hat{\theta}(t)$, and $\hat{\sigma}(t)$, respectively.

3.2. Adaptation Laws

The adaptive process is governed by the following projection-based adaptation laws:

$$\dot{\hat{\theta}}(t) = \Gamma\ \mathrm{Proj}\left(\hat{\theta}(t), -\tilde{x}^{\mathrm{T}}(t)Pbx(t)\right),$$
$$\hat{\theta}(0) = \hat{\theta}_0,$$
$$\dot{\hat{\sigma}}(t) = \Gamma\ \mathrm{Proj}\left(\hat{\sigma}(t), -\tilde{x}^{\mathrm{T}}(t)Pb\right),$$
$$\hat{\sigma}(0) = \hat{\sigma}_0,$$
$$\dot{\hat{\omega}}(t) = \Gamma\ \mathrm{Proj}\left(\hat{\omega}(t), -\tilde{x}^{\mathrm{T}}(t)Pbu(t)\right),$$
$$\hat{\omega}(0) = \hat{\omega}_0 \qquad \ldots(6)$$

where $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$, $\Gamma \in \mathbb{R}^+$ is the adaptation rate and $P = P^{\mathrm{T}} > 0$ is the solution of the algebraic Lyapunov equation $A_m^{\mathrm{T}}P + PA_m = -Q$ for arbitrary $Q = Q^{\mathrm{T}} > 0$.

In the implementation of the projection operator, one can use the compact set Θ as given in Assumption (I), while one can replace Δ_0 and Ω_0 by Δ and $\Omega \triangleq [\omega_{l0}, \omega_{u0}]$ such that [7, 14]

$$\Delta_0 < \Delta, \quad 0 < \omega_l < \omega_{l0} < \omega_u < \omega_{u0} \qquad \ldots(7)$$

3.3. Control Law

The control signal is generated as the output of the following (feedback) system:

$$u(s) = -kD(s)\left(\hat{\eta}(s) - k_g\, r(s)\right), \qquad \ldots(8)$$

where $r(s)$ and $\hat{\eta}(s)$ are the Laplace transforms of $r(t)$ and $\hat{\eta}(t) \triangleq \hat{\omega}(t)u(t) + \hat{\theta}^{\mathrm{T}}(t)\boldsymbol{x}(t) + \hat{\sigma}(t)$, respectively; $k_g \triangleq -1/c^{\mathrm{T}}A_m^{-1}b$; and $k > 0$ and $D(s)$ are a feedback gain and a strictly proper transfer function leading to a strictly proper stable

$$C(s) \triangleq \frac{\omega k D(s)}{1 + \omega k D(s)} \quad \forall \omega \in \Omega_0 \qquad \ldots(9)$$

with DC gain $C(0) = 1$. One simple choice is $D(s) = 1/s$, which yields a first-order strictly proper $C(s)$ of the form [7, 11]

$$C(s) = \frac{\omega k}{s + \omega k} \qquad \ldots(10)$$

letting $\theta \in \Theta$ [7, 11-14],

$$L \triangleq \max_{\theta \in \Theta} \|\theta\|_1$$
$$H(s) \triangleq (s\mathbb{I} - A_m)^{-1}b$$
$$G(s) \triangleq H(s)\left(1 - C(s)\right) \qquad \ldots(11)$$

The $\mathcal{L}_1$-adaptive controller is defined via (1), (2), (4), subjected to the following $\mathcal{L}_1$-norm condition:

$$\|G(s)\|_{\mathcal{L}_1} L \leq 1 \qquad \ldots(12)$$

The $\mathcal{L}_1$-adaptive control architecture with its main elements is represented in figure (1). In the case of constant $\theta(t)$, the $\mathcal{L}_1$-norm condition can be simplified. For the specific choice of $D(s) = 1/s$, it is reduced to;

$$A_g \triangleq \begin{bmatrix} A_m + b\theta^{\mathrm{T}} & b\omega \\ -k\theta^{\mathrm{T}} & -k\omega \end{bmatrix}, \qquad \ldots(13)$$

A_g being Hurwitz which all its eigenvalues have negative real values for all $\theta \in \Theta$ and $\omega \in \Omega_0$ [7, 11].

3.4. Projection Operator

Consider a convex, compact set with a smooth boundary given by [7, 9]

$$\Omega_c \triangleq \{\theta \in \mathbb{R}^n | f(\theta) \leq c\} \quad 0 \leq c \leq 1, \qquad \ldots(14)$$

where $f: \mathbb{R}^n \to \mathbb{R}$ is the following smooth convex function:

$$f(\theta) = \frac{\theta^T\theta - \theta_{max}^2}{\varepsilon_\theta\, \theta_{max}^2} \qquad \ldots(15)$$

where $0 < \varepsilon_\theta \leq 1$ and θ_{max} is the norm bound imposed on the parameter vector θ, and ε_θ denotes the convergence tolerance of our choice. Let the true value of the parameter θ, denoted by θ^* belong to Ω_o, i.e. $\theta^* \in \Omega_o$.

The special structure of the function f should be interpreted as: if one solves $f(\theta) \leq 1$, which defines the boundaries of the outer set, then one can get that $\theta^T\theta \leq (1 + \varepsilon_\theta)\, \theta_{max}^2$. ε_θ specifies the maximum tolerance the adaptive parameter is allowed to exceed compared to its maximum conservative value. The projection operator is defined as:

$$Proj(\theta, y) \triangleq \begin{cases} y & if\ f(\theta) < 0, \\ y & if\ f(\theta) \geq 0\ and\ \nabla f^T y \leq 0, \\ y - \dfrac{\nabla f}{\|\nabla f\|}\left\langle \dfrac{\nabla f^T}{\|\nabla f\|}, y \right\rangle & if\ f(\theta) \geq 0\ and\ \nabla f^T y > 0, \end{cases}$$

