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Creeping Gait Analysis and Simulation of a Quadruped Robot

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Abstract

A quadruped (four-legged) robot locomotion has the potential ability for using in different applications such as walking over soft and rough terrains and to grantee the mobility and flexibility. In general, quadruped robots have three main periodic gaits: creeping gait, running gait and galloping gait. The main problem of the quadruped robot during walking is the needing to be statically stable for slow gaits such as creeping gait. The statically stable walking as a condition depends on the stability margins that calculated particularly for this gait. In this paper, the creeping gait sequence analysis of each leg step during the swing and fixed phases has been carried out. The calculation of the minimum stability margins depends upon the forward and inverse kinematic models for each 3-DOF leg and depends on vertical geometrical projection during walking. Simulation and results verify the stability insurance after calculation the minimum margins which indicate clearly the robot COG (Center of Gravity) inside the supporting polygon resulted from the leg-tips.

Keywords: *Quadruped robot Creeping Gait, Stability Margins, Quadruped robot Simulation.*

1. Introduction

Leg-based walking systems utilization has turned out to be very familiar with the robotics field. These systems present better adaptability over wheeled systems, particularly when dealing with irregular territories [1]. The ability of legged robots in dealing with rough territories and obstacles which also have the planning technique of the standard walking gaits for a quadruped robot [2, 3]. Among legged robots, quadrupeds have the preferred of having a less complex structure while fulfilling the requirements for usage of a statically stable gait during walking, resulted from the stable support polygons. A lot of researches about the quadruped locomotion frequently have its origins in the investigation of natural biological walking gaits. These studies have prompted to the improvement of both statically stable walking gait and dynamic running gaits for legged robots [4]. The main point is the need to design and implement a robust and statically stable gait during the quadruped robot walking. The property of their gaits would be repeating the ability of step sequence and stability over irregular territories [5]. Step sequences for quadrupeds, and the criteria for stability verification on an inclined terrain, as well as the talking about moving between varieties in gait and omnidirectional statically stable walking [6]. Stability criteria have some elements such as COG (Centre of Gravity), Support Polygon and Stability Margin which is also talked about the gait transitions performed by the creeping gaits [7, 8]. The generation and the sequence of such leg motion in the quadruped robot is called gait [9]. There are many motions that quadruped robots doing, such as stair climbing with many performances [17], flying trotting [19], jumping [20], and walking/trotting over obstacles [18].

Generally, the gait of quadruped robots can be separated for two groups: the first one is the statically stable gait and the second one is the

Dynamic running gaits, depending on the gait type [10]. Quadruped body must be stable and have the ability during any transition gait for moving from one position to another. The statically stable gait is simpler and slower contrasted with dynamic gaits. It indicates that the COG projection of quadruped robot is on the ground at instance time is inside support polygon during the walking. Among these properties, the quadruped robot meets the necessity of both force and moment adjustment during the walking gait and here the quadruped robot is said to be statically stable. Dynamic gaits such as the trotting gait [11] aim to the reality that if the COG projection of the quadruped robot on the ground is not inside the support polygon framed by the leg tips. In this case, the quadruped robot is in the unbalanced period and has a tendency of dumping. Then the quadruped robot in case of the dynamic running gaits needs to satisfy the dynamically stable requirement rather than static stability analysis. Most of the quadruped robots can be considered as statically stable, when there are three legs at least should be stayed in contact with the ground at the same instant time while another leg is swinging in the air. The most biological statically stable gait that matches the animals or insects is the creeping gait.

In this paper, the problem of static stability analysis is achieved. This is sufficient for ensuring safety walking. During the movement of the quadruped robot, the motion will be stable if and only if the body center of gravity COG is either inside the supporting triangle in case of one leg in a transition phase or inside the supporting polygon in case all legs are on the ground. At the time period of the quadruped robot is walking the problem is how to produce and control sequence of putting and lifting all the legs extend at any time moment. The quadruped robot walks with stable steps if the stability margins have positive values, another case the robot will be unstable and the walking gait will fail.

2. Quadruped Robot Description

To concentrate on the general guidelines of quadruped robot motion gaits, though, different models with some acknowledgment attributes is embraced. These attributes have three criteria [13]

1. The standard quadruped walking at constant speeds, at steady state movement, over an even supporting surface. In this case, the legs of the quadruped robot are repeated periodically during the locomotion.
2. The standard quadruped robots are symmetrical in both longitudinal and lateral directions. This property leads to the facts that the design is adapted to have a similar structure for every leg with less the mechanical complexity of the system and then less in cost .
3. In case of a regular shape quadruped robot, COG located at the robot geometrical center of the quadruped robot body, which is known as COG the center of gravity.

The quadruped robot is shown in Figure (1) demonstrates the typical model of a quadruped robot which represented the above three criteria. In this research, the motion task assumption includes that the quadruped robot walks towards the X-axis only. This means that there is no change in the Y-axis stride. So the vertical projection of the stride for all legs is a line. These stride lines are situated in the longitudinal direction to fulfill the condition of forwarding walk.

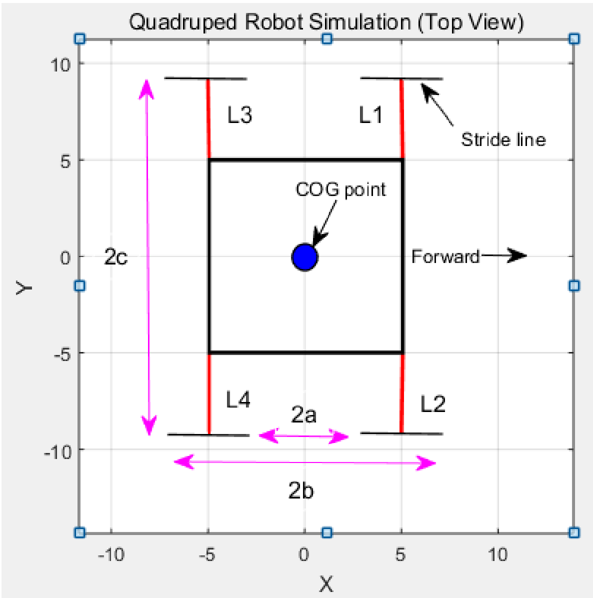


Fig. 1. Quadruped robot (top view) illustrate the length (2b) and the width (2c) and the COG point

From Figure (1), the distance for all legs from the closest point of the stride line to the COG projection is (**a**), the distance for all legs from the farthest point of the stride line to the COG projection is (**b**)and (**c**) is the distance between two front stride line or two back stride line. The definition of stride line for every leg in X-axis is [13]

Stride line $(\lambda) = b - a$ …(1)

When the quadruped robot body is moved the COG is moved, correspondingly followed by the four legs moving. According to the gait sequence of quadruped robot creeping gait, there is no necessity to move the robot body and legs at the same time [14]. The forward walking is towards the X-axis. In the first origin support pattern, the positions of the four legs at (x1, c) (x2,-c) (x3, c) (x4,-c), and the length of stride is λ. By this manner, the positions of every leg after one step movement are located at (x1+ λ, c) (x2+ λ, -c) (x3+ λ, c) (x4+ λ,-c). These locations should be matching the stride constraint as follows:

$$a \le x_1 \le b$$

$$a \le x_2 \le b$$
$$-b \le x_3 \le -a$$
$$-b \le x_4 \le -a$$
$$a \le x_1 + \lambda \le b$$
$$a \le x_2 + \lambda \le b$$
$$-b \le x_3 + \lambda \le -a$$
$$-b \le x_4 + \lambda \le -a$$

Along these considerations, the largest stride of the reachable area for a quadruped robot can be calculated from equation (1). Most of the quadruped robots have the value of length equals to 2 b and the value of width equals to 2 c. henceforth, the ratio between the length and width is equal to c/b.

