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# Minimizing error in robot arm based on design optimization for high stiffness to weight ratio.

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**Abstract:**

In this work the effect of choosing tri-circular tube section had been addressed to minimize the end effector's error, a comparison had been made between the tri-tube section and the traditional square cross section for a robot arm, the study shows that for the same weight of square section and tri-tube section the error may be reduced by about 33%.

A program had been built up by the use of MathCAD software to calculate the minimum weight of a square section robot arm that could with stand a given pay load and gives a minimum deflection. The second part of the program makes an optimization process for the dimension of the cross section and gives the dimensions of the tri-circular tube cross section that have the same weight of the corresponding square section but with less deflection.

**Key word**: robot arm stiffness, flexible manipulator, robot structure analysis, flexible link robot.

## Introduction:

The links of serial manipulators are usually over designed in order to be able to support the subsequent links on the chain and the pay load to be manipulated. However, increasing the size of the links unnecessarily requires the use of larger actuators resulting in higher power requirements. Optimum robot design has been addressed by many researchers as found in the open literature; Shiakolas and koladiye [1] discuss the application and comparison of the evolutionary techniques for optimum design of serial link robot manipulators based on task specifications. The objective function minimizes the required torque for a defined motion subjected to various constraints which considering kinematics, dynamic and structural conditions. The design variables examined are the link parameters and the link cross sectional characteristics, the developed environment was employed in optimizing the design variables for a SCARA and an articulated 3-DOF PUMA type manipulators. In the work developed by Marcus Pettersson et al. [2] an optimization problem are formulated to minimize the weight of the gearboxes, by choosing different discrete gear boxes, and changing the lengths of the arms continuously, subjected to a few requirements on acceleration capability reach and pay load capacity. Analysis of stiffness of manipulator link can be found in Abdel malek, K. and Paul, B.[3] where aspects of the structural design of the manipulator arm are presented. Prismatic joints of manipulator arm are based upon a cross sectional design of the links that provides a high stiffness to weight ratio compared with a hollow round cross-section.

The case that we study in this work is the robot that consists of three arms as shown in fig. (1). Where the first arm is vertical and the second and third arm are horizontal this gives the maximum reach (completely stretched out) for the robot arm and will yield the maximum deflection for the robot.

**Prismatic joints:**

Most manipulator link cross- section are either hollow round or hollow rectangular. Hollow links provide convenient conduits for electric power and communication cables, hoses, power transmission members, etc. Rivin[4] has studied the influence of cross-sections on the deflections both in bending and torsion. He had compared hollow square with hollow circular cross sections. Rivin states that a square cross section can provide a 69 to 84 percent increase in bending stiffness over a circular hollow cross section with only a 27 percent increase in weight.

In this paper a different cross-section is introduced, consisting of three tubes centered on the vertices of an equilateral triangle. This cross section is referred to as a tri-tube configuration the hollow square link will be referred to as a uni-tube configuration, as shown in fig. (2).

**Deflection due to pure bending:**

Links with an open end manipulator are normally modeled as cantilevers. Consider a simple cantilever with solid or hollow cross – section as shown in fig.(3).To study the proposed cross –section, we use the following equations for moments of inertia (2nd moment of area) about any diametrical axis through the centroid of area.

**Uni –tube:**

For the uni- tube depicted fig.(2,a) the moment of inertia about the neutral axis is

$$I_{uni-tube} = \frac{B^4 - b^4}{12} \quad , \quad b = B - 2t$$

Where B, b is the outer and inner sides, respectively for the uni-tube construction, t is the thickness of the uni-tube and the area is

$$A_{uni-tube} = B^2 - b^2$$

**Tri-tube:**

For the tri-tube depicted in fig.(2,b) the moment of inertia about the neutral axis is

$$\boldsymbol{I_{tri-tube} = \frac{3\pi}{64}\left(D^4 - d^4\right) + \frac{\pi}{4}\left(D^2 - d^2\right)\left(\frac{2}{3}h\sin 60\right)}$$

$$\boldsymbol{+ 2\frac{\pi}{4}\left(D^2 - d^2\right)\left(\frac{1}{3}h\sin 60\right)^2}$$

$$d = D - 2t$$

Where D and d are the out side and inside diameters, respectively, for each tube on the equilateral triangle, t is the thickness of each tube in the tri – tube construction.