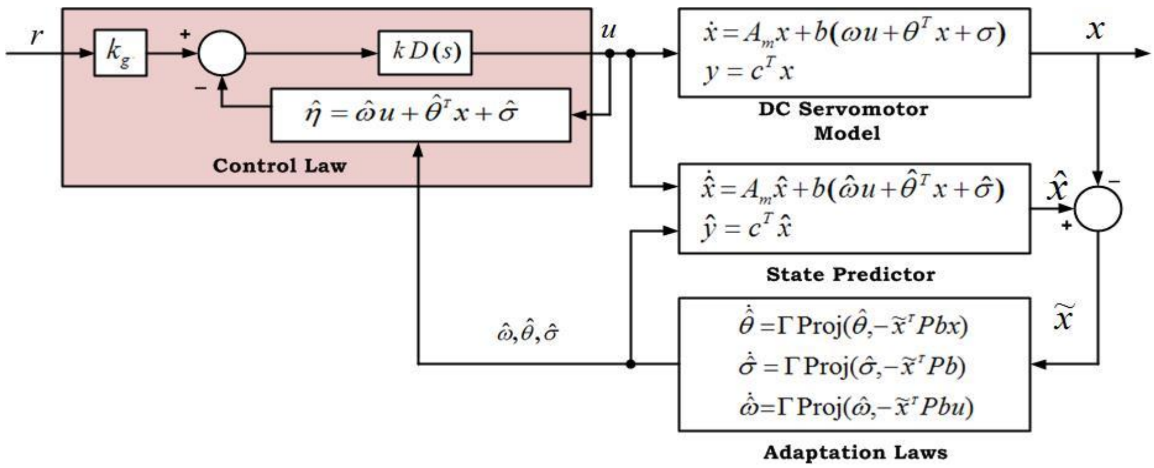


Fig. 1. Closed-loop L_1-Adaptive System [4].

4. Modeling of DC Motor

In the present work, a separately excited DC motor was considered, as shown in Figure (2),

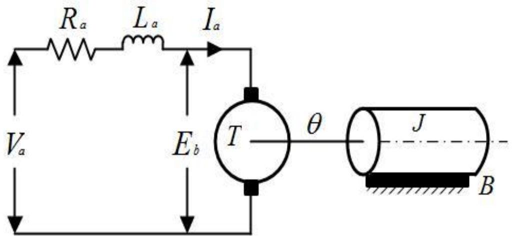


Fig. 2. A Separately Excited DC Motor Model.

The equations describing the dynamic behavior of the DC motor are given by [15]:

$$\frac{di_a(t)}{dt} = \frac{1}{L_a} e_a(t) - \frac{R_a}{L_a} i_a(t) - \frac{1}{L_a} e_b(t) \quad \ldots(17)$$

$$T_m(t) = K_i i_a(t) \quad \ldots(18)$$

$$e_b(t) = K_b \frac{d\theta_m(t)}{dt} = K_b \omega_m(t) = K_b \dot{\theta}_m(t) \quad \ldots(19)$$

$$T(t) = J\frac{d\omega(t)}{dt} + B\ \omega_m(t) + T_1(t) = K_t\ i_a(t) \quad \ldots(20)$$

where the $T_1(t)$ accounts for nonlinear disturbances which includes the nonlinear friction, torque disturbances and other nonlinearities of the system,

$$T_1(t) = k_v \theta_m(t) + T_f(\theta_m(t)) + T_L(t) \quad \ldots(21)$$

$$T_f(\theta_m(t)) = (F_s - F_c) sgn(\theta_m(t)) e^{-(\dot{\theta}_m(t)/\dot{\theta}_s)^2} + F_c sgn(\theta_m(t)) \quad \ldots(22)$$

where, $\omega_m(t)$ is the rotational speed, θ_m is the angular displacement, $i_a(t)$ armature circuit current, $T_f(t)$ is the friction torque, $T_L(t)$ is the load torque, R_a armature circuit resistance, B coefficient of viscous-friction, K_t torque coefficient, J moment of inertia, and L_a armature circuit inductance, k_v is the Viscous friction coefficient, F_s is the stribeck friction constant, F_c is the Colomb friction level, $sgn(\cdot)$ is the signum function, and $\dot{\theta}_s$ is the stribeck angular constant velocity.

For $L_a \ll R_a \Rightarrow L_a$ can be ignored so the model in equations (17) to (20) can be simplified to the following: (For simplicity we will omit the t and s parameters from the equations)

$$0 = e_a - R_a i_a - K_b \dot{\theta}_m \quad \ldots(23)$$

$$i_a = \frac{1}{R_a} e_a - \frac{K_b}{R_a} \dot{\theta}_m \quad \ldots(24)$$

$$\ddot{\theta}_m = -\frac{B}{J} \dot{\theta}_m + \frac{K_t}{J} i_a - \frac{1}{J}(T_1) \quad \ldots(25)$$

By substituting equation (24) into equation (25) this will lead to:

$$\ddot{\theta}_m = -\left(\frac{R_a B + K_t K_b}{J R_a}\right) \dot{\theta}_m + \frac{K_t}{J R_a} e_a - \frac{k_v}{J} \dot{\theta}_m - \frac{1}{J}\left(T_f(\dot{\theta}_m) + T_L\right) \quad \ldots(26)$$

Let $x_1 = \theta_m$ and $x_2 = \dot{\theta}_m$, the state space form can be written in the following form;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\left(\frac{R_a B + K_t K_b}{J R_a}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(\frac{K_t}{J R_a} u - \right.$$
$kvJθm−1JTfθm+TL$ …(27)

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

If equation (27) had been compared with equation (1),

$$\dot{x}(t) = A_m x(t) + b\left(\omega\, u(t) + \theta^{\mathrm{T}}(t)\, x(t) + \right.$$
$$\sigma t$$

By induction, one may easily find that

$\sigma(t) = -\frac{1}{J}\left(T_f(\dot{\theta}_m) + T_L\right),$

$\theta^{\mathrm{T}} = [0 \quad -k_v/J]$, $\omega = \frac{K_t}{JR_a}$,

$b = [0 \quad 1]^T$ and $c = [1 \quad 0]$

where $\boldsymbol{A}_m = \boldsymbol{A} - bK_m$ and $K_m = [k_1 \quad k_2]$ is the state feedback gain necessary for making the state matrix $\boldsymbol{A}$ being Hurwitz which all its eigenvalues have negative real values.

