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Trajectory Tracking Control for a Wheeled Mobile Robot Using Fractional Order PI^aD^b Controller

Mouwafak A. Tawfik* **Emad N. Abdulwahb**** **Salah M. Swadi *****

,Department of Machines and Equipment Engineering / University of Technology*
****Department of Electrical Engineering / University of Mustansiriyah.*
*Email: drmat19853@yahoo.com
**Email: emadnaabd@yahoo.com
***Email: sallah.mahdi99@gmail.com

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Abstract

Nowadays, Wheeled Mobile Robots (WMRs) have found many applications as industry, transportation, inspection, and other fields. Therefore, the trajectory tracking control of the nonholonomic wheeled mobile robots have an important problem. This work focus on the application of model-based on Fractional Order PI^aD^b (FOPID) controller for trajectory tracking problem. The control algorithm based on the errors in postures of mobile robot which feed to FOPID controller to generate correction signals that transport to torque for each driven wheel, and by means of dynamics model of mobile robot these torques used to compute the linear and angular speed to reach the desired pose. In this work a dynamics model of mobile robot was driven for the case where the centroid of mobile robot platform is not coincide with reference frame of mobile robot (i.e. reference frame is located at midpoint of driven wheels axis), while the inertia is counted for. The Evolutionary Algorithm has been used to modified the parameters (K_p, K_d, K_i,a, and b) of the FOPID controller for wheeled mobile robot. Simulation results show the effectiveness of the proposed control algorithm: that is demonstrated by applied this controller at four case studies (Circular trajectory, S-shape trajectory, Infinity trajectory, and Line trajectory at two cases, with presences of disturbance and without), these results shows good matching between desired trajectory and simulation one while error in posture goes to zero rapidly.

***Keywords**: Mobile robot, trajectory tracking, nonholonomic systems, fractional order PID controller.*

1. Introduction

Wheeled mobile robots are increasingly present in industrial and service robotics, particularly when flexible motion capabilities are required, on reasonably smooth grounds and surfaces. Several mobility configurations (wheels number and type, their location and actuation, single or multi-body vehicle structure) can be found in the application . The most common for single-body robots are differential drive and synchro drive (both kinematically equivalent to a unicycle)

The navigation problem may be divided into three basic problems [1]: tracking reference trajectory, following a path, and point stabilization. Earlier literatures focus on kinematic model of mobile robot (e.g. steering system) of mobile robot, and perfect velocity is assumed to generate the actual vehicle control inputs [2], but there are three problems with this approach;

1- The perfect velocity tracking assumption does not hold in practice.
2- Disturbances are ignored.
3- Complete knowledge of the dynamics is needed [3].

Recently literatures focus on integration of the nonholonomic kinematic controller and the dynamics of the mobile robot. Where a sliding mode control scheme is proposed to solve the trajectory tracking problem based on the exact discrete time model of the robot [4]. A backstepping control approach tracking into account the specific vehicle dynamic to convert a steering system command into control inputs for the actual vehicle was implemented in [5]. Fuzzy logic control (FLC) is based on a back-stepping approach to asymptotic stabilization of the robots position and orientation around the desired trajectory proposed in [6]. The tracking control problem for a wheeled mobile robot, where the error in posture of mobile robot used as input control was the topic of this work.

In this paper a simple and significant controller based on FOPID for a two active wheel nonholonomic mobile robot subjected to pure rolling and no side slipping condition was proposed. The developed dynamic model includes inertia moment of driving wheels, the total mass and inertia moment which has a centroid offset from the connection center of wheeled mobile robot through the compute torque method, and a FOPID controller are applied for stabilizing the robot around its reference trajectory even in presence of disturbance/noises and modeling uncertainties.

2. Kinematic & Dynamic Modeling of Nonholonomic WMR

2.1. Kinematic of Nonhlonomic Mechanical System

In mobile robotics the mechanical behavior of the robot must be understood, both in order to design suitable robots for task and to understand how to generate control software, for instance mobile robot hardware [7]. In general kinematics deals with study of motion without considering the force that affects it. In spite of the publish work considered a reference point of the plat for of mobile robot at c.g., it is more it is more confidence to attached platform frame at **c** as shown in Figure 1 which is located in mid axis that connect driven wheels, (in other word the midpoint of the drive wheels axis **c** is considered as a reference point of plat form of mobile robot that will follows the desired trajectory and the inertia of the mobile robot is accounted for). The deferential drive robot platform as a rigid body on wheels, operating on a horizontal plane, is shown in Figure 1.

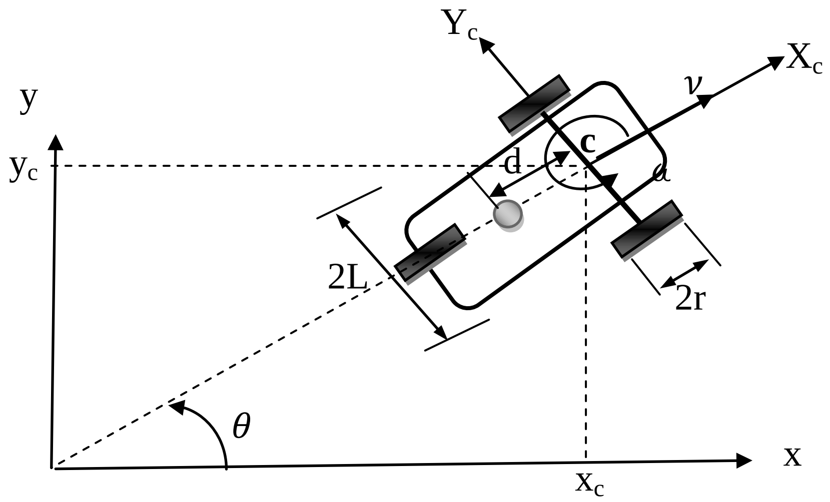

Fig. 1. A nonholonomic mobile platform.