In this paper, each leg contains three joints 3-DOF called (Coxa-joint, Femur-joint, and Tibia-joint). So the total number of DOFs is equal to 12. The overall the quadruped robot is showing in Figure (2). Where in Figure (2-a) shows the main part in the quadruped robot is represent a mimic of the natural anatomy of legged animals. From Figure (2-a), Leg1 is the right front (RF) side and Leg3 is the right rear (RR) side and Leg2 is the left front (LF) side and Leg4 is the left rear (LR) side. In this Figure, it is illustrated and guaranteed to be inside the supporting polygon. The forward direction of motion of the quadruped robot, in case of the forward direction of motion, is set to be with the X-axis direction and the lateral motion of the quadruped robot will be in Y-axis. The quadruped body has a symmetrical dimension in X and Y axes. This gave the robot better stability during the walking and making it statically stable. In Figure (2-b) shows each leg has three links and joints (Coxa, Femur, and Tibia).

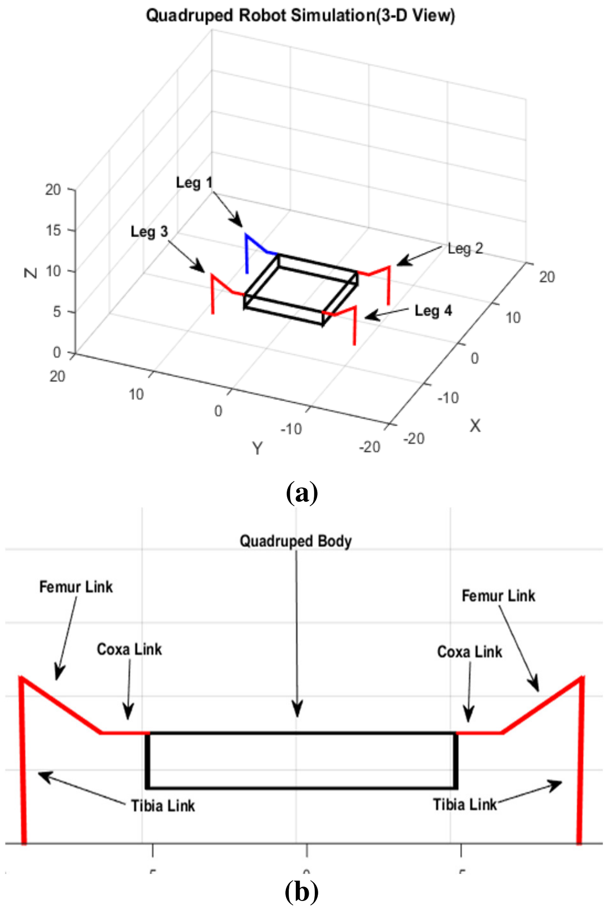


Fig. 2. Quadruped Robot Simulation Using Matlab (a) quadruped robot with four legs labeling (b) showing the three joints (Coxa, Femur, Tibia) in each leg.

To grantee that the quadruped robot is at stable situation during its movement, the position of each leg plays a very important rule of the stability calculations. In order to ensure the static stability condition of the quadruped robot, the forward kinematics and inverse kinematics is needed priory before analysis.

3. Creeping Gait Analysis and Sequence Description

The quadruped robot gaits can be classified into three types of main gaits depending also on the value of the duty factor (β_i), where i=1,..4, like: crawl gait, running gait, and the galloping gait. The duty factor (β_i) is the ratio between the time period of one leg spend in the air to the time period spend on the ground, according to the creeping gait sequence, creeping gait has duty factor equal to 0.75. The running gait has duty factor ranging between 0.5-0.75, and the galloping gait has a duty factor less than 0.5. These gaits that are utilized by quadruped animals have regularly been given names, which generally known as creeping gait, trotting gait and bounding gait [١6]. Some of these gaits are utilized by some mammals. For a very slow walking, animals such as elephants are using the creeping gait. The advantage of this gait, that it can be generally statically stable movement and it can be used at the range of low-speed motions. The creeping gait needs at least three legs on the ground at the translation phase of the other leg in the air. This condition is important for achieve a statically stable gait. The creeping gait used the mechanism that the back leg is touching-down the ground followed by the front leg in the same side leaving the ground. After the front leg is touching-down the ground, the back leg in the other side of the body will begin to leave the ground, and so on, this sequence is repeated continuously during the locomotion of the quadruped robot.

There are six types of legs sequences for quadruped robot. In this paper, the leg swinging sequence is (RR→RF→ LR→LF) where (R: right, L: left, R: rear, F: front). Where RR is leg4, RF is leg2, LR is leg3, and LF is leg1. The main feature of this specific sequence is to have a safe walking that ensures the body robot of moving forward at the instance time [12]. There are two Cartesian coordinate reference frames in the quadruped robot locomotion: the first one is the ground frame, the second one is the coxa joint frame related to the robot body. In these coordinate frames, the front side of the body is set towards the X-axis, while the body left side direction is set towards the Y-axis. In this way, these two frames are important for the computations that ensure the COG location is inside the robot body projection during leg placing and lifting. [13]

4. Kinematics Modeling of a Quadruped Robot

4.1 Forward Kinematics

Depends on the leg design, a quadruped robot relies on the configuration of each leg. Since all parts of walking are represented by the physical constraints of the leg. Each leg has three-revolute joints named in the kinematical chain (r1, r2, r3). These joints have been chosen for every leg mechanism. The basic idea is to model the robot leg structure as the biological leg structure for animals or insects. The forward kinematics is applied to drive the geometrical model for every leg related to the center of the robot body, and then find the position and orientation of the end-tip (xi, yi, zi) where i=1,2,3, and 4, for each leg, the leg structure appeared in Figure (3)

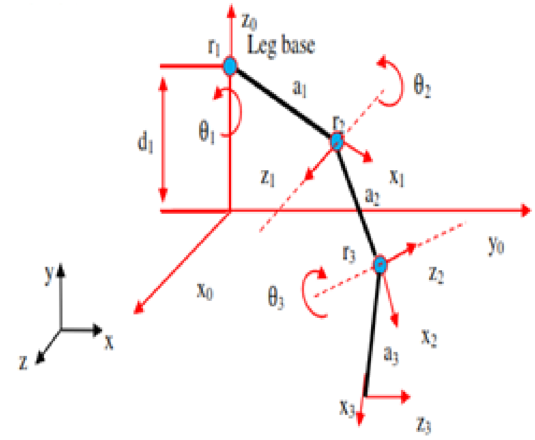


Fig. 3. Coordinate frames for one leg of a quadruped robot. [9]

The D-H (Denavit-Hartenberg) parameters of the leg are shown in Table (1). The leg's frame of this robot begins with the link zero which is the point on the robot body. The leg is jointed as link one is the coxa link, link two is the femur link and link three is the tibia link. Legs are set to be symmetrically around the coordinate axis in the direction of movement. In our case, the direction of motion is the X-axis. The general frame for the transformation matrix from a link i to a link (i-1) utilizing the D-H parameters is given in Equation (2.1) [9]:

The definition of the transformation matrix is a multiple series of transformation along:

1. Translate d_i along z_{i-1} axis.
2. Rotate θ_i about z_{i-1} axis.
3. Translate α_i along x_{i-1} axis.
4. Rotate α_i about x_{i-1} axis.