The area of each tube is

$$A_{tri-tube} = \frac{\pi}{4}\left(D^2 - d^2\right) \quad , \quad d = D - 2t$$

To demonstrate the deflection due to loading, consider the third arm beam depicted in fig.(3,c).

$$\delta_3 = \frac{1}{EI_3}\left(\frac{W_3\, L_3^{\,3}}{3} + \frac{q_3\, L_3^{\,4}}{8}\right)$$

$$q = \frac{M \times g}{L} = \frac{\rho\, g\, L\, A}{L} = \gamma\, A \qquad , W = m\, g$$

Where M is the mass of each arm, m is the mass of the gear box and the mass of the load to be manipulated at the end of the arm, q is the weight per unit length of the beam, W is the load in Newton, g is the gravitational acceleration and L is the length of the arm, A is the cross sectional area of the beam, $\gamma$ is the specific density.

To get the reactions (force and moment) at the fixed end of the third arm we equate the summation of forces and moment to zero i.e.

$$\sum F_y = 0 \qquad F_3 = W_3 + q_3\, L_3$$

$$\sum M = 0 \qquad MO_3 = \frac{q_3\, L_3^{\,2}}{2} + W_3\, L_3$$

The same thing may be said for the second arm (fig.3-b) taking in to account the effect of moment in calculating the deflection i.e.

$$\delta_2 = \frac{1}{EI_2}\left(\frac{\left(F_3 + W_2\right)L_2^{\,3}}{3} + \frac{q_2\, L_2^{\,4}}{8} + \frac{MO_3\, L_2^{\,3}}{2}\right)$$

The reactions at the fixed end will be

$$F_2 = q_2\, L_2 + W_2 + F_3$$

$$MO_2 = \frac{q_2\, L_2^{\,2}}{2} + W_2\, L_2 + F_3\, L_2 + MO_3$$

For the first arm (fig.3-a) we assume that the deflection at the free end is due to bending moment only and the effect of compressive loads on the whole deflection is neglected therefore

$$\delta_1 = \frac{MO_1\, L_1^{\,2}}{2EI_1}$$

The total deflection at the end effecter of the robot manipulator arm will be

$$\delta_{total} = \sqrt{(\delta_3 + \delta_2)^2 + \delta_1^{\,2}}$$

The sequence of analysis in this work is to calculate the weight of the lightest structure that has a square hollow section and with stand the given loading condition this may be achieved by letting the stress in each arm reaches the maximum allowable stress to avoid failure of the structure, the equation for calculating the stress in the third arm may be written as

$$\sigma_3 = \frac{M\,Y}{I} = \frac{MO_3 * B_3/2}{\frac{B_3^{\,4} - b_3^{\,4}}{12}}$$

By letting the stress equal the allowable stress and assuming the thickness of the tube walls to be 2mm we may found the dimension of the third arm, this had been done by the aid of a program built up using MathCAD software. The stress in the second arm may also be calculated in the same way i.e.

$$\sigma_2 = \frac{MO_2 * B_2/2}{\frac{B_2^{\,4} - b_2^{\,4}}{12}}$$

The dimension of the first arm fig.(3.a) is calculated by equating the maximum stress induced in it with the maximum allowable, this maximum stress is found by the Rankine-Gorden formula [5] which is a combination of the Euler and crushing loads for a strut

$$\frac{1}{F_R} = \frac{1}{F_e} + \frac{1}{F_c}$$

For very short strut $F_e$ is very large, $\frac{1}{F_e}$ can therefore be neglected and $F_R = F_c$, for very long struts $F_e$ is very small and $\frac{1}{F_e}$ is very large so that $\frac{1}{F_c}$ can be neglected. Thus $F_R = F_e$. The Rankine formula is therefore valid for extreme values of slenderness ratios. It is also found to be fairly accurate for the intermediate values. Thus, re – writing the formula in terms of stresses

$$\frac{1}{\sigma_R\,A} = \frac{1}{\sigma_e\,A} + \frac{1}{\sigma_Y\,A}$$

$$\frac{1}{\sigma_R} = \frac{1}{\sigma_e} + \frac{1}{\sigma_Y} = \frac{\sigma_e + \sigma_Y}{\sigma_e\,\sigma_Y}$$

$$\sigma_R = \frac{\sigma_e\,\sigma_Y}{\sigma_e + \sigma_Y} = \frac{\sigma_Y}{1 + (\sigma_Y/\sigma_e)}$$

For a strut with one end free and the other fixed

$$F_e = \frac{\pi^2 E\,I}{4L^2} \quad \text{and} \quad \sigma_e = \frac{\pi^2 E\,I}{4L^2\,A}$$

The crushing load on the first arm is

$$F_c = F_1 = F_2 + W_1$$

$$\sigma_Y = \frac{F_c}{A}$$

The final stress $(\sigma_1)$ on the first arm is thus the sum of the direct stress calculated by Rankine formula and that due to bending generated by the exerted moment $(MO_2)$ as was explained in figure (3-a and b)

$$\therefore \sigma_1 = \sigma_R + \sigma_{bending} = \frac{\sigma_Y}{!+(\sigma_Y/\sigma_e)} + \frac{MO_2 * B_1/2}{I_1}$$