From Figure 1, kinematic equation of mobile robot in world frame {o,x,y} can be described as follows:

$$\dot{x_c}(t) = \nu(t)cos\theta(t) \qquad \dots(1)$$

$$\dot{y_c}(t) = \nu(t)sin\theta(t) \qquad \dots(2)$$

$$\dot{\theta}(t) = \omega(t) \qquad \dots(3)$$

The kinematic equation 1,2,and 3 can be represented as follows[8,9,10] :

$$\dot{q} = S(q)V(t) \qquad \dots(4)$$

Where:

$$\dot{q} = [\dot{x}\ \dot{y}\ \dot{\theta}\ ]^T \qquad \dots(5)$$

$$V = [\nu\ \ \omega]^T$$

$$S(q) = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \quad ...(6)$$

The nonholonomic constraint states that the mobile robot can only move on the direction normal to the axis of the driving wheels, i.e, the mobile base satisfies the conditions of pure rolling and non slipping [11,12].

$$\dot{y}_c cos\theta - \dot{x}_c sin\theta = 0 \quad ...(7)$$

The constraint equation (7) can be written in matrix form:

$$A(q)\dot{q} = 0 \quad ...(8)$$

Where

$$A(q) = [\,-sin\theta(t) \quad cos\theta(t) \quad 0\,] \quad ...(9)$$

From (4 and 7) it is found that S(q) is a full rank matrix (n-m) formed by a set of smooth and linearly independent vector fields spanning the null space of A(q), i.e.

$$S^T(q)A^T(q) = 0. \quad ...(10)$$

The mobile robot shown in Figure 1 is a typical example of a nonhlonmonic mechanical system. It consists of a vehicle with two driving wheels mounted on the same axis, and a front free wheel. The motion and orientation are achieved by independent actuators, e.g., DC motors providing the necessary torques to the rear wheels.

The position of the robot in an inertial Cartesian frame {o,x,y} is completely specified by the vector q = $[x_c \quad y_c \quad \theta\,]^T$ where x_c,y_c are the coordinates of the center of mass of the vehicle, and θ is the orientation of the basis{ c , X_c, Y_c } with respect to the inertial basis.

2.2. Dynamics of Nonholonomic Mechanical System

As represented by R. Fierro, at el [5], many nonholonomic mechanical system can be describes by the following dynamic equations:

$$M(q)q\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q)\lambda \quad ...(11)$$

Where $M(q) \in R^{n\times n}$ is a symmetric positive definite inertia matrix ,$V_m(q,q^{\cdot}) \in R^{n\times n}$ is the centeriptal and corilis matrix, $F(\dot{q}) \in R^{n\times 1}$ denotes the surface friction , $G(q) \in R^{n\times 1}$ is the gravitational vector ,τ_d denotes bounded unknown disturbances including unstructured *unmodeled* dynamics, $B(q) \in R^{n\times r}$is the input transformation matrix, $\tau \in R^{n\times 1}$ is the input vector, $A(q) \in R^{m\times n}$ is the matrix associated with the constraints, and $\lambda \in R^{m\times 1}$ is the vector of constraint forces.

The Lagrange formalism is used to find the dynamic equations of wheeled mobile robot. Assume the trajectory of the mobile robot is constrained on the horizontal plane and there is no friction, then G(q) = 0 and F($\dot{q}$) = 0 [13]. Since the system cannot change its vertical position, its potential energy, U, remains constant. Now from the system shown in Figure 3, kinetic energy is equal to:

$$K.E = \frac{1}{2}m[(\dot{x}_c + d\dot{\theta}sin\theta)^2 + (\dot{y}_c - d\dot{\theta}cos\theta)^2] + \frac{1}{2}I_c\dot{\theta}^2 \quad ...(12)$$

The Lagrange's formula is given by:

$$L = K.E - U \quad ...(13)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = Q_i \qquad i = 1,2,\dots,n \quad ...(14)$$

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{x}_c}\right) = m\ddot{x}_c + md*\ddot{\theta}sin\theta + md*\dot{\theta}^2cos\theta \quad ...(15)$$

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{y}_c}\right) = m\ddot{y}_c - md*\ddot{\theta}cos\theta + md*\dot{\theta}^2sin\theta \quad ...(16)$$

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{\theta}}\right) = md*\ddot{x}_c sin\theta + md*\dot{x}_c\dot{\theta}cos\theta + md^2\ddot{\theta} - md*\ddot{y}_c cos\theta + md*\dot{y}_c\dot{\theta}sin\theta + I_c\ddot{\theta} \quad ...(17)$$

$$\frac{\partial K.E}{\partial \theta} = md*\dot{x}_c\dot{\theta}cos\theta + md*\dot{y}_c\dot{\theta}sin\theta \quad ...(18)$$

$$Q_x = \frac{\tau_R}{r}cos\theta + \frac{\tau_L}{r}cos\theta + \lambda\, sin\theta \quad ...(19)$$

$$Q_y = \frac{\tau_R}{r}sin\theta + \frac{\tau_L}{r}sin\theta - \lambda\, cos\theta \quad ...(20)$$

$$Q_\theta = \frac{\tau_R}{r}L - \frac{\tau_L}{r}L \quad ...(21)$$

Neglecting the disturbance term in Eq.(11), dynamical equation of motion for wheeled mobile robot in Figure 1. can be expressed in matrix form as follows:

$$\begin{bmatrix} m & 0 & mdsin\theta \\ 0 & m & -mdcos\theta \\ mdsin\theta & -mdcos\theta & I \end{bmatrix}\begin{bmatrix} \ddot{x}_c \\ \ddot{y}_c \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & md\theta^{\cdot}cos\theta \\ 0 & 0 & md\theta^{\cdot}sin\theta \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} \frac{1}{r}\begin{bmatrix} cos\theta & cos\theta \\ sin\theta & sin\theta \\ L & -L \end{bmatrix}\begin{bmatrix} \tau_R \\ \tau_L \end{bmatrix} + \begin{bmatrix} sin\theta \\ -cos\theta \\ 0 \end{bmatrix}\lambda \quad ...(22)$$

Where :

$I = I_c + md^2$

To find Lagrange multiplier (λ) Newten-Euler formula in lateral direction of mobile robot will be

$$\sum F_{Y_c} = m\, a_{Y_c} \qquad \ldots (23)$$

$$md\ddot{\theta} + m\ddot{x}_c sin\theta - m\ddot{y}_c cos\theta = \lambda \qquad \ldots(24)$$

Differentiating equation (7), (i.e. the constraint equation of nonholonomic wheeled mobile robot).

$$\ddot{y}_c cos\theta \; - \; \ddot{x}_c \; sin\theta \; = \dot{y}_c\dot{\theta} \; sin\theta + \dot{x}_c\dot{\theta}cos\theta \qquad \ldots(25)$$

Substitution (24) in (23) yields:

$$\lambda = md\ddot{\theta} - m\dot{\theta}\;(\dot{x}_c cos\theta + \dot{y}_c sin\theta) \qquad \ldots(26)$$

From the orthogonality properties of the null-space matrix to the constraint matrix ($S^T(q)A^T(q) = 0$) helps to eliminate the Lagrange multipliers. Differentiating equation (4), substituting this result in equation (11), and then multiplying by ($S^T(q)$), the complete equations of motion of the nonholonomic mobile platform are obtained as follows:

$$\dot{q} = S(q)V(t) \qquad \ldots(4)$$

$$S^T M S\dot{V} + \; S^T\big(M\dot{S} + \; V_m S\big)V + \; S^T\tau_d = \; S^T B\tau \qquad \ldots(27)$$