So, for one link and joint the translations and rotations are illustrated in the transformation matrix as [21]:

$$T_i^{i-1} = \begin{bmatrix} cos\theta_i & -sin\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(2)$$

The link parameter table of one leg in the quadruped robot is

Table 1,
The Denavit-Hartenberg parameters table for one leg in our quadruped robot

Link No.	Link name	α_i (deg)	a_i (cm)	d_i (cm)	θ_i (deg)
1	Coxa	90	a_1	d_1	θ_1
2	Femur	180	a_2	0	θ_2
3	Tibia	0	a_3	0	θ_3

Where d_1 = (9 cm), a_1=2.5 cm a_2= 5 cm, and a_3= 9 cm. d_1 is the distance from Coxa joint to the ground. a_i are the lengths of the leg links.

Now, obtaining the overall transformation matrixes by making a product of three transformation matrixes as the following:

$$T_{coxa}^{base} = T_{coxa}^{femur} * T_{femur}^{tibia} \quad \ldots(3)$$

From Equation (2), the transformation matrixes for one quadruped's leg are describing as below:

$$T_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(4)$$

$$T_2^1 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(5)$$

$$T_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(6)$$

By multiplication the Equations (4) and (5) ($T_1^0 * T_2^1$) yields:

$$\begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1c_2 & c_1s_2 & -s_1 & c_1(a_2c_{2+}a_1) \\ s_1c_2 & s_1s_2 & c_1 & s_1(a_2c_{2+}a_1) \\ s_2 & -c_2 & 0 & a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(7)$$

In the same method and by multiplication Equations (7) by (6) $(T_1^0 * T_2^1) * T_3^2$ yielding equation (8):

$$\begin{bmatrix} c_1(c_{2-3}) & c_1s_{2-3} & -s_1 & a_3c_1(c_{2-3}) + c_1(a_2c_{2+}a_1) \\ s_1(c_{2-3}) & s_1(s_{2-3}) & c_1 & a_3s_1(c_{2-3}) + s_1(a_2c_{2+}a_1) \\ s_{2-3} & -(c_{2-3}) & 0 & a_3(s_{2-3}) + a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(8)$$

Where:

$c_i = cos\theta_i$ and $s_i = sin\theta_i$, for i=1,2, and 3.

$(c_{2-3})= \cos(\theta_2 - \theta_3)$

$(s_{2-3})= \sin(\theta_2 - \theta_3)$

From the Figure (3) and by using the equation (8), the coordinate position of each leg-tip can be calculated as the following:

$$x_i = cos\theta_1(a_3(cos\theta_{2-3}) + (a_2cos\theta_{2+}a_1)) \quad (9)$$

$$y_i = sin\theta_1(a_3(cos\theta_{2-3}) + (a_2cos\theta_{2+}a_1)) \quad (10)$$

$$z_i = a_3(sin\theta_{2-3}) + a_2sin\theta_2 + d_1 \quad (11)$$

Where: (x_i,y_i,z_i) is the coordinates of the leg-tip, i=1…4.

4.2 Inverse kinematics

The inverse kinematics method formulated the joint angles from a given position and orientation of the legs tip. These position and orientation are achieved from the forward kinematics.

The objective is to find the leg three joint angles θ_1,θ_2 and θ_3, relating to the required of the leg tip position [15] as shown in Figure (4).

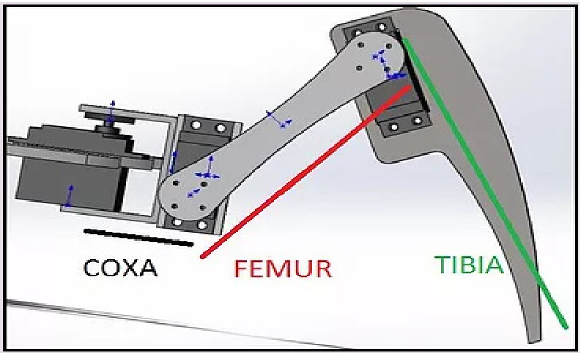


Fig. 4. Front view showing one leg of the quadruped and its links (Coxa, Femur, Tibia) [15]

To find the inverse kinematics in each leg joint, the method simplifies this problem from 3D problem view into two 2D problem view, after that, using the geometrical analysis to solve and find the solution for (θ_1,θ_2 and θ_3), as showing in Figure (5)

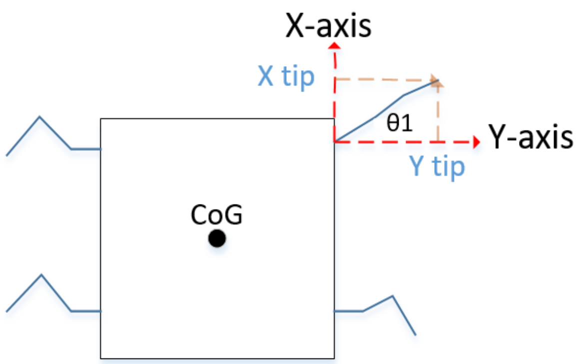


Fig. 5. Quadruped top view showing θ_1

From Figure above, θ_1 can be calculated, as the following:

$\frac{x}{y} = \tan(\theta_1)$

Thus,

$$\theta_1 = tan^{-1}(\frac{x}{y}) \qquad ...(12)$$

To find the other two angels which are θ_2 and θ_3 . These angles are in the same (Y-Z) plane. To find θ_2, divide this angle into $\alpha_1$and α_2 to simplify the problem. To find α_1 firstly working out on the distance L to get the angle α_1 as showing in the following Figure (6).

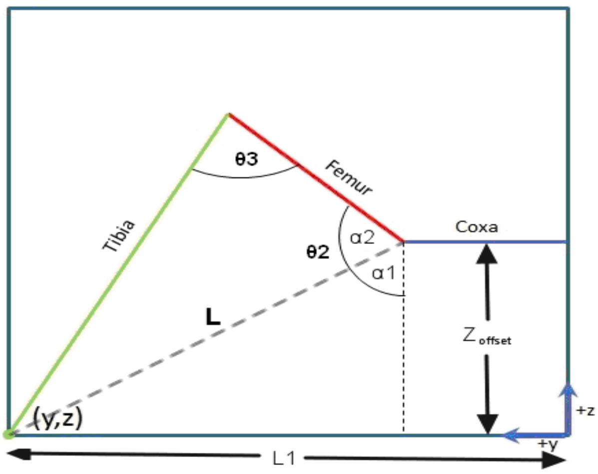


Fig. 6. Quadruped robot one leg side view showing links (Coxa, Femur, Tibia) and θ_2 and θ_3 [15]

From the Figure above calculate the followings:

$$L = \sqrt[2]{Z_{offset}{}^2 + (L_1 - c)^2} \qquad ...(13)$$

Where c is the Coxa link length, and:

$$\alpha_1 = cos^{-1}(\frac{Z_{offset}}{L}) \qquad ...(14)$$

From the Cosine rules, α_2 and θ_3 angles can be calculated. From Figure (6) shown above the three sides of a triangle are (Femur, Tibia, and L). To calculate α_2 :

$$T^2 = F^2 + L^2 - (2 * F * L)\cos(\alpha_2)$$

Thus:

$$\alpha_2 = cos^{-1}\left(\frac{F^2+L^2-T^2}{2*F*L}\right) \qquad ...(15)$$

Where: F is the length of Femur link.