5. Controllability Condition Pole-Placement

The requirement for applying pole placement is that the system must be completely state controllable. The state and input matrix of DC motor are given, respectively,

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\left(\frac{R_a B + K_t K_b}{J R_a}\right) \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = [0 \quad 1]^T$$

The controllability matrix is given by [16]

$$\bar{\boldsymbol{M}} = [\boldsymbol{b} \quad \boldsymbol{A}\,\boldsymbol{b}] = \begin{bmatrix} 0 & 1 \\ 1 & -\left(\frac{R_a B + K_t K_b}{J R_a}\right) \end{bmatrix} \qquad \ldots(28)$$

The controllability matrix is given by [16]

It is evident that the rank of controllability matrix is equal to 2, which is equal to the system order. Therefore, the system is completely controllable and the pole placement could be applied.

Numerically, if the system has the following values in Table (1), the eigenvalues for this system is

$s_1 = 0$ and $s_2 = -59.202$

Since the system is completely state controllable, one can arbitrary select the desired poles to be

$s_1 = -40$ and $s_2 = -50$

The elements of state feedback gain K_m which performs pole placement requirements is given by

$\boldsymbol{K}_m = [k_1 \quad k_2] = [2000 \quad -30.8]$

This transformation is achieved by

$$\boldsymbol{A}_m = \boldsymbol{A} - bK_m = \begin{bmatrix} 0 & 1 \\ -2000 & -90 \end{bmatrix}$$

Table 1,
System Model Parameters For DC Servomotor.

Parameter	value
R_a	5 Ω
B	0.136 N.m.s
K_t	0.245 N.m/A
K_b	0.245 V.s/rad
J	0.0025 $kg.m^2/_s^2$
L_a	0.01

6. Simulink Modeling of Adaptive Controller for DC-servo motor

Based on equation (27), complete Simulink implementation adaptive controller for DC servomotor is depicted in Figure (3). The overall Simulink model consists of different blocks that combines together to achieve a suitable simulation of single axis positioning table.

Signals block contains the main reference signals and for the different types of inputs. It also comprises the uncertainties resulting from load and friction forces. The elements of the signals block can be illustrated in Figure (4).

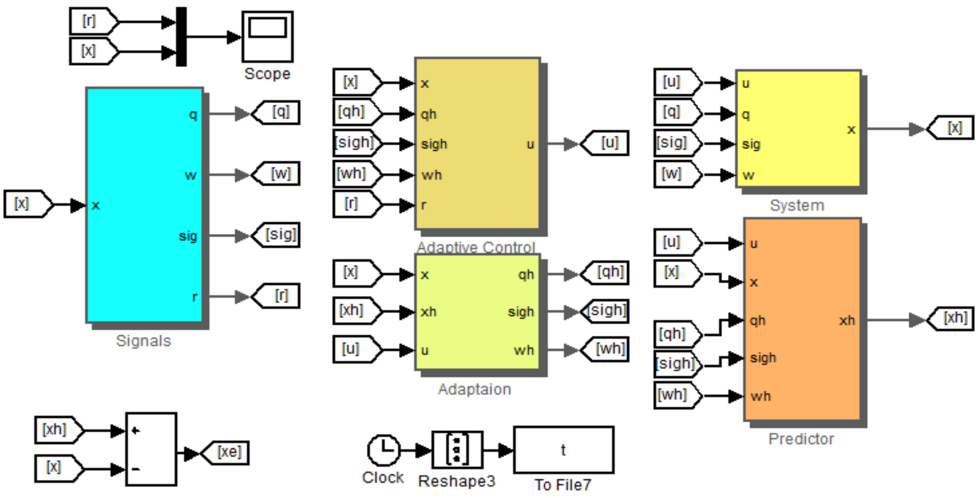


Fig. 3. Overall Simulink Blocks DC Servomotor

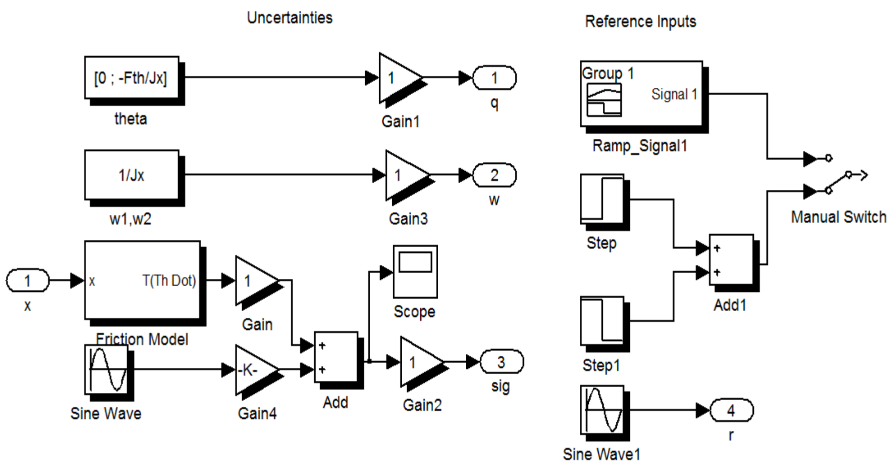


Fig. 4. Signals Block.

The contents of L_1-adaptive controller block are simulated in Figure (5).