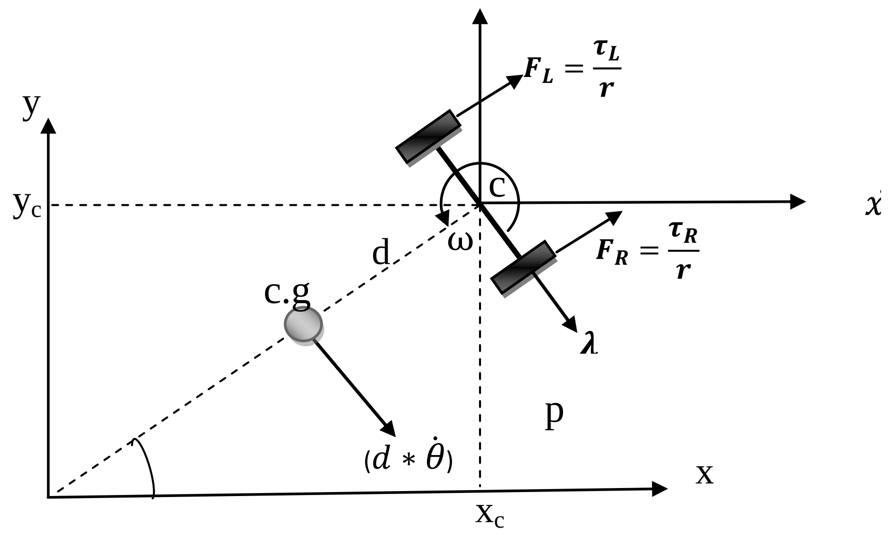

Fig. 2. Kinetic diagram of nonholonomic mobile robot.

3. Control Algorithm

3.1. Computed Torque (CT) Controller

In general, the trajectory tracking problem aims at tracking a reference mobile robot with known posture $q_r=[x_r,y_r,\theta_r]^T$. Therefore the errors between the actual and desired postures are:

$$e(t) = \; q_r - q = [x_r - x, y_r - y, \theta_r - \theta]^T \qquad \ldots(28)$$

Equation (22) can be written as follows:

$$(S^T B)^{-1} S^T\;(M\,\ddot{q} + \; V_m\dot{q}\,) = \; \tau \qquad \ldots (29)$$

the Brunovsky canonical form can be developed by differentiating e(t) twice and writing it in terms of the state x [14]:

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \qquad \ldots(30)$$

From equations 28, 29, and 30 one can found:

$$\tau = (S^T B)^{-1}(S^T M\;(\ddot{q_r} - u) + \; S^T\; V_m\;\dot{q}) \qquad \ldots(31)$$

To develop the CT controller, a linear system design procedure will be used in order to select a feedback control u(t) that stabilizes the tracking error equation, then compute the torques for the motors are computed using Eq.(31). A PID feedback for u(t) with a derivative gain matrix K_v ,a proportional gain matrix K_p ,and integral gain matrix K_i produce PID CT controller where the motor torque is equal to:

$$\tau = \big(S^T B\big)^{-1}\;\Big(S^T M\,\big(\ddot{q_d} + \; K_d\dot{e} + \; K_p e + K_i \textstyle\int e\big) + \; S^T\; V_m\;\dot{q}\Big) \qquad \ldots(32)$$

In PID controller case, three parameters K_p, K_d, and K_i should be tuned to design the controller. One of the possibilities to improve PID controllers is to use fractional – order PI^aD^b

controllers with real index of derivative and integral.

3.2. Fractional Order PI^aD^b Controller

Equation of fractional order controller was described as [15].

$$u(t) = -(K_p e(t) + K_d D_t^a e(t) + K_i D_t^{-b} e(t)) \quad \ldots(33)$$

Where e(t) is the error between a measured process output variable and a desired set point, and u(t) is the control signal. Equation (33) illustrates that the five control parameters (K_p, K_i, K_d, a, and b) should be tuned to design an accurate controller. The integral order and derivative order add more flexibility to design an FOPID controller. Continuous transfer function in s - domain of FOPID controller is be given by [15]:

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s^a} + K_d\, s^b \quad \ldots(34)$$

The orders a and b can be any real numbers. From equation (3.4) the classical PID controller can be obtained by setting a=b=1. When a=1 (or 0) and b=0 (or 1) a normal PI (or PD) controller can be obtained. Figure 3 shows that the fractional order PI^aD^b controller generalizes and expands the classical PID controller from a point to plane.

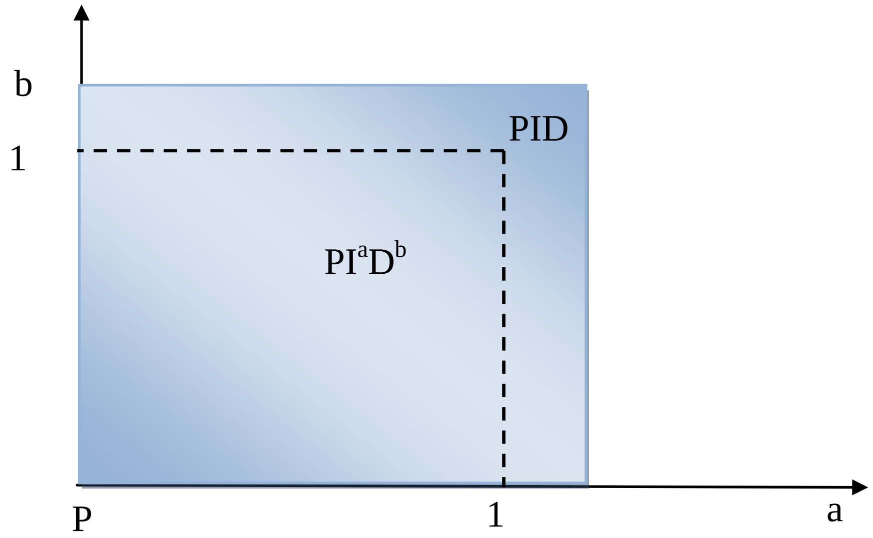

Fig. 3. Generalized FOPID controller

3.3. Evolutionary Algorithm

The Evolutionary Algorithm (EA) is an optimization algorithm that is used to search for optimal solutions to a problem [16]. This algorithm operates on a population of potential solutions applying the principle of survival of the fittest to produce best approximations to a solution. Evolutionary algorithms provide a universal optimization technique that mimics the type of genetic adaptation that occurs in natural evolution [16,17]. Unlike specialized methods designed for particular types of optimization tasks, they require no particular knowledge about the problem structure other than the objective function itself. At each iteration step, a new set of approximations is assumed by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators, such as mutation, crossover and selection borrowed from natural genetics in order to generate the new generations [18]. It shares the same features with the genetic algorithm (GA) in terms of selection.