From this equation we may find the dimension of the first arm. After knowing the dimension of each arm the weight of each arm may be found and also the total weight of the manipulator structure will be determined. The next step in the analysis is to input those information to the program to began the process of changing the dimension of the cross section to minimize the total error (deflection) at the end of the robot arm this process gives many generations of the dimensions of the arm cross section which satisfies the conditions specified for the maximum and minimum error allowed at the end effecter and also the permissible increase in the weight of the robot structure specified from us, from between all those generation the program select the best generation or probability that gives the lightest weight and the less deformation. The next step in the analysis is to input the new weight of the robot arms to the optimization process for the tri –tube cross section shown in figure(2-b) and trying to find the best dimensions that gives the highest moment of inertia for the cross section so as to minimize the deflection in each arm and also the total deflection of the robot at the end effecter, i.e. the mass of uni –tube section found by the program should be equal to the mass of the tri –tube section which is equal to

$$M_3 = \rho V = \rho \left[ \begin{array}{l} L_3 \frac{3\pi}{4}(D^2 - d^2) + 2S(\pi R^2 \\ -3\frac{D^2\pi}{4}) \end{array} \right]$$

$$R = \frac{2}{3} h \sin\frac{\pi}{3} + D/2$$

Where $\rho$ is the density of robot arm metal, $R$ is the radius of the two flanges (stiffeners) welded at each end, $S$ is the thickness of the flanges and $h$ is the distance between the vertices of the equilateral triangle.

The optimization problem is defined as follow

$M_{tri-tube} = M_{uni-tube}$

$\max imize(I_{tri-tube})$

$h \geq D \quad , D \geq 2t$

In the optimization problem the thickness of the tubes $(t)$ and the thickness of the flanges $(S)$ are assumed to be 2mm.

The results of the optimization problem showed that the tri –tube section that have the same weight (mass) of a uni –tube may improve the stiffness of the robot and minimize the total deflection in about 33%, this results means that we may construct a robot having tri –tube section which is less in weight from that of uni –tube section and both of them having the same end effecter deformation.

***Results:***

In order to verify the analysis of the previous section a run had been done which has the following characteristics for the robot arm $E = 200 \times 10^9 \ N/m^2$

$\sigma_{all} = 120 \times 10^6 N/m^2$, $g = 9.81 m/\sec^2$, $L_1 = .5m, L_2 = .45m, L_3 = .4m$ , $\rho = 7850 \ kg/m^3$ , $\delta_{\max} = 0.0022 m$, $\delta_{\min} = 0.0005 m$, $T = 0.002 m$ ( tube thickness ), $S = 0.002m$ (stiffener thickness), $m_1 = 9.5kg$ (mass of the first gear box), $m_2 = 4.4kg$ (mass of the second gear box), $m_3 = 50kg$ (manipulated mass). The available gear boxes for the application are given in the list of table 1

The results of the program shows that for the given configuration the minimum weight for the structure of the uni –tube robot is ($W_{min}$=19.986 N), the robot with such structural weight could manipulate the load with out failure because the stress in each arm is less than or equal to the allowable stress, but the deflection of the end point effecter is very large. The iteration process for increasing the dimension of the section to minimize the deflection and letting it be within the range (0.0005< $\delta$ <0.0022) shows that there are 22 generation all of which has a deflection (0.0005< $\delta$ <0.0022) and also a weight (W<Factor * $W_{min}$ ) the permissible weight factor (Fac.) for increasing the weight was chosen to be (Fac.=1.35). The dimensions of the inner side of the uni –tube section for the 22 generation are shown in fig.(4). The relation between the total deflection at the end -effecter and number of generation is shown in fig.(5). The relation between the new weight of the robot structure and its generation is shown in fig.(6). The program chooses the best generation which has the less variable (variable= weight*deflection), the relation between the variable and the generation is shown in fig(7) it is obvious that the generation no. 16 has the minimum value, the dimensions of the section for that generation are $B_1$ =0.07464m, $B_2$ =0.06397m, $B_3$=0.05657m and has a deflection $\delta_{total}$ =1.87125*$10^{-3}$ m and a total structural weight $W_{total}$ =26.917N. Those results are the input for the next step in the program for calculating the dimensions of the tri –tube section in which an optimization problem where solved to maximize the moment of inertia for the section in terms of the dimensions h and D, the results of the program are shown in table (2)