T is the length of Tibia Link.

Although, $\theta_2$is equal to:

$$\theta_2 = \alpha_1 + \alpha_2$$

$$\theta_2 = cos^{-1}\left(\frac{Z_{offset}}{L}\right) + cos^{-1}\left(\frac{F^2+L^2-T^2}{2*F*L}\right) \qquad ...(16)$$

Finally, θ_3 is calculated as:

$$\theta_3 = cos^{-1}\left(\frac{F^2+T^2-L^2}{2*F*T}\right) \qquad ...(17)$$

5. Stability Analysis for Quadruped Robot Creeping Gait

5.1 Static Stability Criterion

With a specific goal for the quadruped robot to walk with statically stable movements, the COG projection on the ground should be inside the supporting polygon framed by the leg tips. This condition is considered as the main criteria for the statically stable walking. When this condition is completely satisfied, the quadruped robot won't overturn because of the gravity. Furthermore, this analysis can be made for the legs gait sequence as discussed to be (leg4→leg2→leg3→leg1). During the swing period of leg4, the area of support triangle is framed by the tips of the supporting legs (leg1, leg2, leg3), and it has a symbolic as Tr1. In the swing period of the leg2, the support triangle is shaped by the tips of the supporting legs (leg1, leg3, leg4). And this triangle has a symbolic as Tr2. At this moment the intersection of the Tr1 and Tr2 resulted in another triangle named as DST1. In which the COG projection must be inside this triangle to ensure the static stability when leg4 and leg2 are in swinging phase. By the same way, the edges of supporting triangles Tr3 and Tr4 resulted during the period of swing leg3 and leg1, respectively. The intersection of the Tr3 and Tr4 resulting another triangle named as DST2. By the same condition that the COG must be inside the triangle DST2 triangle when leg3 and leg1 are in swinging phase. Many series of DSTs (Double Supporting Triangles) will be shaped during the movement of the quadruped robot, thus, to ensure the statically stable condition the planned trajectory of the COG projection must be planned along the DST's progression sequence:

DST1→DST2→DST3→DST4.... ...(18)

This grantee that the locomotion of the quadruped robot is statically stable [10]. The DST sequence of the quadruped robot motion is illustrated in Figure (7).

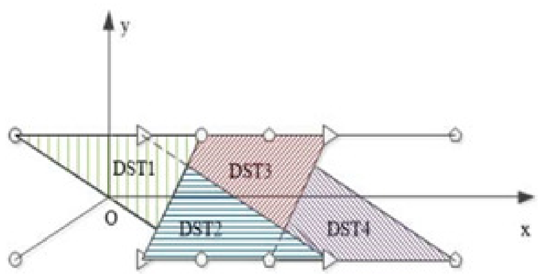
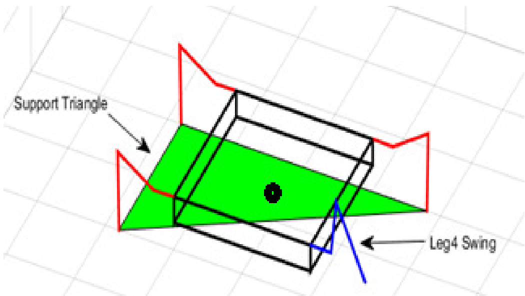


Fig. 7. Series of Double Support Triangles on the ground frame [10].

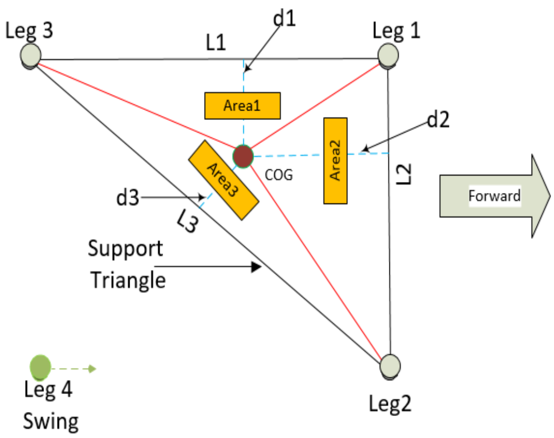
The main advantage of making the walking locomotion of the quadruped periodically repeated is to achieve a constant velocity which leads to making the acceleration on the quadruped body is equal to zero, which reduces the disturbances on the swinging legs.

5.2 Stability Margin Analysis

In case of quadruped robot creeping gait walking, and according to static stability condition, the stability margins are utilized the shortest distance between the COG projection of the quadruped robot to the boundary limits of the supporting pattern [13]. The following Figures explaining the mathematical model of a quadruped robot with the analysis of stability margins where the calculations of these stability margins as follows



(a)



(b)

Fig. 8. Quadruped robot when leg4 is swinging (a) quadruped robot (b) Quadruped support triangle

The first case, shown in Figure (8-a) when the quadruped robot leg4 swing in the air and legs (1, 2, and 3) on the ground. The supporting area will have three triangles, each triangle has a defined area (Area1, Area2, Area3) and the minimum distance from one of the supporting lines (L1, L2, L3) to the perpendicular COG projection on the ground which is denoted as (T1, T2, T3). Then the analysis of the stability margins at this period is showing in Figure (8-b) as:

$$Area_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ x_{cog} & x_1 & x_3 \\ y_{cog} & y_1 & y_3 \end{bmatrix}$$

Where (x_{cog}, y_{cog}) is the coordinate of the center of gravity of the quadruped robot on the ground. (x_1, y_1) is the coordinate of leg1 position. (x_3, y_3) is the coordinate of leg3 position.

So, by expanding and simplify this matrix $Area_1$ can be calculated as the following:

$$Area_1 = \frac{1}{2}\{(x_1 - x_{cog})(y_3 - y_{cog}) - (x_3 - x_{cog})(y_1 - y_{cog})\}$$

$$L_1 = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2}$$

$$d_1 = 2 * (\frac{Area_1}{L_1}) \qquad ...(19)$$

By the same way, $Area_2$ and $Area_3$ can be calculated:

$$Area_2 = \frac{1}{2}\{(x_1 - x_{cog})(y_2 - y_{cog}) - (x_2 - x_{cog})(y_1 - y_{cog})\}$$

$$L_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d_2 = 2 * (\frac{Area_2}{L_2}) \qquad ...(20)$$

$$Area_3 = \frac{1}{2}\{(x_2 - x_{cog})(y_3 - y_{cog}) - (x_3 - x_{cog})(y_2 - y_{cog})\}$$