3.3.1. Structure of Evolutionary Algorithm

By using the Evolutionary Algorithm, the natural processes such as selection, crossover, mutation and reinsertion are modeled. Figure 4 depicts the structure of a generic evolutionary algorithm[19].

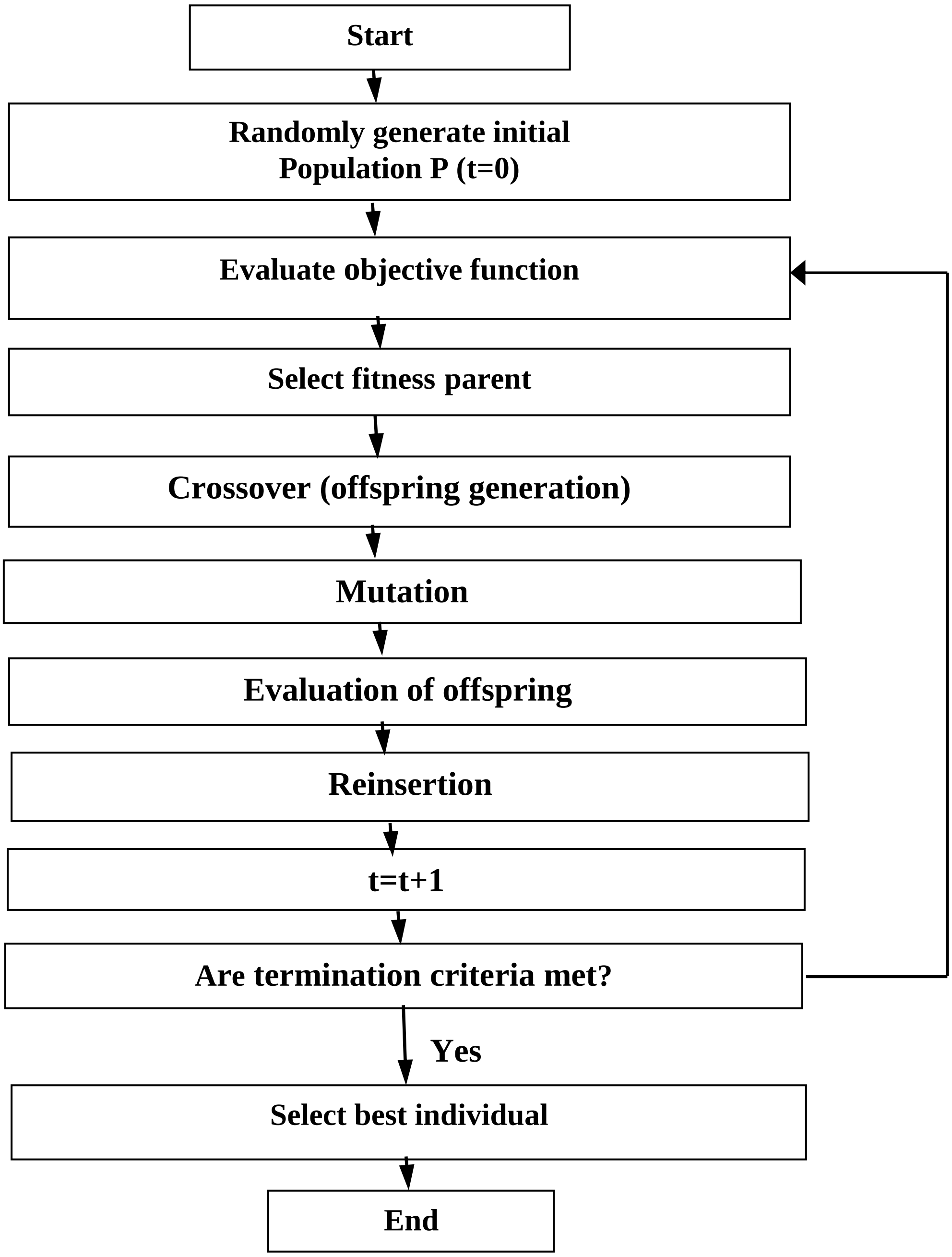

Fig. 4. Structure of an E.A. method.

3.4. Implementation of the FOPID Controller with Proposed System Using MATLAB/SIMULINK & Optiy Packages

Figure 5 shows the layout of the Simulink model of FOPID controller with controlled system. The Evolutionary Algorithm has been used to modify the parameters of the FOPID controller for mobile robot the OptiY program is used to obtain the optimal parameters of the FOPID controller. From equation 36, five parameters K_p, K_d, K_i, a and b. Each individual vector has five parameters (five-dimension variables vector). For reducing the time of optimization, the ranges of FOPID parameters are selected as:
$K_p \in [0\ 20], K_d \in [0\ 10], K_i \in [\ 0\ 5], a \in [0\ 1]$, and $b \in [0\ 1]$. To evaluate the objective function, the following function will be used:

$$J_f = \sum_{i=1}^{3} |y_i - 1|$$

The initial values and optimal values of the FOPID controller parameters are shown in Table 1 Figures 6 and 7 demonstrate the changing of the control parameters (values of K_p , K_d , K_i , a and b) during the optimization steps. After 3957 optimization steps, the parameters of the FOPID controller reach to steady state values (the optimal values) while the objective function $\mathbf{J_f}$ value reach to small value as illustrated in Figure 8.