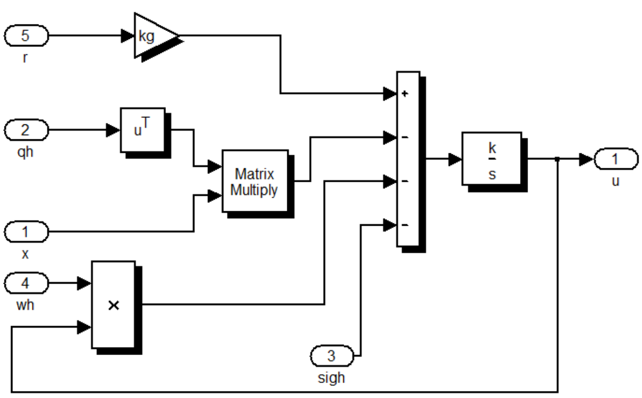


Fig. 5. Adaptive Controller Block Contents

The Simulink block shown in Figure (6) gives the components which simulate the model of the single axis positioning table.

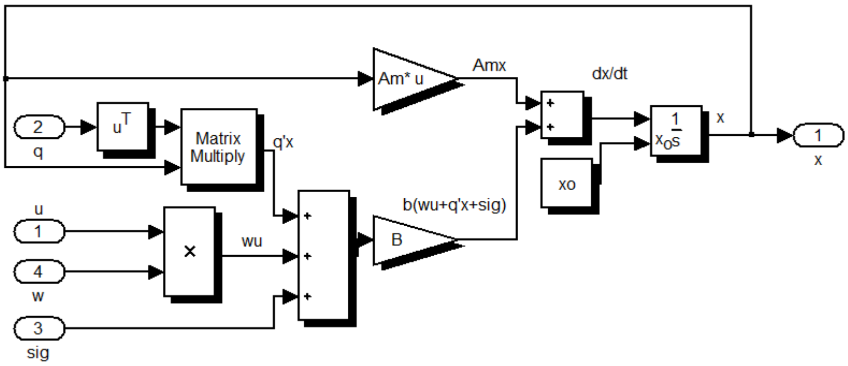


Fig. 6. Modeling of Single Axis Positioning Table.

Adaptation block implements the adaptation techniques used in the present work. It also includes projection block to perform the task of projection operator. This operator used to keep the system parameters bound in a known defined region. The contents of the adaptation block are shown in Figure (7).

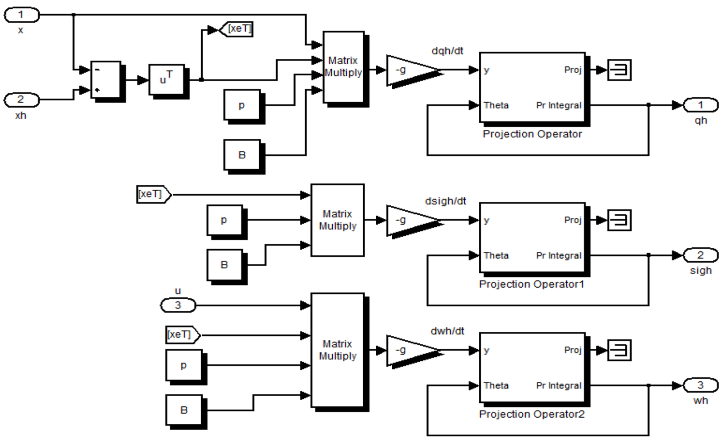


Fig. 7. Simulink Simulation of Adaptation Algorithm with Projection Operator.

The elements of predictor block responsible for predicting both parameters and errors are depicted in Figure (8).

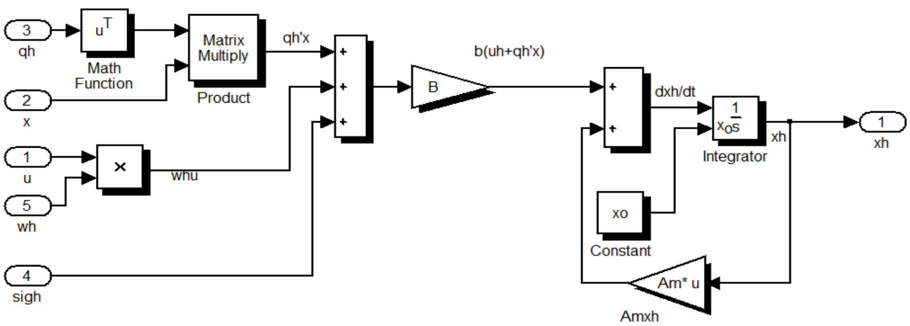


Fig. 8. Contents of Predictor Block.

7. Simulated Results

For simulation purposes, two different architectures of adaptive control were taken: $\mathcal{L}_1$-adaptive control and MRAC. Three types of inputs: ramp, sinusoidal, and step inputs were used to compare between the two architectures. In design of $\mathcal{L}_1$-adaptive controller, the filter of the controller is selected as $D(s) = K/s$ and the parameter of gain K and adaptation gain Γ have been set to $K = 100$ and Γ = 10^4 using trial-and-error procedure.

The system model parameters are listed in Table (1) above. The uncertainty $\sigma(t)$ considered in this system has the following form,

$$\sigma(t) = -\frac{1}{J}\left(T_f\left(\dot{\theta}_m(t)\right) + T_L(t)\right)$$

Also,

$\omega = \frac{K_t}{JR_a}$, $\theta^{\mathrm{T}} = [0 \quad -k_v/J]$

Substituting the values of friction model parameters, the uncertainty bound and the value of parameter ω can be given by

$\sigma(t) \in \Delta = [-1.2, 2.1996]$ (N.mm),

$\omega = 19.6$ (N.s^2/kg.mm.A), $\theta^{\mathrm{T}} = [0 \quad -0.8]$

To show the robustness of the $\mathcal{L}_1$-adaptive control, four cases of different values of uncertainties and disturbances were listed in table (2),

The following normalized friction parameters have been considered for simulation;

$k_v =$ 0.002 N.s/mm, $F_c =$ 0.0002 N.mm, $F_s =$ 0.003 N.mm, $\dot{\theta}_s$= 0.0002 rad/s

Table 2,
Cases for Disturbance Amplitude and Frequency.

