



## Modeling and Filtering for Tracking Maneuvering Targets

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### Abstract

A new mathematical model describing the motion of manned maneuvering targets is presented. This model is simple to be implemented and closely represents the motion of maneuvering targets. The target maneuver or acceleration is correlated in time. Optimal Kalman filter is used as a tracking filter which results in effective tracker that prevents the loss of track or filter divergency that often occurs with conventional tracking filter when the target performs a moderate or heavy maneuver. Computer simulation studies show that the proposed tracker provides sufficient accuracy.

**Keywords:** Kalman Filter, Modelling and Filtering, Tracking Maneuvering Targets.

### 1. Introduction

For years lots of effort has been spent on the development of sophisticated digital filtering algorithms for tracking maneuvering targets. These algorithms can be classified into classical and modern. The classical algorithms include least squares and polynomial filter [1, 2, 3], Wiener filter [4], and  $\alpha - \beta$  filter [5, 6]. The two-point extrapolator is considered as a non-recursive filter and can be implemented without any need for a storage device [3]. The function of this filter is simply obtained through the use of the last two data points. The other simple approach is the Wiener filter. It is a constant gain filter which is equivalent to the steady state gain of the regular Kalman filter [7]. Wiener filter does not require the calculation of the covariance elements; thus this filter does not account for the variation and the statistics in the target maneuver. Furthermore, this scheme incurred the problem of tracking both the maneuvering and non maneuvering targets with the same accuracy, as well as might even loose the track or diverge.

The  $\alpha - \beta$  filter is another classical tracking scheme extensively utilized in most modest tracking scenarios [5, 6]. It is designed to minimize the mean square error in the filtered state under the assumption that the target moves along straight line trajectory, so it has small

capability to track severely maneuvering targets. For this reason, various maneuvering detectors are often attached to facilitate its job against evasive vehicles.

The modern algorithms involve the use of state space estimation and adaptive Kalman filtering [8]. Gurfil et. al. [9] suggest an attractive alternative method to the standard Kalman filter to optimally estimate three dimensional states of maneuvering target in two steps: the first is linear and the second is nonlinear. Another technique described by Sinha et. al. [10], involves switching between the Kalman-levy filter and the standard Kalman filter. The Kalman-levy filter is more effective in response to large error due to the onset acceleration or deceleration; while the performance of this filter is worse in the non-maneuvering portion. For this reason the system switched to the standard Kalman filter.

In this paper a simple and accurate target model is developed. The maneuver equations are derived for the actual continuous time target motion and then expressed in discrete time according to the standard discretization procedure providing accurate statistical representation of the true target behavior [11]. The remaining parts of this paper are devoted to dynamic equation of target maneuver, discrete time target equations of motion, optimal Kalman tracking filter and computer simulation.

## 2. Dynamic Equations of Target Maneuver

The model stimulated in this section is based on the fact that without maneuver the target under consideration, e.g. aircraft, generally follows a straight line constant speed trajectory. Turn, evasive maneuvers and accelerations due to atmospheric turbulence may be viewed as perturbations on this flying trajectory. The continuous time target equation of motion may be represented by [11]:

$$\frac{d}{dt} X'(t) = F' \cdot X'(t) + G' \cdot a(t) \quad \dots (1)$$

where: a(t) is target acceleration

$$X'(t) = \begin{cases} r(t) & \text{target range at time } t \\ v(t) & \text{target velocity at time } t \end{cases}$$

$$F' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad G' = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The acceleration term a(t) is assumed to be white Gaussian noise. The normality assumption of the noise is one of the necessary conditions for applying the theory of optimum Kalman filter [7]. However, the whiteness here seems to be inappropriate justification for the real-world auto-commanded vehicles. For such vehicles, the target acceleration and hence the target maneuver are correlated in time: namely, if the target is accelerated at time t, it is likely to be accelerated at time (t + τ) for sufficiently small τ. For example, a lazy turn will often give rise to correlated acceleration inputs for up to one minute; evasive maneuvers will provide correlated acceleration inputs for periods between ten to thirty seconds and atmospheric turbulence may provide correlated acceleration inputs for one to two seconds. A typical representative model of the correlation function c(τ) associated with the target acceleration is assumed to be:

$$c(\tau) = E \{ a(t) \cdot a(t + \tau) \} = \sigma_m^2 e^{-b|\tau|} \quad b \geq 0 \quad \dots (2)$$

where,  $\sigma_m^2$  is the variance of the target acceleration and b is the reciprocal of the maneuver (acceleration) time constant.