The total deflection for the tri –tube configuration

$(\delta_{Total})_{Tri-tube} = \sqrt{(\delta_3 + \delta_2)^2 + \delta_1^{\ 2}}$ =1.31462*10$^{-3}$ m

The deflection for the tri –tube configuration is less than that for the uni –tube which was found to be ( $\delta_{uni-tube} = 1.8725 * 10^{-3}$ ) this results shows that the tri –tube section reduces the deflection in a bout 29.7% from that of uni –tube section, the result may be improved to reach a value of 33.38% if we change the weight factor (Fac.) to make it equal to (1.3) on the other hand if (Fac.) is increased to(1.65) the improvement in deflection would be less and equal to (6.02%) those results for iteration and optimization are shown in table(3)

Another interesting feature in the design of the uni –tube section is that if the weight factor (Fac.) was chosen to be 1.3 and the range of deflection is limited to $\delta$<0.0022 we would find only five generation which satisfies the previous mentioned configuration and if (Fac.) was changed to 1.65 and $\delta$< 0.00104 we would find only 4 generation these results which are shown in table (4) shows the band of limits of the design of robot in other word we can not find a robot with a weight factor less than (Fac.=1.3) and has a deflection less than 0.0022m or we cannot find a robot with a weight factor less than (Fac.=1.65) and had a deflection less than 0.00104m those results of iteration are shown in table(4).

A flow chart of the program used is shown in fig.(8).

**Conclusions:**

This paper presents a method for optimization of robot design in the conceptual design stage. The robot is modeled in the MathCAD package and the optimization problem is formulated as to determine the dimension of robot arm in order to minimize the weight and maximize stiffness this formulation can be interpreted as to design the cheapest possible robot that will still meets the design demands. The optimization method showed good capability in finding an optimum set of dimension of the arm of robot manipulator with three degree of freedom.

The optimization method shows that the tri –tube is superior to uni –tube section in minimizing deflection in about 33%.

The presented work provides a good support for conceptual robot design.

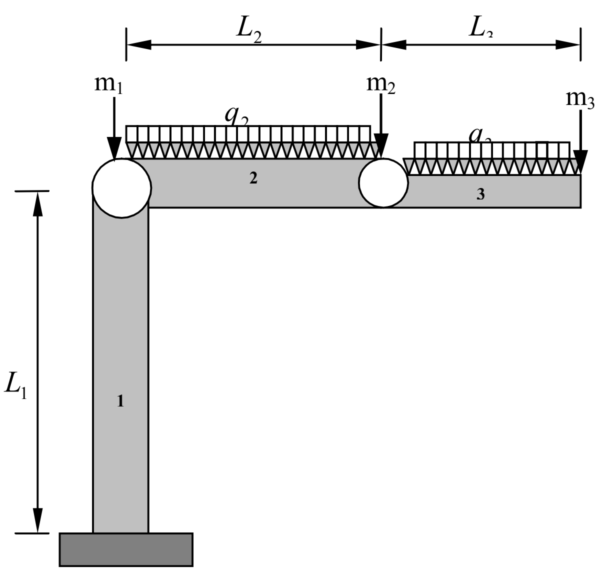


Fig. (1) robot configuration

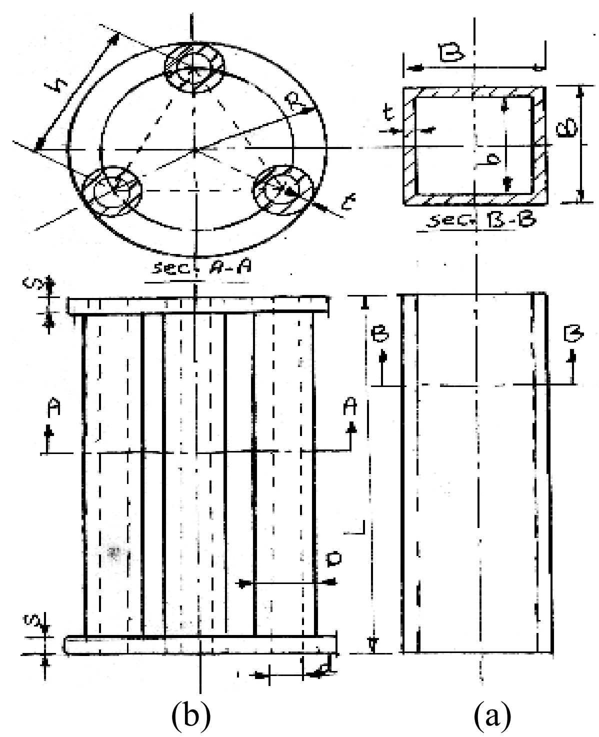


Fig.(2) (a) uni-tube configuration
(b) tri-tube configuration.

