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Dynamic Analysis of Thin Composite Cylindrical and Spherical Shells

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Abstract

In this work, an investigation for the dynamic analysis of thin composite cylindrical and spherical shells is presented. The analytical solution is based upon the higher order shear deformation theory of elastic shells from which the developed equations are derived to deal with orthotropic layers. This will cover the determination of the fundamental natural frequencies and mode shapes for simply supported composites cylindrical and spherical shells.

The analytical results obtained by using the derived equations were confirmed by the finite element technique using the well known Ansys package. The results have shown a good agreement with a maximum percentage of discrepancy, which gives a confidence of using this solution in prediction the dynamic analysis of cylindrical and spherical shells.

Keywords: Dynamic, Composite, Cylindrical, Spherical, Shells.

1. Introduction

A shell is a three-dimensional body which is bounded by two closely spaced curved surfaces, the distance between the surfaces, being small in comparison with the other dimensions [1]. A shell is considered to be a thin shell when the shell thickness is less than 1/20 of the wavelength of the deformation mode and/or radius of curvature and at the small time, it is assumed that both shear deformation and rotary inertia are ignored.

The dynamic analysis of shells has been expanding rapidly due to the importance of shell structures in civil, mechanical and aerospace engineering. The thin composite cylindrical and spherical shells are found in many aerospace and aircraft industrial applications such as aircraft wings and fuselage radomes, EWACS and fuel tanks. Also, the composite cylindrical shells are found in compressor blades, ships and rocket.

Humayun R.H. Kabir [2] investigated analytically the free vibration of composite shallow cylindrical shells with simply supported boundary conditions using Kirchoff-Love theory.

H.M. Wang [3] investigated the dynamic solution of a multi layered hollow cylinder in a state of axisymmetric plane strain. The solution is divided into two parts: one is quasi-static and the other is dynamic. The qusai static is solved by the state space method, and the dynamic part is obtained by the separation of variables coupled with the initial parameter method .

Rong-Tyai & Zung-Xian Lin [4] presented the formulation of governing equations for a symmetric cross-ply laminated cylindrical shell with a circumferential stiffener.

Penzes and Burgin [5] were the first to solve the problem of the free vibrations of thin isotropic spheroid al shells by using Galerkins method using membrane theory and harmonic axisymmetric exitation. Al-Najim F. [6] used the Rayliegh method to obtain the natural frequencies and mode shapes of axisymmetric vibrations of thin elastic oblate spheroidal shells theoretically and experimentally. He showed that the Rayleigh's method is suitable to deal with such types of problems.

From the previous literatures, it is found that there is still a chance to work in this field especially for the analysis of composite cylindrical and spherical shells and therefore this work will be devoted in this respect analytically using the higher order shear deformation theory and numerically using the finite element method.

Mathematical Analysis

The review of literature reveals that the governing equations for thin spherical shells are not available. The following formulation gives the free formulation for the thin shells of revolution which can be applied for both cylindrical and spherical shells. Based upon the Third-order theory of Reddy using the displacement field Ref. [7]:

$$u(x,y,z,t) = u_0(x,y,t) + z \times \phi_1(x,y,t) + z^3 \times \left(\frac{-4}{3 \times H^2}\right) \times \left(\phi_1 + \frac{\partial w_0}{\partial x}\right)$$

$$v(x,y,z,t) = v_0(x,y,t) + z \times \phi_2(x,y,t) + z^3 \times \left(\frac{-4}{3 \times H^2}\right) \times \left(\phi_2 + \frac{\partial w_0}{\partial x_2}\right)$$

$$w(x,y,z,t) = w_0(x,y,t)$$

The resulting strain-displacement relations are:

$$\varepsilon_1 = \frac{\partial u_0}{\partial x_1} + z \times \frac{\partial \phi_1}{\partial x_1} - z^3 \times \left(\frac{4}{3 \times H^2}\right) \times \left[\frac{\partial \phi_1}{\partial x_1} + \frac{\partial^2 w_0}{\partial x_1^{\,2}}\right]$$

$$\varepsilon_2 = \left(\frac{\partial v_0}{\partial x_2} + \frac{w_0}{R_2}\right) - z \times \frac{\partial \phi_2}{\partial x_2} - z^3 \times \left(\frac{4}{3 \times H^2}\right) \times \left[\frac{\partial \phi_2}{\partial x_2} + \times \frac{\partial^2 w_0}{\partial x_2^{\,2}}\right]$$