Parameter	Case 1	Case 2	Case 3	Case 4
$\sigma(t)$	$0.002\sin(t) + T_f\left(\dot{\theta}_m(t)\right)$	$\sin(t) + T_f\left(\dot{\theta}_m(t)\right)$	$0.002\sin(10t) + T_f\left(\dot{\theta}_m(t)\right)$	$\sin(10t) + T_f\left(\dot{\theta}_m(t)\right)$

The following normalized friction parameters have been considered for simulation;

$k_v =$ 0.002 N.s/mm, $F_c =$ 0.0002 N.mm, $F_s =$ 0.003 N.mm, $\dot{\theta}_s$= 0.0002 rad/s

7.1. Results Based on Ramp Input

For case(1)-case(4) of table (2), the position responses and the control signals are shown in figures (9)-(12). The figures show that $\mathcal{L}_1$-adaptive controller gives a better tracking performance for the ramp input rather than MRAC.

Moreover, the control signals based on MRAC suffer from distortion along tracking period. The steady-state errors for $\mathcal{L}_1$-adaptive controller and MRAC responses are shown in Table (3).

7.2. Results based on step input

It is interesting to examine the effectiveness of both controllers as the system is subjected to a unit step. For the present scenarios, a step input of 0.005 rad height is fed to the system. This step input is inverted after 1.5 sec. such that a square wave input is repeated for every 3 sec. The performance of both controllers for the situations listed in Table (2) will again considered here. The position responses and the control signals based on the four cases are shown in figures (13)-(16). One can easily see that the response based on $\mathcal{L}_1$-adaptive controller could give better performance in terms of transient characteristics than those based on MRAC. Table (4) lists the summary of steady state errors resulting from both controllers for all four cases. It can be concluded that steady state error based on $\mathcal{L}_1$-adaptive controller for all considered cases has nearly zero value. On the other hand MRAC gives considerably large steady state error for all studied cases.

7.3. Results based on sinusoidal input

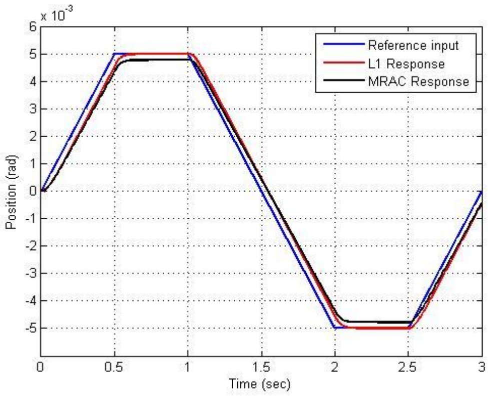
In what follows, the simulation is made for sinusoidal type of input. Table (2) is considered for the uncertainty structures. The procedure followed earlier will be repeated here for all cases of Table (2). It is clear from the figures (17)-(20) that $\mathcal{L}_1$-adaptive controller is better than MRAC in terms of position tracking. Again the control signals based on MRAC suffer from distortion along tracking period. It can be seen from table (5) that for all cases the steady state errors based on $\mathcal{L}_1$-adaptive controller are lower than those based on MRAC.

Table 3,
Steady-State Errors for Different Cases of Ramp Input.

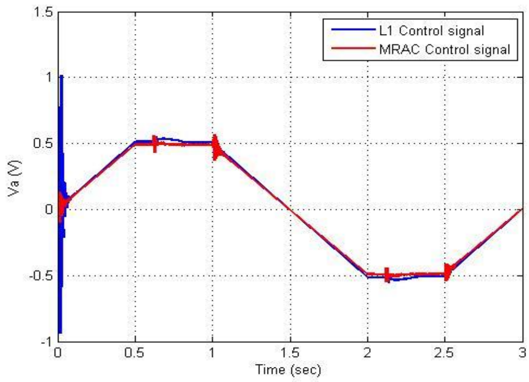
	Steady state error (rad)			
	Case 1	**Case 2**	**Case 3**	**Case 4**
L_1-controller	0.0086	0.0065	0.0089	0.108
MRAC	0.2248	0.2068	0.2247	0.2128

Table 4,
Steady-State Errors for Different Cases of Step Input.

	Steady state error (rad)			
	Case 1	**Case 2**	**Case 3**	**Case 4**
$\mathcal{L}_1$-controller	0.01	0.0073	0.01	0.0071
MRAC	0.23	0.2058	0.23	0.2129

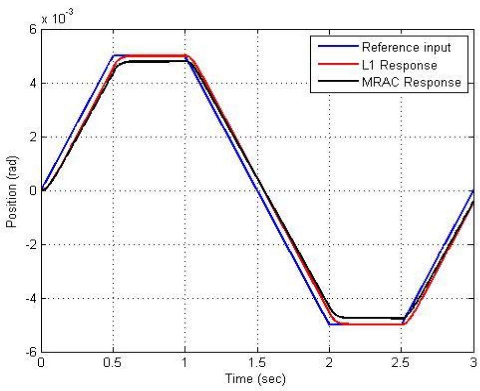


(a) Ramp Responses

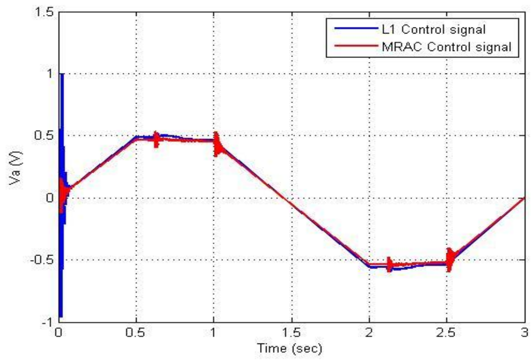


(b) Control Signals

Fig. 9. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Ramp Input (case 1).