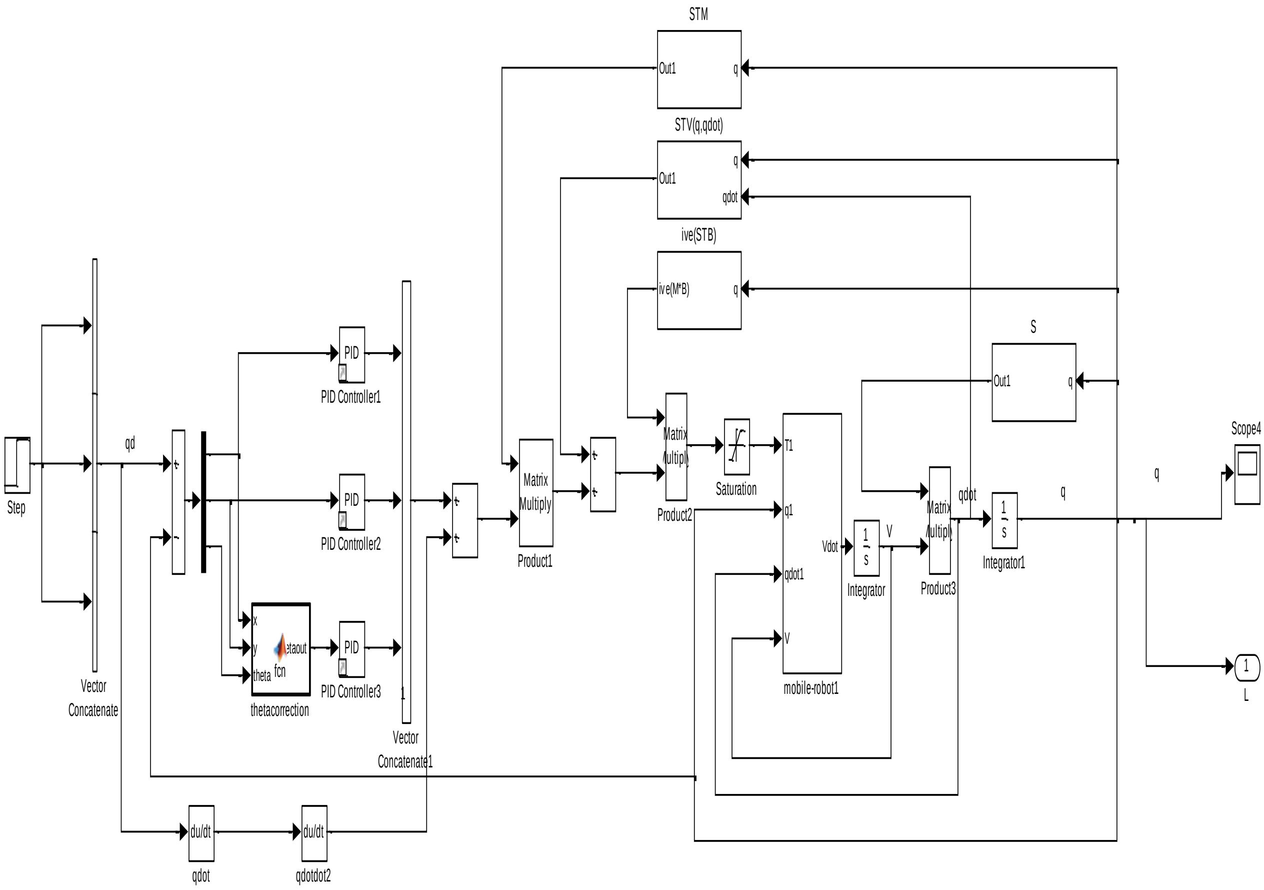

Fig. 5. Simulink model of FOPID Controller with Proposed.

Table 1,
Initial and Optimal Value of FOPID controller.

Parameter	$\mathbf{K_p}$	$\mathbf{K_i}$	$\mathbf{K_d}$	**a**	**b**
Initial Value	1	1	1	0.7	0.1
Optimal Value	5.7632	0.005596	3.296	0.893	0.394

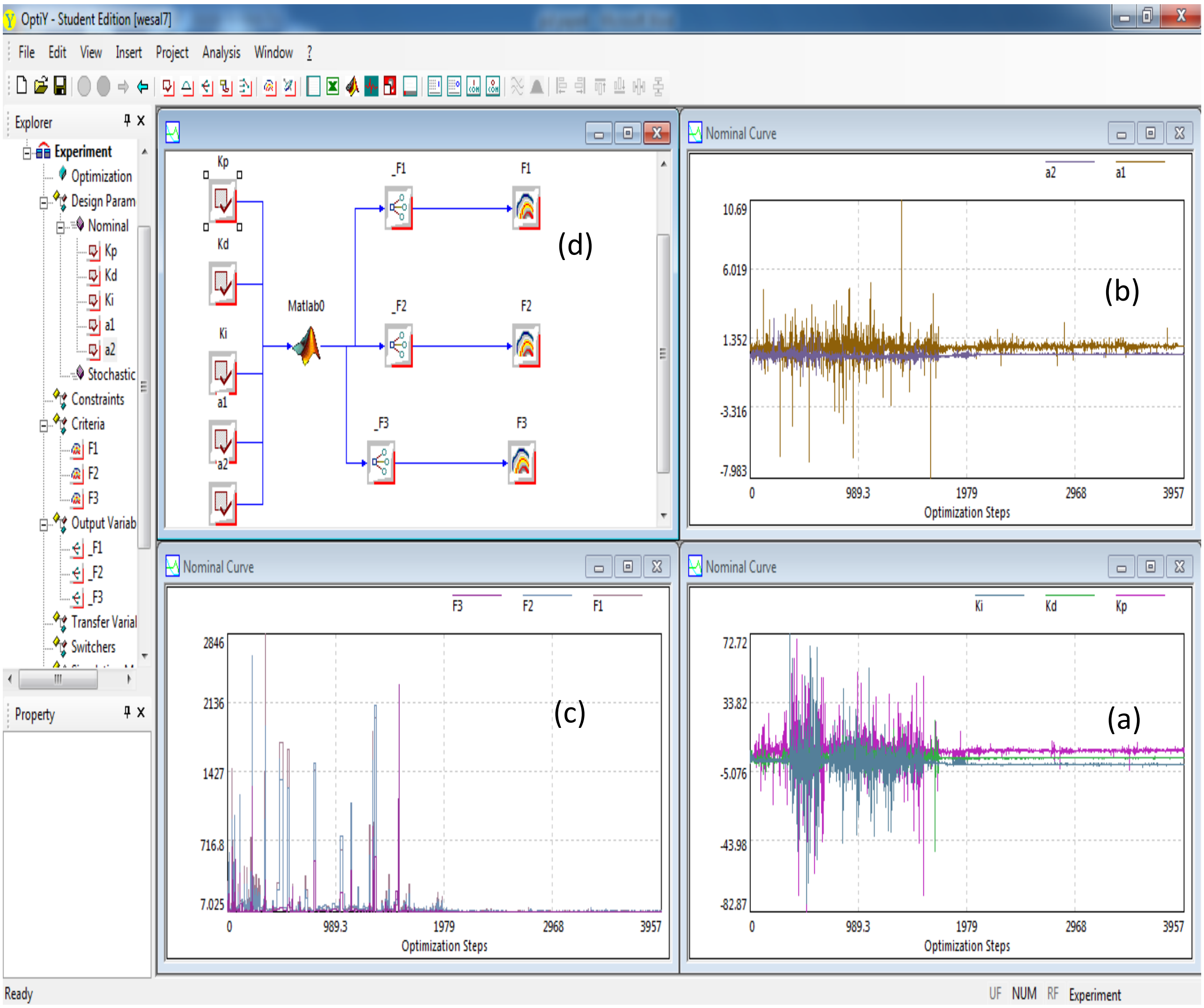

Fig. 6. Optiy program results where: a) variation of K_p, K_d, and K_i parameters during optimization process, b) variation of a1,and a2 parameters during optimization process, c) F1, F2, and F3 are objective functions, d) Optiy program workflow window.