$$\varepsilon_3 = 0$$

$$\varepsilon_6 = \frac{\partial v_0}{\partial x_1} + \frac{\partial u_0}{\partial x_2} + z \times \left(\frac{\partial \phi_1}{\partial x_2} + \frac{\partial \phi_2}{\partial x_1}\right) - z^3 \times \left(\frac{4}{3H^2}\right) \times \left[\left(\frac{\partial \phi_2}{\partial x_1} + \frac{\partial \phi_1}{\partial x_2}\right) + 2 \times \frac{\partial^2 w_0}{\partial x_1 \partial x_2}\right]$$

$$\varepsilon_4 = \left(\phi_2 + \frac{\partial w_0}{\partial x_2}\right) - z^2 \times \left(\frac{4}{H^2}\right) \times \left(\phi_2 + \frac{\partial w_0}{\partial x_2}\right)$$

$$\varepsilon_5 = \phi_1 + \frac{\partial w_0}{\partial x_1} - z^2 \times \left(\frac{4}{H^2}\right) \times \left(\phi_1 + \frac{\partial w_0}{\partial x_1}\right) \quad ...(1)$$

Appling Hamilton's principles, the resulting equations contain double and triple integrals and because of the variation principle of displacements, the coefficient of the variations displacements is zero from which the following equations of motion are obtained:

$$\int_{t2}^{t1} (\delta U - \delta K)\partial t = 0 \quad ...(2)$$

where:

$$\delta U = \int_A \int_z \sigma_1 \delta\varepsilon_1 + \sigma_2 \delta\varepsilon_2 + \sigma_3 \delta\varepsilon_3 + \sigma_6 \delta\varepsilon_6 + \sigma_5 \delta\varepsilon_5 + \sigma_4 \delta\varepsilon_4 \times R d\theta dz dx$$

$$\delta K = -\int_t \iiint_V R\rho\left(\ddot{u}\,\delta u + \ddot{v}\,\delta v + \ddot{w}\,\delta w\right) dz dx d\theta dt$$

Carrying out the above integrations will give the followings:

$$\frac{\partial N_1}{\partial x_1} + \frac{\partial N_6}{\partial x_2} = \left(I_1 + \frac{2}{R_1}\right)\ddot{u} + \left(I_2 + \frac{I_3}{R_1} - \left(\frac{4}{3H^2}\right)\left(I_4 + \frac{I_5}{R_1}\right)\right)\ddot{\phi}_2 - \left(\frac{4}{3H^2}\right)\left(I_4 + \frac{I_5}{R_1}\right)\frac{\partial \ddot{w}}{\partial x} \quad ...(3)$$

$$\frac{\partial N_2}{\partial x_2} + \frac{\partial N_6}{\partial x_1} = \left(I_2 + \frac{I_3}{R_2} - \left(\frac{4}{3H^2}\right)\left(I_4 + \frac{I_5}{R_2}\right)\right)\ddot{\phi}_2 + \left(I_1 + \left(\frac{2I_2}{R_2}\right)\right)\ddot{v} + \left(\left(\frac{4}{3H^2}\right)\left(I_4 + \frac{I_5}{R_2}\right)\right)\frac{\partial \ddot{w}}{\partial x_2} \quad ...(4)$$

$$\left(\frac{4}{3H^2}\right)\left(\frac{\partial^2 S_2}{\partial x_2^{\,2}} + \frac{\partial^2 S_1}{\partial x_1^{\,2}} + 2\frac{\partial^2 S_6}{\partial x \partial \theta}\right) - \frac{N_1}{R_1} - \frac{N_2}{R_2} - \left(\frac{4}{H^2}\right)\left(\frac{\partial K_4}{\partial x_1} + \frac{\partial K_5}{\partial x_2}\right) + \left(\frac{\partial Q_4}{\partial x_1} + \frac{\partial Q_5}{\partial x_2}\right) + q =$$

$$\left(\left(\frac{4I_4}{3H^2}\right) - \left(\frac{16I_5}{9H^4 R_2}\right)\right)\frac{\partial \ddot{v}}{\partial x_2} + I_1 \ddot{w}$$