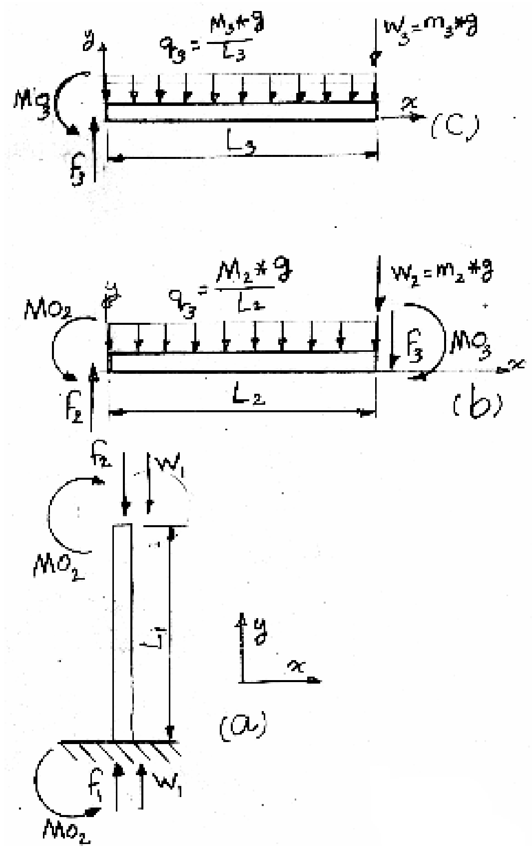


Fig.(3) modulation of manipulator links

Table 1 torque–mass relation for available gear boxes [2]

| Out put torques (N.m) | Mass (kg) |
|---|---|
| 101 | 2.5 |
| 231 | 4.4 |
| 572 | 9.5 |
| 1088 | 12.7 |
| 1499 | 18 |
| 2176 | 28 |
| 4361 | 47 |
| 6135 | 69 |

Table (2) dimension of the tri –tube section

| arm No. | h (m) | D (m) $*10^{-3}$ | $I_{tri-tube}(m^4)$ $*10^{-7}$ | Deflection ($\delta$) m $*10^{-4}$ |
|---|---|---|---|---|
| 3 | 0.10028 | 7.52652 | 1.75056 | 8.09087 |
| 2 | 0.10440 | 9.22085 | 2.48207 | 7.35704 |
| 1 | 0.10440 | 11.92 | 3.42024 | 3.0044 |

Table (3) results for iteration and optimization problem

| Fac. | $\delta_{Tri-tube}$ (m)*10-3 | $\delta_{Uni-tube}$ (m)*10-3 | Improvement In deflection |
|---|---|---|---|
| 1.3 | 1.39973 | 2.10114 | 33.38% |
| 1.35 | 1.31462 | 1.87125 | 29.7 % |
| 1.4 | 1.2189 | 1.6237 | 24.9 % |
| 1.45 | 1.16048 | 1.48623 | 21.9 % |
| 1.5 | 1.09686 | 1.3437 | 18.37% |
| 1.55 | 1.04631 | 1.22646 | 14.68% |
| 1.6 | 0.985197 | 1.08422 | 9.13 % |
| 1.65 | .948732 | 1.00955 | 6.02 % |

Table (4) limits of deflection-weight factor for robot design

| Fac. | $\delta$ max (m) | generations |
|---|---|---|
| 1.3 | 0.0022 | 5 |
| 1.35 | 0.0019 | 2 |
| 1.4 | 0.0017 | 3 |
| 1.45 | 0.0015 | 2 |
| 1.5 | 0.0014 | 3 |
| 1.55 | 0.0013 | 0 |
| 1.6 | 0.0011 | 2 |
| 1.65 | 0.00104 | 4 |