4. Simulation Result

The trajectory tracking problem for nonholonomic vehicles is posed as follows. Let there be prescribed a reference cart

$$\dot{x_r} = v_r\, cos\theta_r,\;\; \dot{y_r} = v_r sin\theta\;, \dot{\theta} = \omega_r\;, q_r = [x_r y_r\; \theta_r\;]^T \qquad \ldots(35)$$

With $v_r > 0$ for all t, find CT such that $\lim_{t\to\infty}(q_r - q) = 0$, where v_r and ω_r are the reference velocity vector , and reference orientation respectively. The proposed controller is verified by means of computer simulation using **MATLAB/SIMULINK.** The kinematic and dynamic model of the nonholonomic mobile robot described in section 2 is used. The simulation is carried out by tracking a desired position (x,y) and orientation angle (θ) with circular , S-shape, and lemniscates trajectories in the tracking control of the robot. The parameters values of the robot model are taken from [20,21], which are as follows:

m=0.65 kg, I = 0.36 kg.cm^2, L = 0.105 m, r =0.033 m, and tacking d = 0.01m for this mobile robot.

The resulting mobile robot trajectory tracking, obtained by the proposed FOPID controller is shown in Figures 7 to 16 including trajectory tracking , tracking error, and linear and angular velocity of mobile robot, the sampling period was set to T_0 = 0.1 s. In these cases, there are no external disturbances.

1- The circular trajectory was generated from the desired linear velocity $v_d = 0.1\ m/s$ and angular velocity $\omega_d = 0.1\ rad/s$. Thus the desired circular trajectory, which has continuous gradient with rotation radius constant, can be describe as follows:

$$\theta_d(t) = \frac{\pi}{2} + \frac{t}{10} \quad \ldots(36)$$

$$x_d(t) = 1 + \cos(\frac{t}{10}) \quad \ldots(37)$$

$$y_d(t) = \sin\left(\frac{t}{10}\right) \quad \ldots(38)$$

The initial posture of the desired trajectory is $q_d(0) = [2\ ,0, \frac{\pi}{2}\ ]^T$, while the actual initial posture of the robot is $q(0) = [\ 2.1, -0.1, \pi]^T$, circular trajectory tracking simulation and posture error curves are shown in Figures 7 to 8 respectively. It's very clear that good tracking performance that achieved by means of the proposed controller.

2- the S-shape trajectory compose of nine segments five of them are line segment has constant linear and angular velocity $v_d = 0.1\ \frac{m}{s}$, and $\omega_d = 0$, while the arc segment has linear and angular velocity $v_d = 0.01 \frac{m}{s}$, and $\omega_d = 0.1\ rad/s$. The initial posture of reference trajectory is set at $q_d = [0\ ,0,0]^T$ while the actual initial posture of robot is $q(0) = [-0.1, -.1, \frac{\pi}{6}]^T$. Simulation results of both trajectory tracking and the posture error curves are shown in Figures 9 and 10, although S-shape trajectory has sudden change in path, the proposed controller shows good trajectory tracking and the posture error goes to zero in short time.

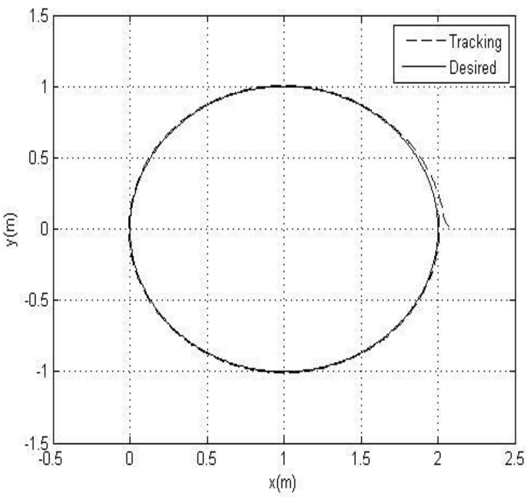

Fig. 7. Circular trajectory tracking.

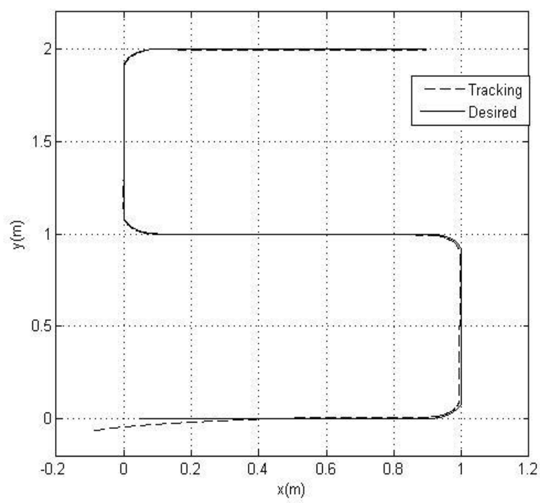

Fig. 9. S-shape trajectory tracking.

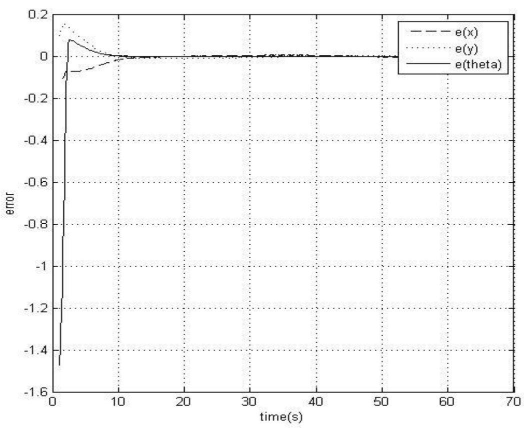

Fig. 8. Trajectory tracking error.