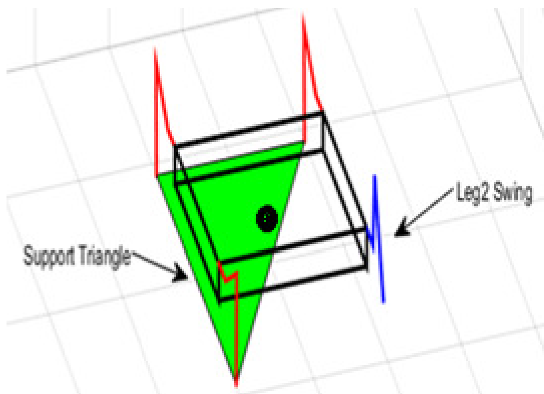
$$L_2 = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$$

$$d_3 = 2 * (\frac{Area_3}{L_3}) \qquad ...(21)$$

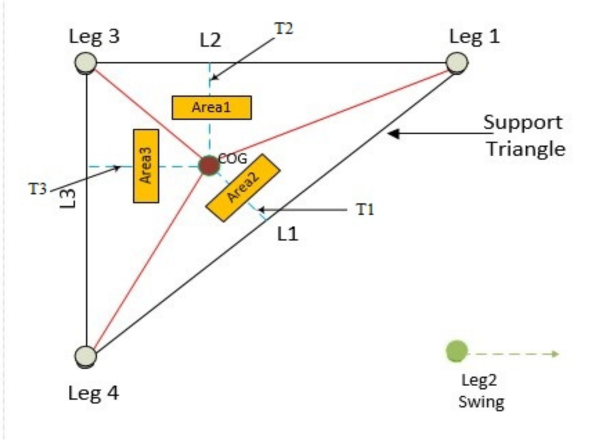
Finally, the first stability margin sm_1 is the minimum of these three margins:

$$sm_1 = min(d_1, d_2, d_3) \qquad ...(22)$$

According to the quadruped robot creeping gait sequence, the robot swinging leg 2 while the other legs (leg1, leg3, and leg4) are on the ground, sm_2 can be found as the following



(a)



(b)

Fig. 9. Quadruped robot when leg2 is swinging (a) quadruped robot 3D view (b) Quadruped support triangle.

The second case, showing in Figure (9-a) when the leg2 is swinging in the air and the other legs (1, 3, and 4) will be on the ground. Then the analysis of the stability margins at this period is showing in Figure (9-b) as:

$$Area_1 = \frac{1}{2}\{(x_1 - x_{cog})(y_3 - y_{cog}) - (x_3 - x_{cog})(y_1 - y_{cog})\}$$

$$L_1 = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2}$$

$$T_1 = 2 * (\frac{Area_1}{L_1}) \quad \ldots(23)$$

By the same way, $Area_2$ and $Area_3$ can be calculated:

$$Area_2 = \frac{1}{2}\{(x_1 - x_{cog})(y_4 - y_{cog}) - (x_4 - x_{cog})(y_1 - y_{cog})\}$$

$$L_2 = \sqrt{(x_1 - x_4)^2 + (y_1 - y_4)^2}$$

$$T_2 = 2 * (\frac{Area_2}{L_2}) \quad \ldots(24)$$

$$Area_3 = \frac{1}{2}\{(x_3 - x_{cog})(y_4 - y_{cog}) - (x_4 - x_{cog})(y_3 - y_{cog})\}$$

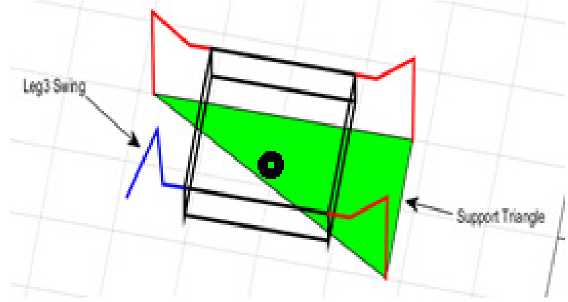
$$L_2 = \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}$$

$$T_3 = 2 * (\frac{Area_3}{L_3}) \quad \ldots(25)$$

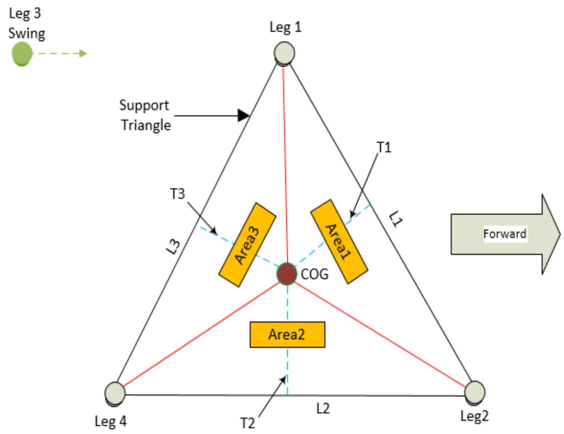
Finally, the second stability margin sm_2 is the minimum of these three margins:

$$sm_2 = \min(T_1, T_2, T_3) \quad \ldots(26)$$

According to the quadruped robot creeping gait sequence, the robot swinging leg 3 and the other legs (leg1, leg2, and leg4) are on the ground, the sm_3 can be found as the following



(a)



(b)

Fig. 10. Quadruped robot when leg3 is swinging (a) quadruped robot 3D view (b) Quadruped support triangle.

The Third case, showing in Figure (10-a) when the leg3 is swinging in the air and the other legs (1, 2, and 4) will be on the ground. Then the analysis of the stability margins at this period is showing in Figure (10-b) as:

$$\mathrm{Area}_1 = \frac{1}{2}\{(x_1 - x_{COG})(y_2 - y_{COG}) - (x_2 - x_{COG})(y_1 - y_{COG})\}$$

$$L_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$T_1 = 2 * (\frac{\mathrm{Area}_1}{L_1}) \quad \ldots(27)$$

By the same way, $Area_2$ and $Area_3$ can be calculated:

$$\mathrm{Area}_2 = \frac{1}{2}\{(x_2 - x_{COG})(y_4 - y_{COG}) - (x_4 - x_{COG})(y_2 - y_{COG})\}$$

$$L_2 = \sqrt{(x_2 - x_4)^2 + (y_2 - y_4)^2}$$

$$T_2 = 2 * (\frac{\mathrm{Area}_2}{L_2}) \quad \ldots(28)$$

$$\mathrm{Area}_3 = \frac{1}{2}\{(x_1 - x_{COG})(y_4 - y_{COG}) - (x_4 - x_{COG})(y_1 - y_{COG})\}$$

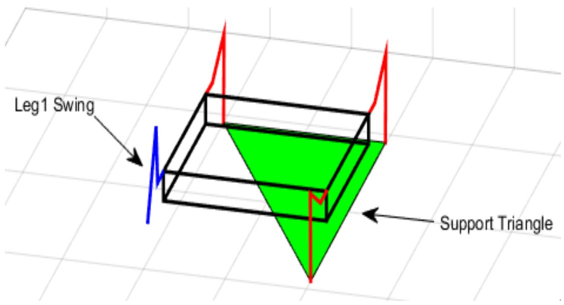
$$L_3 = \sqrt{(x_1 - x_4)^2 + (y_1 - y_4)^2}$$

$$T_3 = 2 * (\frac{\mathrm{Area}_3}{L_3}) \quad \ldots(29)$$

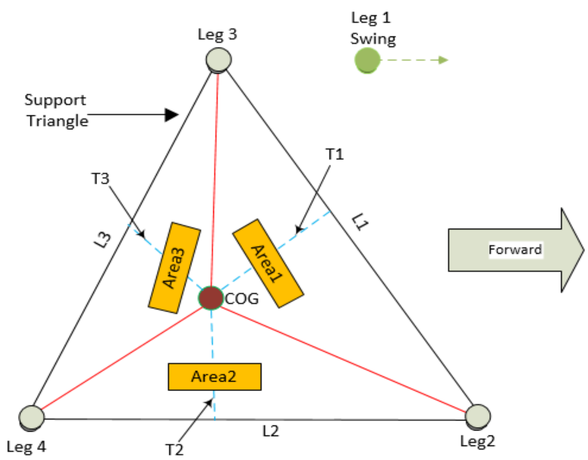
Finally, the Third stability margin sm_3 is the minimum of these three margins:

$sm_3 = \min(T_1, T_2, T_3)$ ...(30)

According to the quadruped robot creeping gait sequence, the robot swinging leg 1 and the other legs (leg2, leg3, and leg4) are on the ground, the sm_4 can be found as the following



(a)



(b)

Fig. 11. Quadruped robot when leg3 is swinging (a) quadruped robot 3D view (b) Quadruped support triangle.

The Fourth case, showing in Figure (11-a) when the leg1 is swinging in the air and the other legs (2, 3, and 4) are on the ground. Then the analysis of the stability margins at this period is showing in Figure (11-b) as:

$$\mathrm{Area}_1 = \frac{1}{2}\{(x_2 - x_{COG})(y_3 - y_{COG}) - (x_3 - x_{COG})(y_2 - y_{COG})\}$$

$$L_1 = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$$

$$T_1 = 2 * \left(\frac{\mathrm{Area}_1}{L_1}\right) \quad \text{...(31)}$$

By the same way, $Area_2$ and $Area_3$ can be calculated:

$$\mathrm{Area}_2 = \frac{1}{2}\{(x_2 - x_{COG})(y_4 - y_{COG}) - (x_4 - x_{COG})(y_2 - y_{COG})\}$$

$$L_2 = \sqrt{(x_2 - x_4)^2 + (y_2 - y_4)^2}$$

$$T_2 = 2 * \left(\frac{\mathrm{Area}_2}{L_2}\right) \quad \text{...(32)}$$

$$\mathrm{Area}_3 = \frac{1}{2}\{(x_3 - x_{COG})(y_4 - y_{COG}) - (x_4 - x_{COG})(y_3 - y_{COG})\}$$

$$L_3 = \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}$$

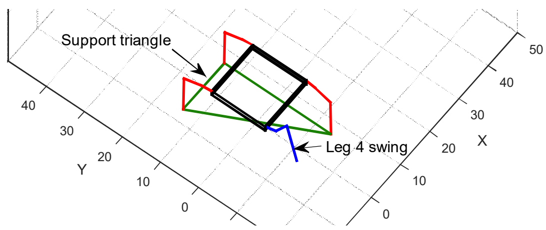
$$T_3 = 2 * \left(\frac{\mathrm{Area}_3}{L_3}\right) \quad \text{...(33)}$$

Finally, the Fourth stability margin sm_4 is the minimum of these three margins:

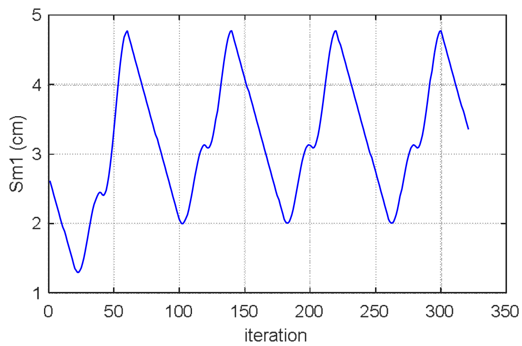
$sm_4 = \min(T_1, T_2, T_3)$ ... (34)

6. Simulation and Results

This section is showing the simulation results of the analyzed stability margins which are necessary for the quadruped robot walking stability. According to the robot legs sequence, the results are achieved as the following

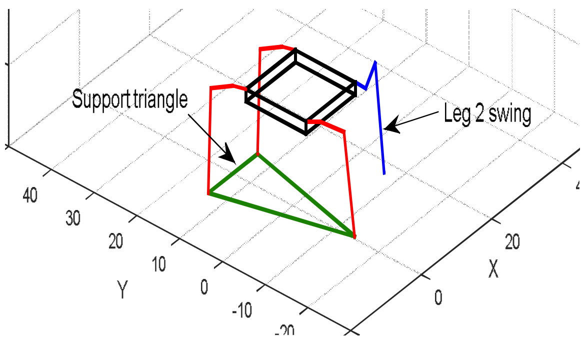


(a)

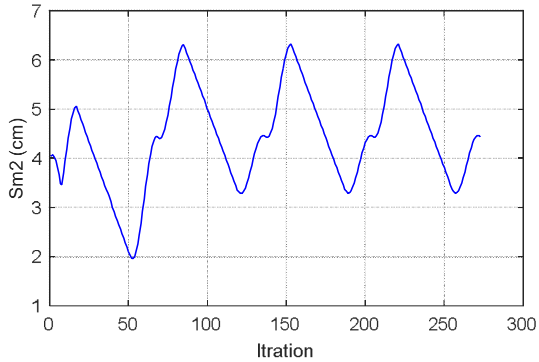


(b)

Fig. 12. Showing (a) Leg 4 in swinging phase (b) The stability margin Sm_1(cm) when leg4 is swing.

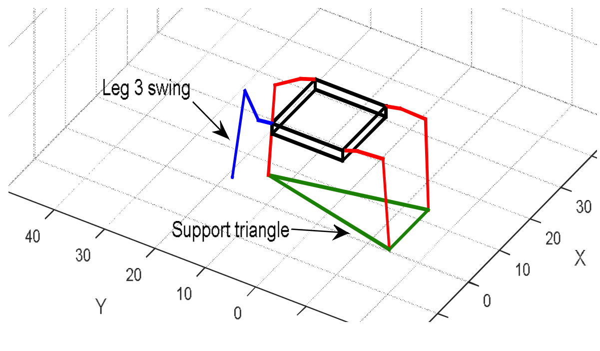


(a)

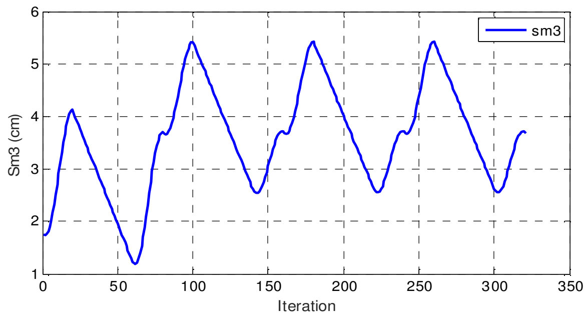


(b)

Fig. 13. Showing (a) Leg 2 in swinging phase. (b) The stability margin Sm_2(cm) when leg2 is swing.