$$\left(-\left(\frac{16I_7}{9H^4}\right) + \left(\frac{4I_5}{3H^2}\right)\right)\frac{\partial \ddot{\phi}_2}{\partial x_2} + \left(\begin{array}{c}\left(\frac{4I_4}{3H^2}\right) \\ -\left(\frac{16I_7}{9H^4}\right)\end{array}\right) \quad ...(5)$$

$$\left(\frac{\partial^2 \ddot{w}}{\partial x_1^{\,2}} + \frac{\partial^2 \ddot{w}}{\partial x_2^{\,2}}\right) + R\left(-\left(\frac{16I_7}{9H^4}\right) + \left(\frac{4I_5}{3H^2}\right)\right)$$

$$\frac{\partial \ddot{\phi}_1}{\partial x_1} + \left(\frac{4}{3H^2}\right)\left(I_4 + \frac{I_5}{R_1}\right)\frac{\partial \ddot{u}}{\partial x}$$

$$\frac{\partial M_1}{\partial x_1} - \left(\frac{4}{3H^2}\right)\frac{\partial S_1}{\partial x_1} + \frac{\partial M_6}{\partial x_2} - \left(\frac{4}{3H^2}\right)\frac{\partial S_6}{\partial x_2} - Q_5$$

$$+\left(\frac{4}{H^2}\right)K_5 = \left(I_2 + \frac{I_3}{R_1} - \frac{4}{3H^2}\left(I_4 + \frac{I_5}{R_1}\right)\right)\ddot{u} + \quad \ldots(6)$$

$$\left(I_3 - \frac{8I_5}{3H^2} + \frac{16I_7}{9H^4}\right)\ddot{\phi}_1 + \left(-\frac{8I_5}{3H^2} + \frac{16I_7}{9H^4}\right)\frac{\partial \ddot{w}}{\partial x_1}$$

$$\frac{\partial M_2}{\partial x_2} - \left(\frac{4}{3H^2}\right)\frac{\partial S_2}{\partial x_2} + \frac{\partial M_6}{\partial x_1} - \left(\frac{4}{3H^2}\right)\frac{\partial S_6}{\partial x_1} - Q_4$$

$$+\left(\frac{4}{H^2}\right)K_4 = \left(I_2 + \frac{I_3}{R_2} - \frac{4}{3H^2}\left(I_4 + \frac{I_5}{R_2}\right)\right)\ddot{v} + \quad \ldots(7)$$

$$\left(I_3 - \frac{8I_5}{3H^2} + \frac{16I_7}{9H^4}\right)\ddot{\phi}_2 + + \left(-\frac{4I_5}{3H^2} + \frac{16I_7}{9H^4}\right)\frac{\partial \ddot{w}}{\partial x_2}$$

$$(Ni, Mi, Si) = \int \sigma_i \left(1, z, z^3\right) \times dz \longrightarrow i = 1,2,3,6$$

$$(Qi, Ki) = \int \sigma_i \left(1, z^2\right) \times dz \longrightarrow i = 4,5$$

$$(I_i) = \int \rho_i \left(1, z, z^2, z^3, z^4, z^5, z^6\right)$$

$$\times dz \longrightarrow i = 1,2,3,4,5,6,7$$

Substituting the above resultant forces in the developed equations and then the assumed displacement components according to Navier′s Solution Ref. [8] (for simply supported boundary conditions), the stiffness and mass matrices are obtained.

Solution of the Developed Equations:

The solution is assumed as follows:

$$u_0(x, \theta, z, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \cos \alpha x \sin \beta\theta e^{i\omega mn}$$

$$v_0(x, \theta, z, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin \alpha x \cos \beta\theta e^{i\omega mn}$$

$$w_0(x, \theta, z, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin \alpha x \sin \beta\theta e^{i\omega mn}$$

$$\phi_1(x, \theta, z, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} D_{mn} \cos \alpha x \sin \beta\theta e^{i\omega mn}$$

$$\phi_2(x, \theta, z, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} E_{mn} \sin \alpha x \cos \beta\theta e^{i\omega mn} \quad \ldots (8)$$

where $\alpha = \left(\frac{m\pi}{L}\right)$, $\beta = n$

The mass and stiffness matrices are obtained from the solution of the eigenvalue equation as follows:

$$\left|[K] - \omega^2 [M]\right|\{A\} = 0 \quad \ldots(9)$$

From which the natural frequencies and mode shapes are obtained.