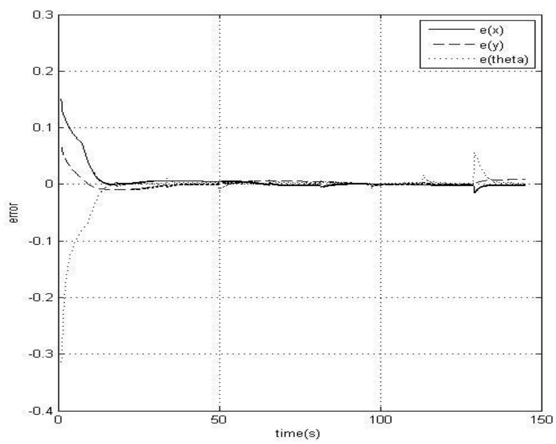

Fig. 10. Tracking error in S-shape trajectory tracking.

3- The lemniscates or infinity represent a most challenge in trajectory tracking cases, which has continuous gradient with rotation radius changes with constant linear velocity $v_d = 0.1$ m/s, this trajectory can be described by the following :

$$x_d(t) = 0.75 + 0.75 \sin\left(\frac{2\pi t}{50}\right) \quad \ldots(39)$$

$$y_d(t) = 0.75 \sin\left(\frac{4\pi t}{50}\right) \quad \ldots(40)$$

$$\theta_d(t) = \begin{cases} \tan^{-1}\frac{8\cos\left(2\sin^{-1}\left(\frac{4x}{3}-1\right)\right)}{3\times\sqrt{\left(1-\left(\frac{4x}{3}-1\right)^2\right)}} & 0 \le t < 12.5\ s \\ -\pi - \tan^{-1}\frac{8\cos\left(2\sin^{-1}\left(\frac{4x}{3}-1\right)\right)}{3\times\sqrt{\left(1-\left(\frac{4x}{3}-1\right)^2\right)}} & 12.5 \le t < 37.5\ s \\ \tan^{-1}\frac{8\cos\left(2\sin^{-1}\left(\frac{4x}{3}-1\right)\right)}{3\times\sqrt{\left(1-\left(\frac{4x}{3}-1\right)^2\right)}} & 37.5 \le t \end{cases} \quad \ldots(41)$$

The robot model starts from the initial posture $q(0) = [\,0.75, 0.2, \frac{\pi}{3}]^T$ as it's initial condition. Trajectory tracking and tracking error are shown in Figures 11 and 12, from these curves one can found very good trajectory tracking and posture error bounded by less than ±0.01 m for position error (i.e. $e_x and\ e_y$) and between ±0.02 rad for orientation error (i.e. e_θ) that's usually happened because of presence of tipping points in such trajectory and these values have a little effect on the trajectory tracking performance as that clear in Figure 11.

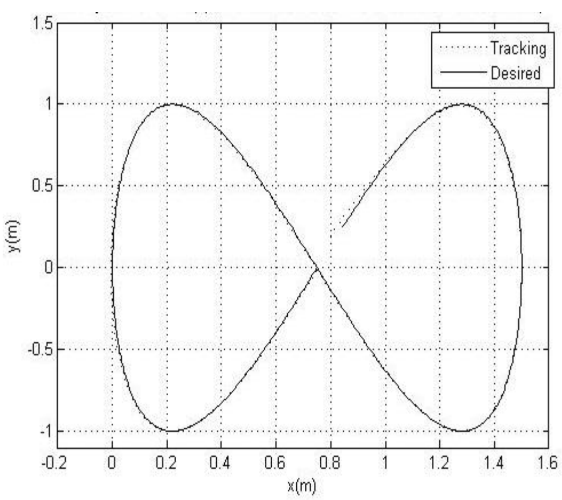

Fig. 11. Lemniscates trajectory tracking.

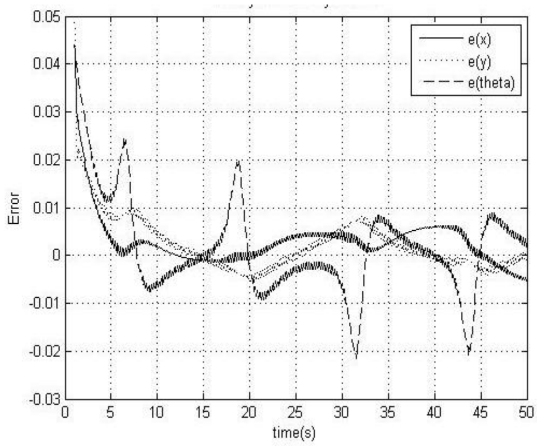

Fig. 12. Tracking error in Infinty trajectory tracking.

4- Simulation is also out for desired line trajectory; in this case a disturbance $\overline{\tau_d} = [0.01\sin(2t)\ \ 0.01\sin(2t)]^T$ [7], is added to the robot system as unmodeled kinematics and dynamics disturbance in order to prove the adaptation and robustness ability of the proposed controller. The robot starts form the initial posture $q(0) = [\,2, 1, {}^{\pi}/_{6}]$ as its initial conditions, simulation result is figured out for both presences of disturbance and without it where Figures (13 and 14) show the trajectory tracking and posture error respectively without presence of disturbance, while Figures (15 and 16) show trajectory tracking and posture error respectively with presence of disturbance. Simulation results confirm the power full of the proposed controller in rejected external disturbance and return to desired trajectory in short time furthermore very good trajectory tracking in both cases.

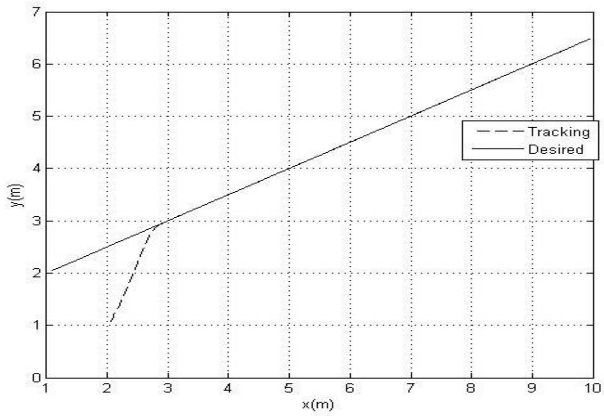

Fig. 13. Line Trajectory tracking.

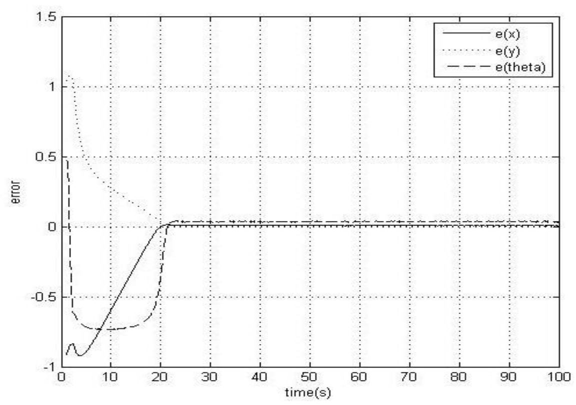

Fig. 14. Trajectory tracking Error.