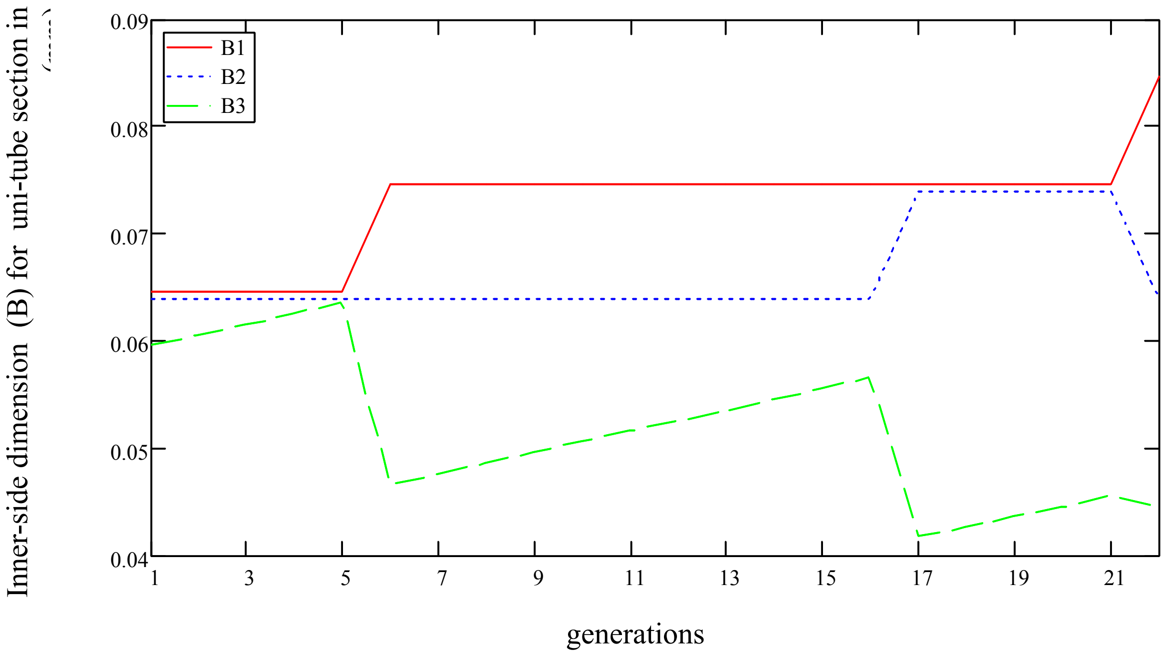


fig.(4) correlation between inner side dimension and no. of generations.

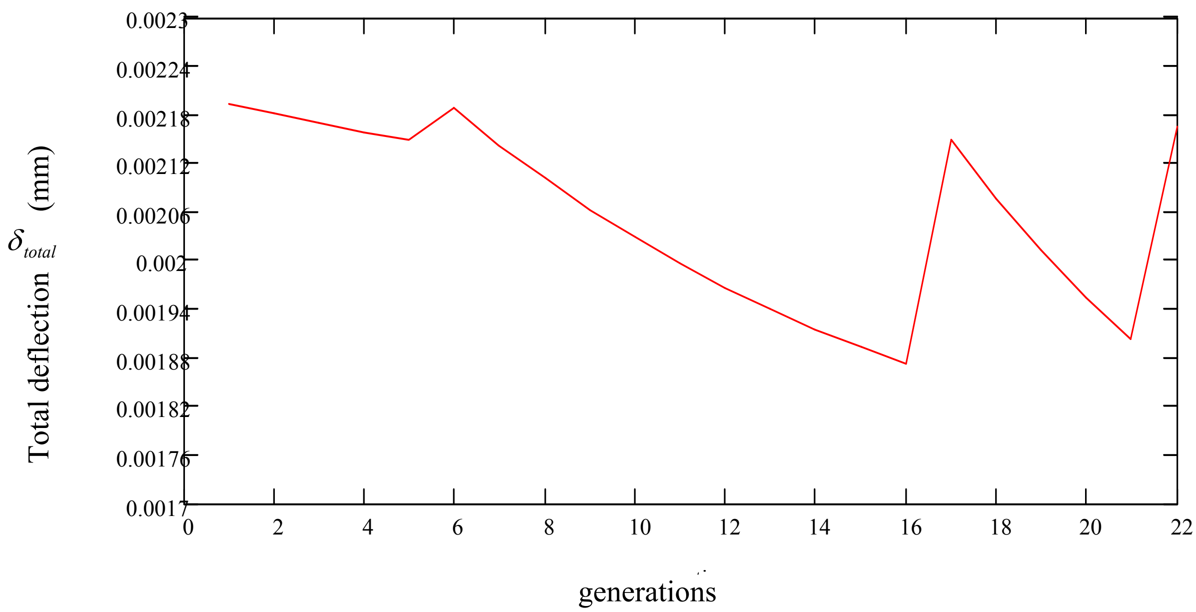


Fig.(5) correlation between the deflection and no. of generations.