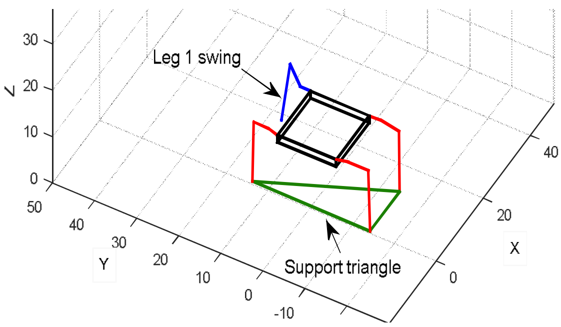


(a)

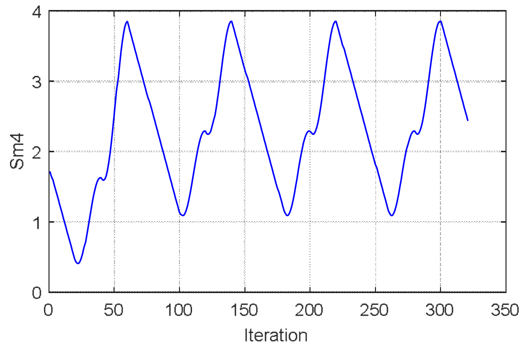


(b)

Fig. 14. Showing (a) Leg 3 in swinging phase. (b) The stability margin Sm_3(cm) when leg3 is swing.



(a)

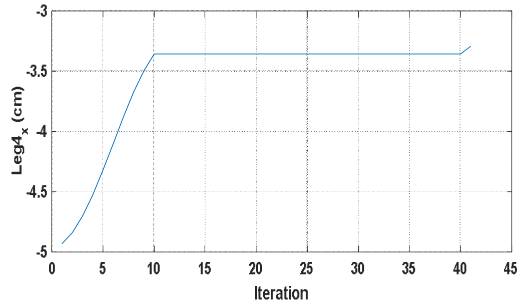


(b)

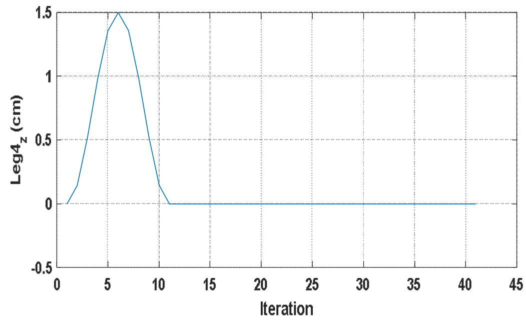
Fig. 15. Showing (a) Leg 1 in swinging phase. (b) The stability margin Sm_4(cm) when leg1 is swing.

From figures shown above, it can be seen that all stability margins (Sm_1, Sm_2, Sm_3, and Sm_4) have positive values. These values are sufficient to satisfy a stable walking for the quadruped robot creeping gait as assumed in this paper.

The results of the stable creeping gait sequence for all legs have been converted into joint angles (θ_1,θ_2,θ_3) for each leg by using inverse kinematics analysis. So, the results contain the changing of each leg three angles and the change of distance of each leg-tip along X-axis direction and Z-axis direction during the creeping gait walking of quadruped robot. The following Figures (12-15) illustrate the quadruped robot configurations and results.

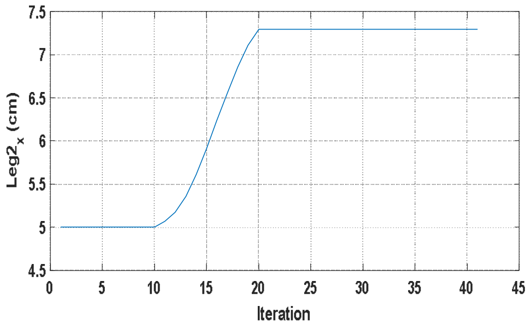


(a)

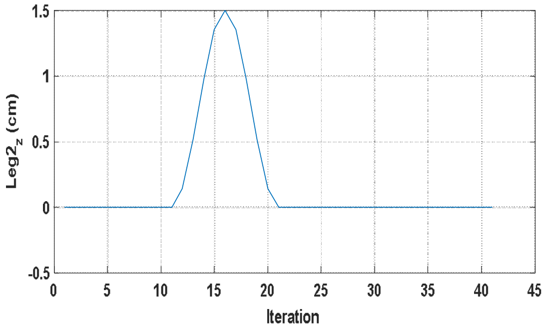


(b)

Fig. 16. When Leg 4 is swinging. (a) Leg 4 moved one step along X-axis. (b) Leg 4 moved one step along Z-axis.

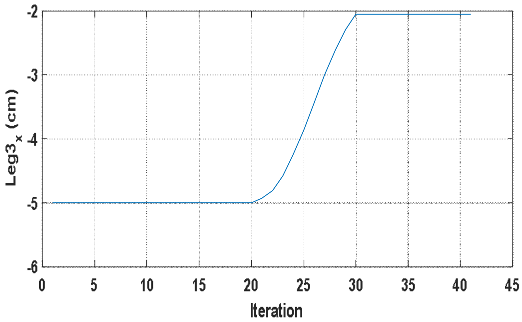


(a)

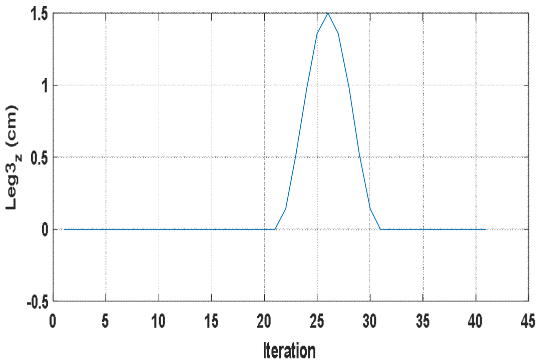


(b)

Fig. 17. When Leg 2 is swinging.
(a) Leg 2 moved one step along X-axis.
(b) Leg 2 moved one step along Z-axis.

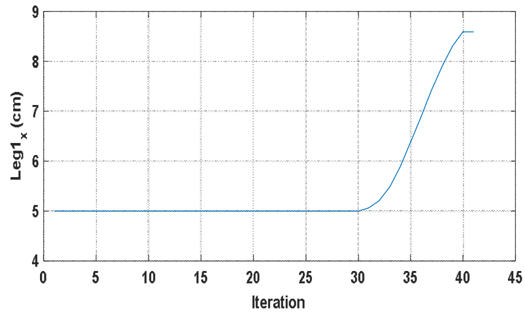


(a)

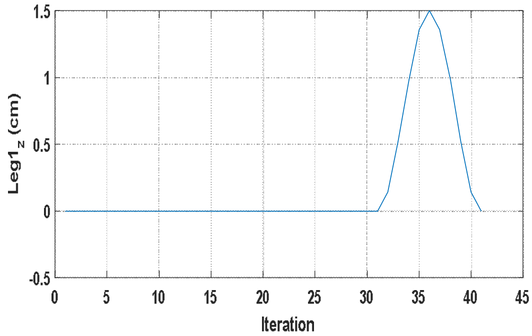


(b)

Fig. 18: When Leg 3 is swinging.
(a) Leg 3 moved one step along X-axis.
(b) Leg 3 moved one step along Z-axis.



(a)



(b)

Fig. 19. When Leg 1 is swinging.
(a) Leg 1 moved one step along X-axis.
(b) Leg 1 moved one step along Z-axis.