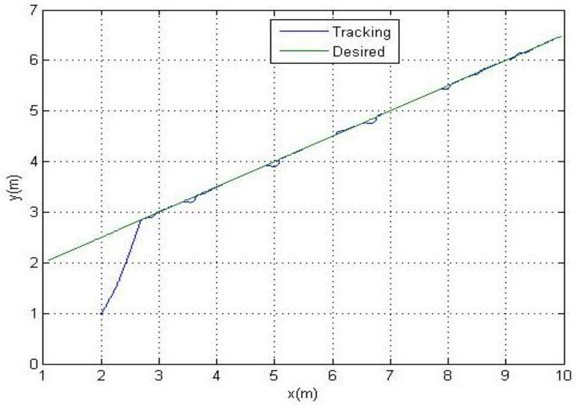

Fig. 15. Line Trajectory tracking with presence disturbance.

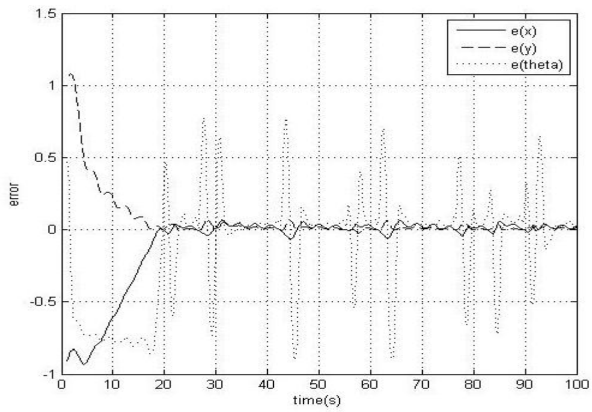

Fig. 16. Trajectory tracking error with presence of disturbance.

5. Conclusion

Evolutionary Algorithm used as optimization method for tuning **FOPID** controller, implemented throughout **Optiy** software. The control algorithm has been applied for four kind of reference trajectory (i.e. circular, S-shape, infinity, and line) these trajectory performed by using Simulation program (i.e**. MATLAB/SIMULINK**) and result showed good agreement. Simulation work demonstrates the effectiveness of the proposed algorithm with two-wheeled mobile robot and proved that this method is characterized by its robustness with disturbance and uncertainties in the system model.

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List of Symbols

Symbol	Definition
a	Integral order.
b	Derivative order.
B(q)	The input transformation matrix.
c	Reference point of mobile platform.
c.g.	Center of gravity.
d	Offset of cintroid from axis of wheels.
e(t)	Error between the actual and reference posture in time domain.
$F(\dot{q})$	Surface friction vector.
G(q)	Gravitational vector.
K_d	Derivative gain.
K_i	Integral gain.
K_p	Proportional gain.
2L	The distance between the two wheels for mobile robot.
M(q)	Inertia matrix of mobile robot.
m	Mass of mobile robot
q	Pose vector of mobile robot.
$\dot{q}$	Time derivative of pose vector of mobile robot.
q_r	Reference trajectory vector.
$\ddot{q}_r$	Time second derivatives of reference trajectory vector.
r	Radius of the wheel for mobile robot.
t	Time domain.
S(q)	Transfer matrix.
S^T	Transfer matrix transpose.
u(t)	Control input in time domain.

V	Velocity vector (linear and angular velocity).
$V_m(q,\dot{q})$	The centripetal and carioles matrix.
X_c and Y_c	Local frame of mobile robot.
x_c and y_c	Pose of mobile robot in global coordinate frame.
$\dot{x_c}$	Velocity of mobile robot in x-direction.
$\dot{y_c}$	Velocity of mobile robot in y-direction.
θ	The robotic orientation angle.
ν	Linear velocity of mobile velocity
λ	The vector of constrained force.
ω	Angular velocity of mobile robot.
τ	Input torque vector.
τ_d	Bonded unknown disturbances vector.

List of Abbreviation

Abbreviation	**Definition**
FOPID	Fractional Order of Proportional, Integral, and Derivative controller.
WMR	Wheeled Mobile Robot.

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سيطرة تتبع الأثر للروبوتات المتحركة بعجلات باستخدام المسيطر الجزئي نوع PID

موفق علي توفيق* عماد ناطق عبد الوهاب صلاح مهدي سوادي*****

**، **قسم هندسة المكائن والمعدات/ الجامعة التكنولوجية*

****قسم الهندسة الكهربائية/ كلية الهندسة/ الجامعة المستنصرية*

*البريد الالكتروني: drmat19853@yahoo.com

**البريد الالكتروني: emadnaabd@yahoo.com

***البريد الالكتروني: sallah.mahdi99@gmail.com

الخلاصة

للروبوتات المتحركة بعجلات (WMRs) العديد من التطبيقات في الوقت الحاضر في مجال الصناعة، النقل، والتفتيش وغيرها من المجالات الاخرى وبالتالي فإن أيجاد منظومة سيطرة لهذا النوع من الروبوتات لتقوم بتتبُع أثر معين تعد من المشكلات المهمة. هذا البحث يركز على على تطبيق انموذج مستند على المسيطر الجزئي نوع PID لحل مشكلة تتبُع الاثر. خوارزمية التحكم اعتمدت على تغذية الخطاء في موضع الروبوت المتحرك الى المسيطر (FOPID) لتوليد إشارات التصحيح التي تحول الى عزم دوران لعجلات القيادة ، تم حساب السرعة الخطية والسرعة الزاوية من خلال حل الانموذج الرياضي للحركة الديناميكية للروبوت المتحرك للوصول الى الوضع المطلوب . اشتق الانموذج الديناميكي للروبوت المتحرك للحالة التي لايتطابق فيها مركز الثقل مع الاطار المرجعي لمنصة الروبوت والتي اختيرت في منتصف محور عجلات القيادة ، كما تم الاخذ بنظر الاعتبار تأثير قوى القصور الذاتي للروبوت على الاطار المرجعي. استخدامت الخوارزمية التطوري لتعديل برامترات (K_p, K_i, K_d, a, and b) المسيطر الجزئي FOPID للروبوت المتحرك بعجلات. أظهرت نتائج المحاكاة فعالية المسيطر المقترح والذي تبين من تطبيق هذه المسيطر على أربعة انواع من المسارات (مسار دائري، مسار شكل S ، مسار إنفينيتي ، و المسار المستقيم ولحالتين بوجود أضطراب خارجي و بدونه) ، كما بينت النتائج مطابقة جيدة لتتبع الاثر المطلوب وأن دالة الخطأ في الموضع تذهب بسرعة إلى الصفر.