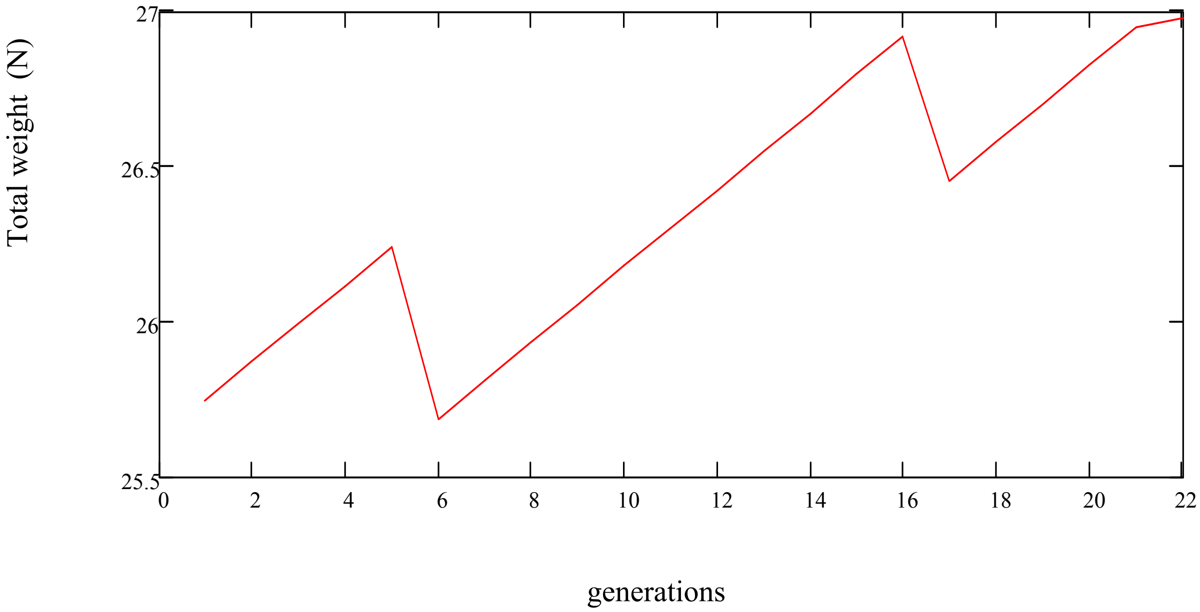


Fig.(6) correlation between robot weight and no. of generations.

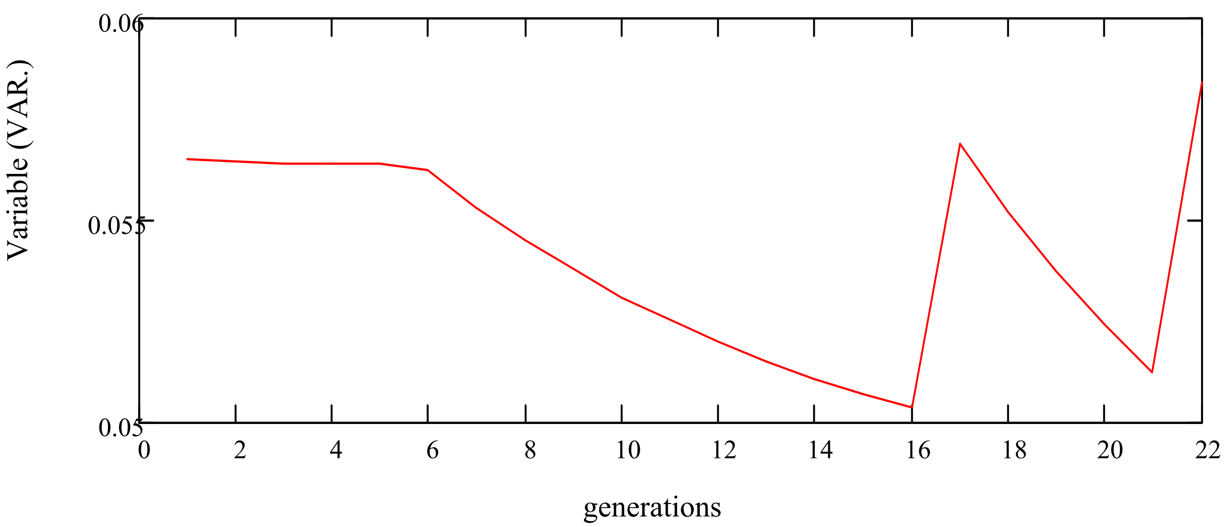


Fig.(7) correlation between (Variable) and no. of generations.