Finite element modeling

Finite element modeling for the laminated shells is done using ANSYS (5.4), following the major steps [9]:

1. Building the structure using quad shell element 99 as shown in Fig. (1).
2. Applying boundary conditions.
3. Solve the natural frequency problem and getting the results.

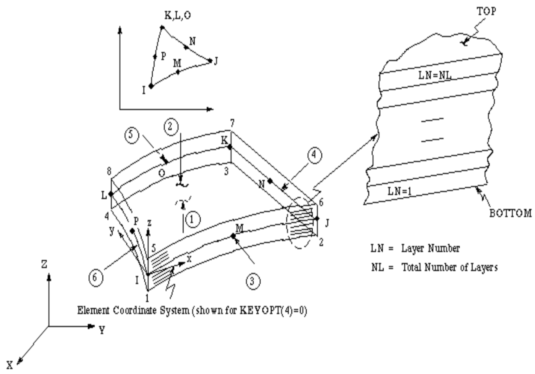


Fig. 1. 100-layer Shell-99 element.

Results and Discussions

The developed analytical solution using the general third order shear deformation theory (HSDT) will be employed to investigate its applicability in investigating the dynamic analysis of symmetric and non-symmetric cross-ply laminated cylindrical and spherical shells. The results are composite plates used by other researchers.

(a) Spherical shell

In order to obtain the fundamental natural frequencies from the developed analytical solution, different ratios of radius-to-side length 5,10 and 20 for both type of shells thin(a/ H) =100 and thick (a/H)=10 for the composites (0,90,0) and (0,90,90,0).The results are shown in table 1. It is seen that the maximum percentages of discrepancy is 8.97%. The results indicate that the thicker shells have lower frequency parameter than the thinner shells, and for shells with smaller (R/a) the frequency parameter is greater than that for larger ratios .In the above calculations the material properties are as followings [10].

E1=2e6, E2=E3=1e6, G12=G13=0.5e6, G23=0.2e6, $\nu_{12} = \nu_{13} = 0.24$, $\nu_{23} = 0$

Another interesting result is that the fundemenral frequency for symmetric shells is greater than that for antisymmetric one. However, Fig. 2 shows the first three modes shapes for the spherical shell of composite type (0,90). The shapes are consistent with the predicted modes. Fig. 3 shows the frequency parameter change $\left(\frac{\omega a^2}{H}\right)\sqrt{\frac{\rho}{E_2}}$ with the ratio (a/H) which shows that the fundamental natural frequency is increased with the increasing (a/H) ,for both types of composites investigated (0,90,0) and (0,90)

(b) Cylindrical Shell

General Third Order Theory (HSDT) is employed to investigate its capability level for dynamic analysis of the symmetric and non-symmetric cross-ply laminated cylindrical shells, and compared with other theories used by other researchers such as FSDT.

In this respect, the fundamental natural frequency was obtained for the ratios of radius to side length (5, 10, and 20) and (a/H)=100 thick shell and (a/H)=10, thin shell using the composite, (0,90,0) and (0, 90, 90, 0).

The results of using higher shear deformation theory are compared with those obtained from using first order shear deformation and the Ansys Package with a percentage of discrepancy 9 %.

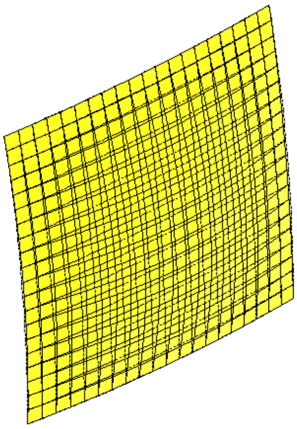
The results indicates a decrease in the frequency parameter, for example for composite (0,90) a decrease from 16.69 for (R/a)= 5 and 10.27 for (R/a)= 20 while in thin cylinder (a/H)= 10, No noticeable change , the frequency parameter decreases from 9.023 for (R/a)= 5 to 8.972 for (R/a)= 2. Similar trends were noticed for the other composites. However Fig. 4 shows theses trends for both composies (0, 90) and (0, 90, 0).