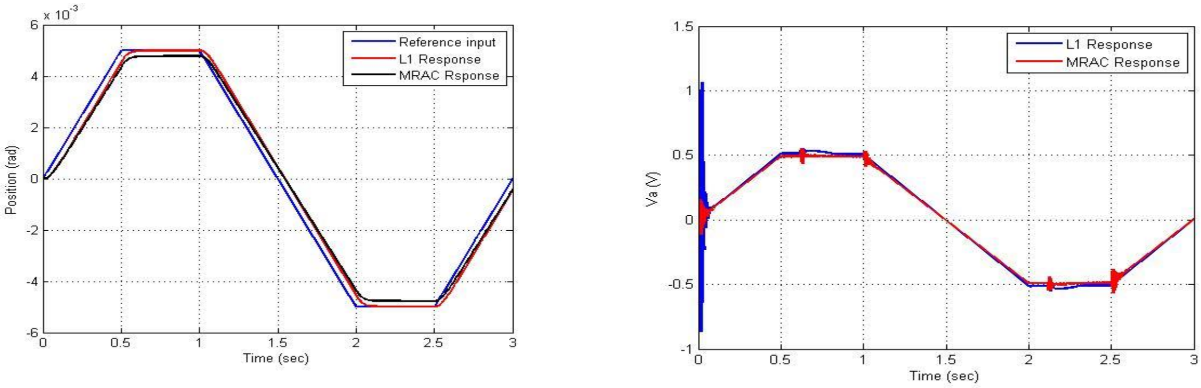


(a) Ramp Responses



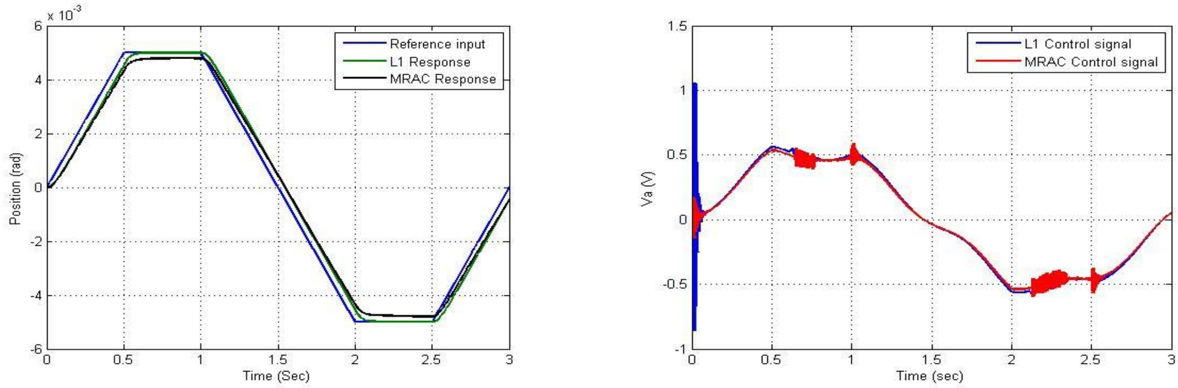
(b) Control Signals

Fig. 10. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Ramp Input (case 2).



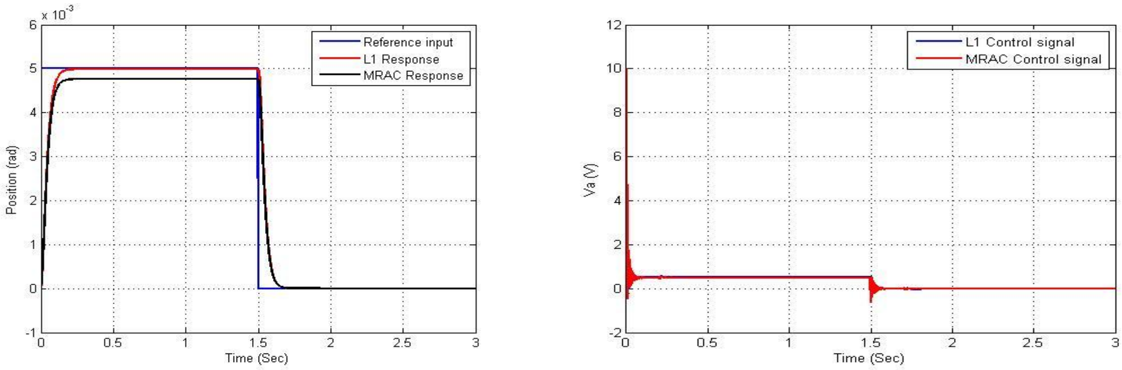
(a) Ramp Responses **(b) Control Signals**

Fig. 11. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Ramp Input (case 3).



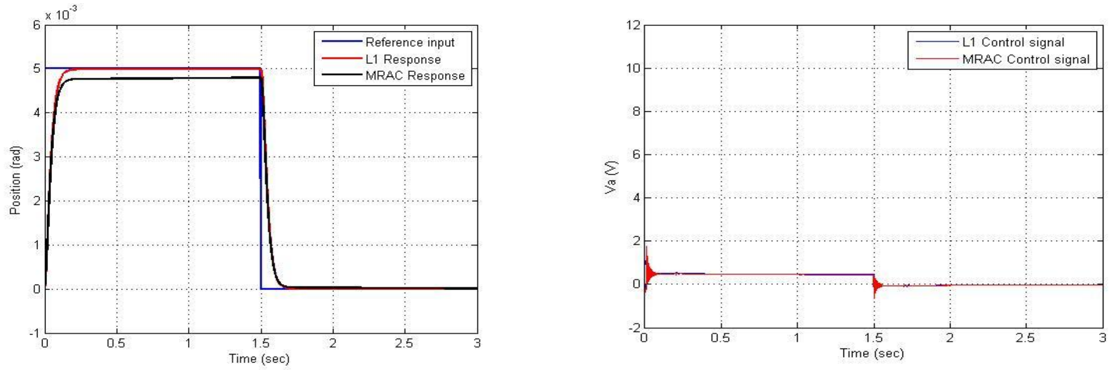
(a) Ramp Responses **(b) Control Signals**

Fig. 12. Transient Responses and Control Signals Based on L_1-Adaptive Controller and MRAC for Ramp Input (case 4).