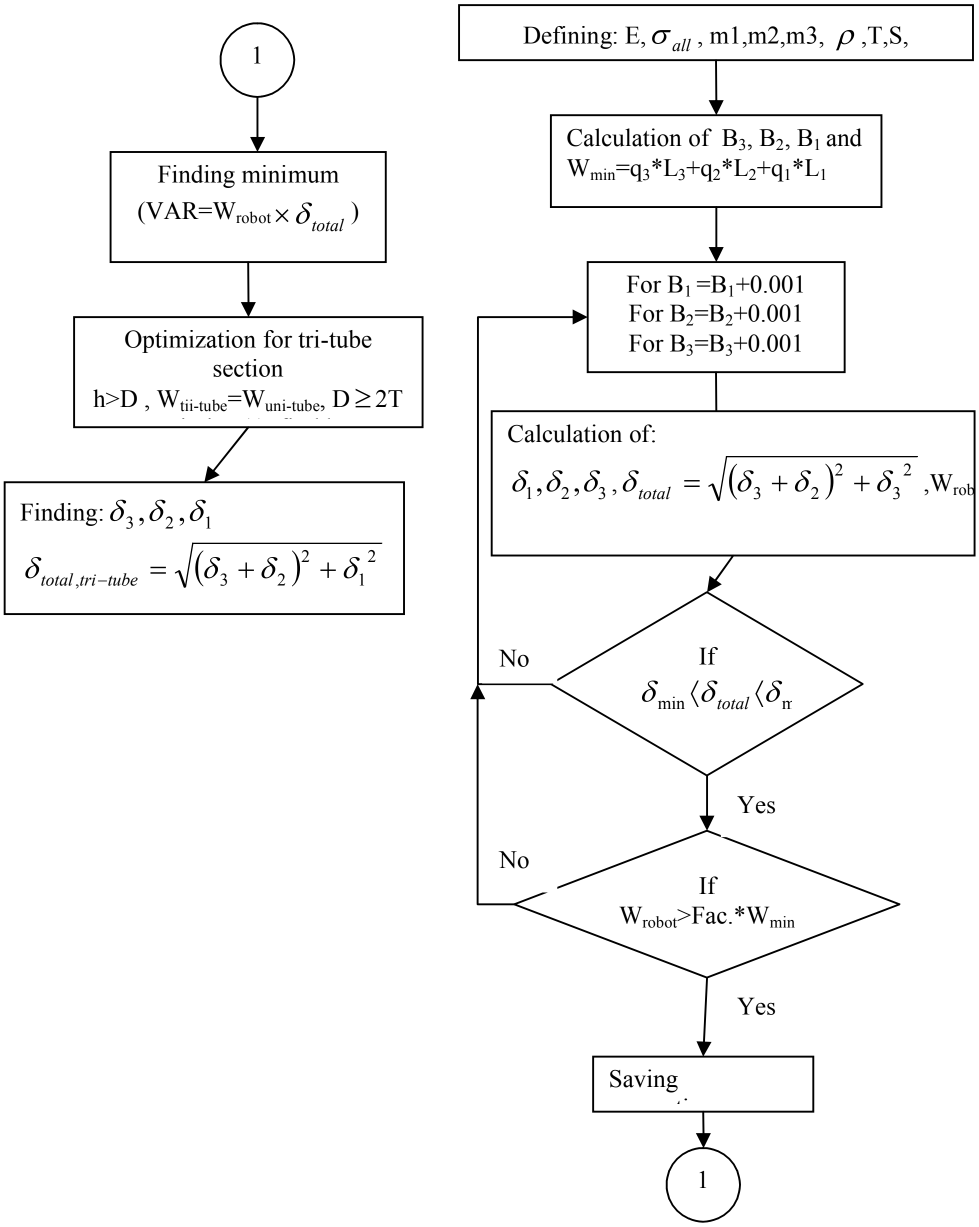


Fig.(8) flow chart of the program built-up by the use of MathCAD soft ware

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# تقليل الخطاء في الذراع الألي (الروبوت) على أساس التصميم الأمثل ولنسبة جساءة الى وزن عالية.

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قسم الميكانيك

**الخلاصة:**

في هذا البحث تم دراسة تاثير استخدام المقطع الثلاثي الأنابيب الدائريه لأجل تقليل الخطاء الطرفي في الذراع الألي ، تم اجراء مقارنه بين المقطع الثلاثي الأنابيب و المقطع المربع التقليدي للذراع الألي ، الدراسه بينت بانه لكلا الذراعين ذات المقطع الثلاثي و المربع والذان لهما نفس الوزن ممكن تقليل الخطاء بحدود ٣٣% .

تم كتابة برنامج باستخدام MathCAD لحساب أقل وزن للذراع الآلي ذي المقطع المربع الذي يمكن أن يتحمل الأوزان المسلطه ويعطي أقل تشوه.

الجزء الثاني من البرنامج يقوم بعملية الأمثليه لأجل ايجاد ابعاد المقطع ذي الأنابيب الثلاثيه الدائريه والذي له نفس وزن الذراع ذي المقطع المربع وله تشوه أقل من نضيره المربع.