Table 1
Nondimensionalized fundamental frequencies versus Radius-to-side length ratios of spherical shell.

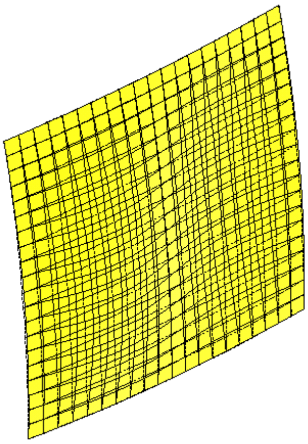
(R/a)	Theory	[0-90]		[0-90-0]		[0-90-90-0]	
		(a/H)=100	(a/H)=10	(a/H)=100	(a/H)=10	(a/H)=100	(a/H)=10
5	FST	28.825	9.230	30.993	12.372	31.079	12.437
	Present Work	28.829	9.307	30.999	12.018	31.083	12.007
	FEM	27.563	8.872	29.253	11.563	30.146	11.683
Discrepancy%		4.3	4.7	5.63	3.67	3	2.6
10	FST	16.706	8.984	20.347	12.215	20.380	12.280
	Present Work	16.710	9.064	20.353	11.853	20.385	11.840
	FEM	16.001	8.254	19.754	11.102	19.831	11.024
Discrepancy%		4.24	9	2.9	6.1	2.71	6.64
20	FST	11.841	8.921	16.627	12.176	16.638	12.240
	Present Work	11.847	9.002	16.634	11.811	16.643	11.798
	FEM	11.011	8.201	16.001	11.310	15.885	11.023
Discrepancy%		7.06	8.97	3.807	4.11	4.55	6.33

Table 2
Nondimensionalized fundamental frequencies versus Radius-to-side length ratios of cylindrical shell.

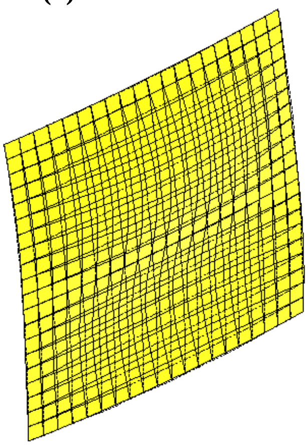
(R/a)	Theory	[0-90]		[0-90-0]		[0-90-90-0]	
		(a/H)=100	(a/H)=10	(a/H)=100	(a/H)=10	(a/H)=100	(a/H)=10
5	FST	16.668	8.9082	20.332	12.207	20.361	12.267
	Present Work	16.690	9.0230	20.330	11.850	20.360	11.830
	FEM	16.001	8.731	19.705	11.203	19.871	11.211
Discrepancy%		4.13	4.99	3.54	2.86	3.09	4.55
10	FST	11.831	8.887	16.625	12.173	16.634	12.236
	Present Work	11.840	8.979	16.620	11.8	16.630	11.79
	FEM	11.151	8.535	16.031	11.451	16.115	11.233
Discrepancy%		5.82	4.99	3.54	2.86	3.09	4.55
20	FST	10.265	8.89	15.556	12.166	15.559	12.23
	Present Work	10.27	8.972	15.55	11.79	15.55	11.78
	FEM	9.891	8.420	15.067	11.247	15.004	11.136
Discrepancy%		3.69	6.2	3.1	4.48	3.509	5.265



(a)- first mode



(b)-second mode



(c)-third mode

Fig. 2 Mode shapes of [0-90] spherical shell.

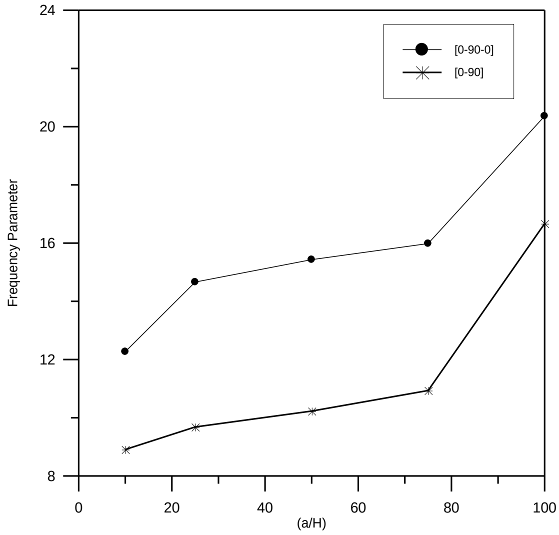


Fig. 3 Frequency parameter $\Omega = \left(\frac{\omega a^2}{H}\right)\sqrt{\frac{\rho}{E_2}}$ **change with (a/H) for cylindrical shell.**