From above Figures it can be seen the following:

1. In Figure (16): leg4 tip is swinging and moving from (-5 cm) to (-3.5 cm) along X-axis and (1.5 cm) along Z-axis.
2. In Figure (17): leg2 swinging and moving from (5 cm) to (7.4 cm) along X-axis, also the height of the lifting tip is (1.5 cm) along Z-axis.
3. In Figure (18): leg3 swinging and moving from (-5 cm) to (-2 cm) along X-axis, also the height of the lifting tip is (1.5 cm) along Z-axis.
4. In Figure (19): leg1 swinging and moving from (5 cm) to (8.5 cm) along X-axis, also the height of the lifting tip is (1.5 cm) along Z-axis.

As discussed in this paper, the quadruped walking along X-axis only while the Y-axis is fixed, there is no changing in the Y-axis during the walking of the quadruped robot. Figure (20) showing this case.

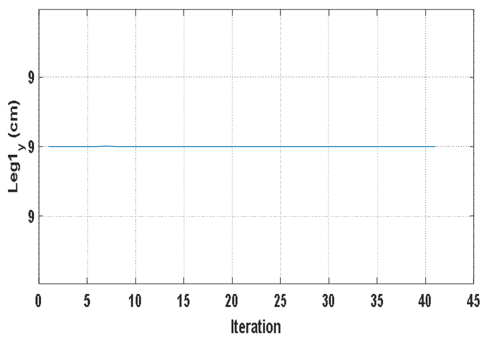


Fig. 20. Showing the changing of the leg tips along Y-axis during the quadruped walking.

During the creeping gait sequence, the quadruped robot leg angles are changing. These angles are (θ_1,θ_2 andθ_3) of each leg. This changing is showing in Figure (21)

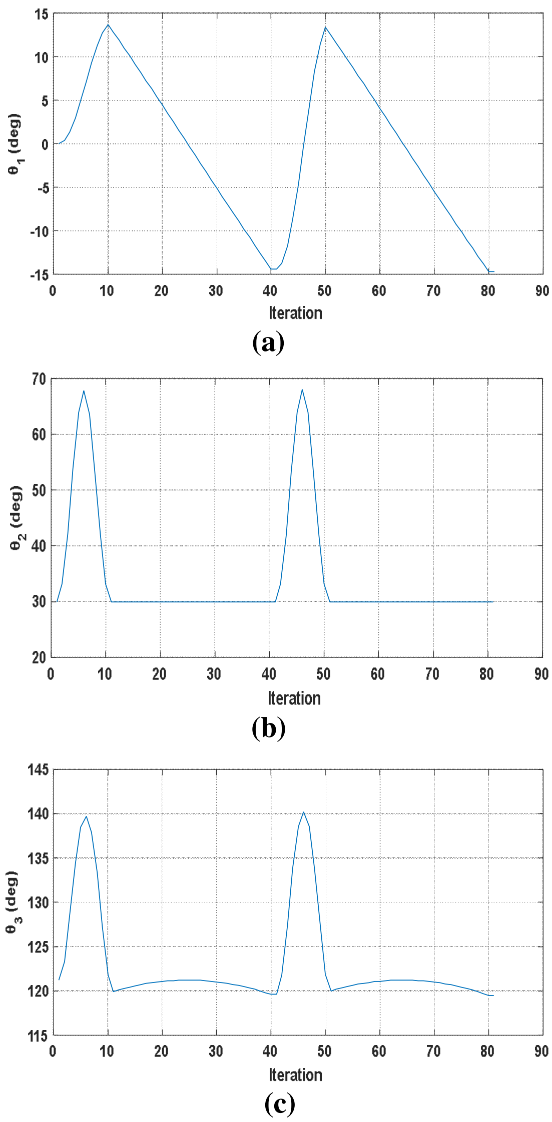


Fig. 21. changing of angles (a) Coxa-angle (b) Femur-angle (c) Tibia-angle

From Figure (21) shown above it can be seen the following:

1. Figure (21-a) is the changing of leg1 angleθ_1. This angle is varying between (15 degree) to (-15 degree).
2. Figure (21-b) is the changing of leg1 angleθ_2. This angle is varying between (30 degree) to (65 degree).
3. Figure (21-c) is the changing of leg1 angleθ_3. This angle is varying between (120 degree) to (140 degree).

7. Conclusion

The main focus of this paper is how to analyze the quadruped robot creeping gait walking and then to derive, verify and prove that the robot is statically stable during walking. In order to satisfy the goal of this work, first, the full forward and inverse kinematics model has been derived and used for walking steps stability validation. Also, the interaction between creeping gait sequence made for this robot and the geometric modeling of robot legs-tip has been fully derived for finding all the static stability margins during walking. The results clearly prove that the stability margins have the positive values which guarantee to keep robot COG inside the supporting triangle during the swing phase of one leg while other legs are on the ground. Furthermore, a future work development is needed to analyze and enhance the quadruped robot walking on the hard and rough terrain.

Notation

COG	Center of gravity
x_{cog}	Center of gravity in X-axis
y_{cog}	Center of gravity in Y-axis
x1	leg1 tip in X-axis
x2	leg2 tip in X-axis
x3	leg3 tip in X-axis
x4	leg4 tip in X-axis
y1	leg1 tip in Y-axis
y2	leg2 tip in Y-axis
y3	leg3 tip in Y-axis
y4	leg4 tip in Y-axis
T_1, T_2, T_3	distance from stability margins to the COG

Greek letter

λ	Stride line

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تحليل و محاكاة روبوت رباعي الارجل في حالة المشي

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الخلاصة

الكوادروبيد ربوت هو روبوت رباعي الارجل له القابلية على القيام بمهمات عديدة منها المشي والانتقال على الارض المسطحة فضلا عن قدرته على المشي على الارض الوعرة. يقوم هذا الروبت على محاكاة الحيوانات او الحشرات رباعية الارجل من خلال تقليد الحركات التي تقوم بها هذه الحيوانات. إن من أشهر الحركات التي يستخدمها هذا الروبوت في الانتقال هي: المشي والركض والركض مع القفز. إن حركة المشي التي تم تنفيذها في هذه الورقة هي عبارة عن تحريك رجل واحدة بعد الاخرى خلال مدة زمنية معينة وبعد وصول الرجل الى المكان المناسب تبدأ رجل اخرى بالحركة. يستخدم هذا الايقاع في الحركة لكي نضمن بقاء ثلاثة أرجل ملتصقة على الارض لتحقيق الاستقرارية خلال مدة انتقال الرجل المرفوعة. خلال عملية رفع احدى الارجل سوف يتكون مثلث ناتج من إسقاط نقاط تثبيت بقية الارجل الثلاث مع سنتر الروبوت على الارض. إن مهمة هذا الروبوت هو البقاء مستقرا خلال هذه الحركات لذلك يحتاج الى تحليل الاستقرارية عن طريق حساب الـ (Stability Margins) في هذه الورقة ايضا تم تحليل حركة الروبوت بالنسبة للمشي البطيء(Creeping Gait) ومن ثم حساب معادلات الكاينماتيك الخاصة للحصول على موقع الارجل على الارض واللازمة لتحقيق الاستقرارية الثابتة للروبوت اثناء المشي. ومن بعدها تم الحصول على النتائج اللازمة لتحقيق هذه الاستقرارية من خلال مواقع الارجل على الارض.