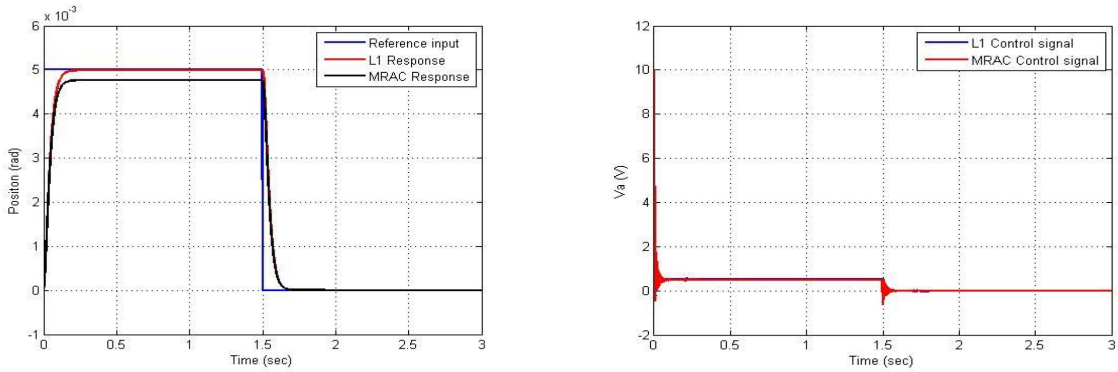
(a) Step Responses **(b) Control Signals**

Fig. 13. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Step Input (case 1).



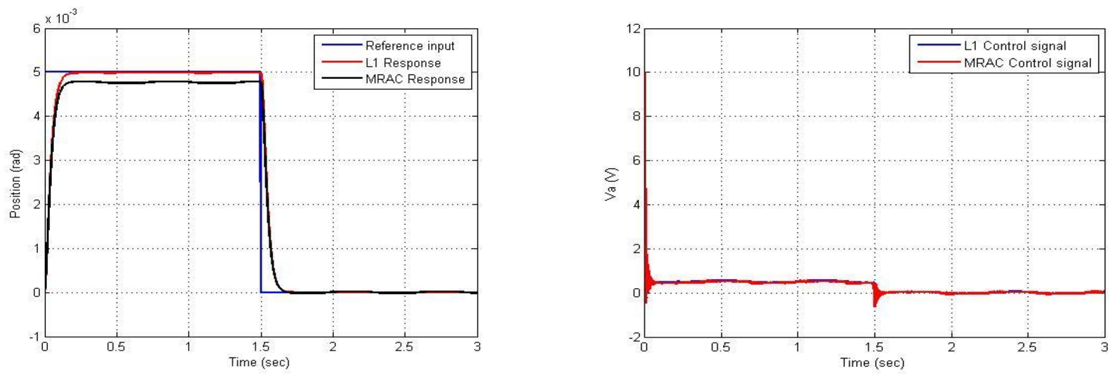
(a) Step Responses **(b) Control signals**

Fig. 14. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Step Input (case 2).



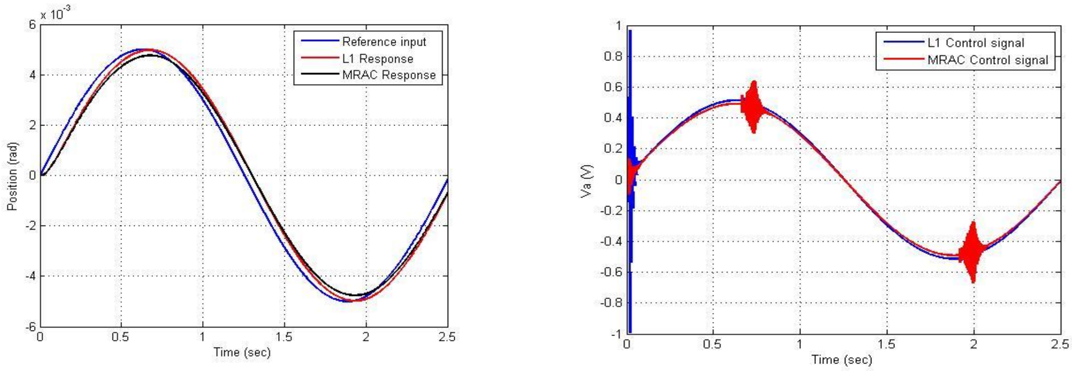
(a) Step Responses **(b) Control Signals**

Fig.15. Transient Responses and Control Signals Based on L1-Adaptive Controller and MRAC for Step Input (case 3).



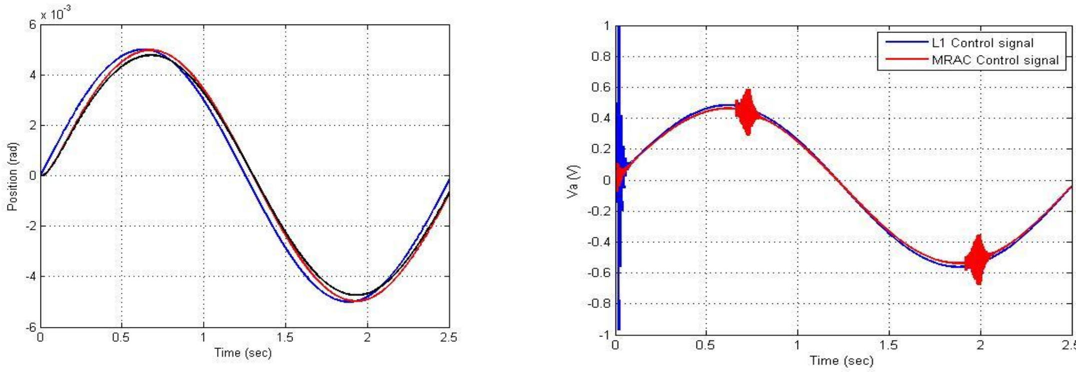
(a) Step Responses **(b) Control Signals**