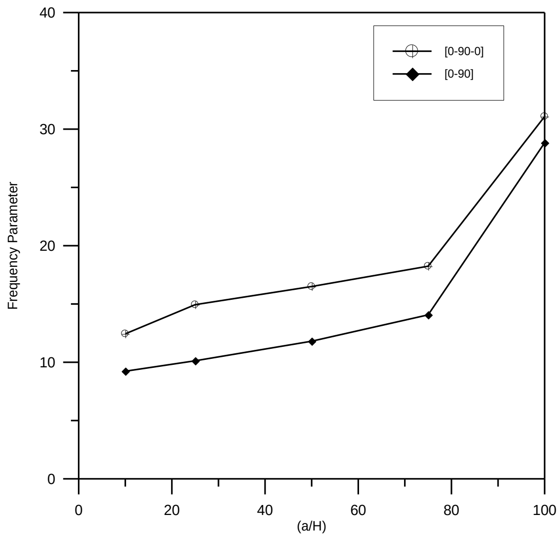


Fig. 4 Frequency parameter $\Omega = \left(\frac{\omega a^2}{H}\right)\sqrt{\frac{\rho}{E_2}}$ **change with (a/H) for spherical shell**

Conclusions

The following points may be summarized from the current work:

1. The developed analytical solution may be used for the dynamic analysis of thin composite shell, spherical or cylindrical and may be extended to the conical shells. The validity obtained between the analytical and numerical results were in good agreement with a maximum discrepancy of 6.2%.
2. The natural frequency tends to increase with the increasing of radius to side length for all the composite shells (0, 90, 0, 90, 0) and (0, 90, 90, 0).
3. The natural frequency is decreased with increasing the radius/side ratio for all the types of composites. (0, 90), (0, 90, 90, 0).
4. The frequency parameter $\Omega = \left(\frac{\omega a^2}{H}\right)\sqrt{\rho / E_2}$ for spherical shell is more than that of the cylindrical shell for the same radius/side ratio.

Nomenclature

a	: radius mm
a	: Side length (mm)
E	: Modulus of elasticity (N/m^2)
G	: Modulus of rigidity (N/m^2)
H	: Thickness (mm)
I_i	: Integrations
M_i	: Resultant moments per unit length N.m/mm
N_i	: Resultant forces per unit length N/m
P	: Density (kg/m^3)
R	: Radius (mm)
t	: Time (sec)
u, v, w	: Displacement components in x, y, and z directions respectively (mm)
δU	: Change in strain energy (N.m)
δK	: Change in kinetic energy (N.m)
ε_i	: Strain component in the principal direction (I, i=1,….6)
σ_i	: Stress component in the principal direction (I, i=1,….6)
Φ_i, θi	: Rotations
ω	: Frequency (rad/s)
ν	: Poisons ratio

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التحليل الديناميكي للقشريات الاسطوانية والكروية الخفيفة المصنوعة من مواد مركبة

قصي حاتم جبر
الكلية التقنية/بغداد هيئة التعليم التقني.

الخلاصة

في هذا البحث تم دراسة التحليل الديناميكي للقشريات الاسطوانية والكروية المركبة . استند الحل التحليلي على نظرية تشوه القص من الرتبة العليا للقشريات المرنة والتي منها نستطيع اشتقاق المعادلات الخاصة بالطبقات الغير سوية الخواص. ان ذلك يشمل ايجاد الترددات الطبيعية وانساق الاهتزازات للقشريات الاسطوانية والكروية المركبة والمسندة ببساطة.

تم اثبات النتائج التحليلية والتي تم الحصول عليها باستخدام المعادلات المطورة بواسطة تقنية العناصر المحددة من خلال برنامجANSY.
بينت النتائج توافقا جيدا وباعظم نسبة تفاوت9% في التحليل الديناميي للقشريات الاسطوانية والكروية.