Fig. 16. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Step Input (case 4)



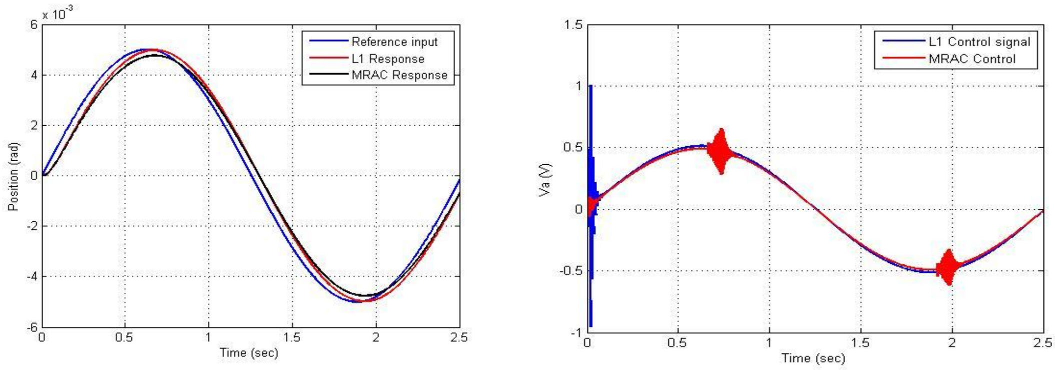
(a) **Sinusoidal Responses** **(b)** **Control Signals**

Fig. 17. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Sinusoidal Input (case 1).



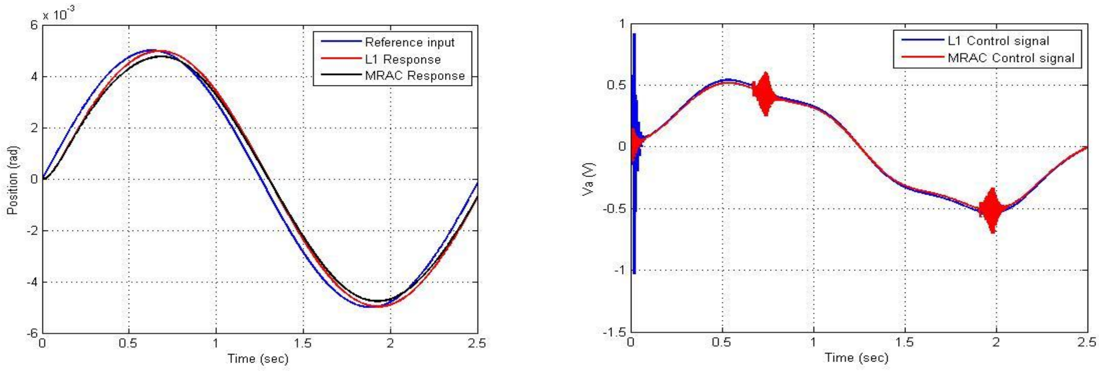
(a) **Sinusoidal Responses** **(b)** **Control Signals**

Fig. 18. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Sinusoidal Input (case 2).



(a) **Sinusoidal Responses** **(b)** **Control Signals**

Fig. 19. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Sinusoidal Input (case 3).



(a) Sinusoidal Responses **(b) Control Signals**

Fig. 20. Transient Responses and Control Signals Based on $\mathcal{L}_1$-Adaptive Controller and MRAC for Sinusoidal Input (case 4).

Table 5,
Steady-State Errors for Different Cases of Sinusoidal Input.

	Steady state error (rad)			
	Case 1	**Case 2**	**Case 3**	**Case 4**
L_1-controller	0.0265	0.0247	0.0266	0.023
MRAC	0.2749	0.2273	0.2419	0.24

8. Conclusions

1. For ramp exciting input, the results showed that $\mathcal{L}_1$-adaptive controller has better tracking performance than MRAC. For some uncertainty structure, the MRAC control signals give distorted response.
2. For step exciting input, $\mathcal{L}_1$-adaptive controller could track and gives better transient characteristics than MRAC.
3. For sinusoidal inputs, $\mathcal{L}_1$-adaptive controller gives satisfactory tracking performance for all structures of uncertainties. However, for some uncertainty structure, the MRAC control signals give distorted response.
4. For all types of inputs and for all structures of uncertainties, the $\mathcal{L}_1$-adaptive controller gives less steady state errors (nearly zero) than the MRAC.

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تصميم ومحاكاة مسيطر تكيفي نوع $\mathcal{L}_1$ للسيطرة الموضعية لمؤازرات محرك التيار المستمر

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الخلاصة

يعرض هذا العمل المسيطر المتكيف نوع ($\mathcal{L}_1$-adaptive controller) للسيطرة على الموضع المؤازر ذو تيار مستمر في حالة وجود معلمات غير مؤكدة القيمة وكذلك في حال وجود معلمات تتغير زمنياً بشكل غير معلوم

لغرض تقويم فعالية المسيطر المقترح للسيطرة الموضعية للمحرك ذي التيار المستمر فقد تم مقارنة اداؤه مع اداء المسيطر المتكيف ذات الانموذج المرجعي (MRAC) للمؤازر لنفسه.

تم دراسة مدى قابلية كلا المسيطرين للحفاظ على الاداء المطلوب مع تغير معلمات المنظومة مع اعطاء اشكال مختلفة لاشارة الادخال(اشارة خطوية، اشارة جيبية واشارة مائلة).

قد اوضحت النتائج ان المسيطر نوع ($\mathcal{L}_1$-adaptive controller) يضمن متابعة انتقالية محددة لمختلف اشكال اشارات الادخال وكذلك يضمن متابعة مستقرة . التمثيل باستخدام الحاسبة للمنظومة تحت اشراف كلا المسيطرين مع وجود اضطراب وخصائص احتكاك معينة واثبتت النتائج صحة التحليلات النظرية.