For example:  $b \approx 1/60$  for a lazy turn,  $b \approx 1/20$  for an evasive maneuver and  $b \approx 1$  for atmospheric turbulence.

Now, taking the Laplace transform of both sides of Eq. (2) and by partitioning the result, one can get

$$C(s) = \Gamma \{ c(\tau) \} = \left[ \frac{-2b}{(s-b)(s+b)} \right] \sigma_m^2 = H(-s) \cdot H(s) \cdot W(s) \quad \dots (3)$$

where:  $\Gamma \{ . \}$  is the Laplace transform operator,

$$H(s) = \frac{1}{(s+b)}$$

and  $W(s) = 2b \sigma_m^2$

The term H(s) is the transfer function of the physical shaping filter for a(t), and W(s) is the transform of the white noise w(t) that drives a(t). The resulting equation of the shaping filter in time domain is

$$\dot{a}(t) = -b \cdot a(t) + w(t) \quad \dots (4)$$

For which  $c_w(\tau)$  is the correlation function of the input white noise which satisfies

$$c_w(\tau) = 2b \cdot \sigma_m^2 \cdot \delta(\tau) \quad \dots (5)$$

This secondary system is blended with the pervious two state per coordinate target model Eq.(1) to obtain the overall augmented target model which is driven by a white noise w(t) as follows :

$$\frac{d}{dt} X(t) = F \cdot X(t) + G \cdot w(t) \quad \dots (6)$$

where:  $X(t) = \begin{bmatrix} r(t) \\ v(t) \\ a(t) \end{bmatrix}$

w(t) is a zero mean white Gaussian noise driving function with covariance equal to  $2b \sigma_m^2$ ,

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & b \end{bmatrix} \quad \text{and} \quad G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

### 3. Discrete Time Target Equations of Motion

The discrete form of the target model can readily be found by discretizing the continuous form of the target equation of motion described in Eq.(6) by simply using the standard discretization procedure explained in [11] . This is done by integrating Eq.(6) over the interval (t , t+T) to get

$$X(t+T) = e^{FT} \cdot X(t) + \int_t^{t+T} e^{F((t+T)-\tau)} G \cdot w(\tau) d\tau \quad \dots (7)$$

Rewriting Eq.(7) in appropriate form, then

$$X(k+1) = \phi(k+1, k) \cdot X(k) + u(k) \quad \dots (8)$$

where  $\phi(\tau_2, \tau_1) = e^{F(\tau_2 - \tau_1)}$

$$u(k) = \int_{kT}^{(k+1)T} e^{F[(k+1)T - \tau]} \cdot G \cdot w(\tau) \cdot d\tau \quad \dots (9)$$

and  $t = kT$

It can be easily verified that the state transition matrix  $\phi(k+1, k)$  is

$$\phi(k+1, k) = \begin{bmatrix} 1 & T & \frac{1}{b^2}[-1 + bT + e^{-bT}] \\ 0 & 1 & \frac{1}{b}[1 - e^{-bT}] \\ 0 & 0 & e^{-bT} \end{bmatrix} \quad \dots (10)$$

And when bT is small, this matrix can be reduced to the Newtonian matrix

$$\phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad \dots (11)$$

Furthermore, the input vector of the maneuver excitation noise u(k) given in the target model Eq.(8) is not equivalent to the sampled version of the continuous time white noise w(t) as it is seen in Eq.(9). After substituting F and G in Eq.(9), the input noise vector is determined as follows :

$$u(k) = \int_{kT}^{(k+1)T} \begin{bmatrix} \frac{1}{b^2}[-1 + b\{(k+1)T - \tau\} + \exp[-b\{(k+1)T - \tau\}]] \\ \frac{1}{b}[1 - \exp[-b\{(k+1)T - \tau\}]] \\ \exp[-b\{(k+1)T - \tau\}] \end{bmatrix} w(\tau) d\tau$$

$$= \int_{kT}^{(k+1)T} \begin{bmatrix} n_1(\tau) \\ n_2(\tau) \\ n_3(\tau) \end{bmatrix} \cdot w(\tau) d\tau \quad \dots (12)$$

Since w(t) is a zero mean white Gaussian noise, then u(k) is a discrete time white Gaussian sequence with zero mean and covariance matrix Q(k):

$$E\{u(k)\} = 0$$

$$E\{u(k) \cdot u^T(j)\} = Q(k) \cdot \delta(k - j) \quad \dots (13)$$

where:  $\delta(\cdot)$  is Kronecker delta symbol, and

$$Q(k) = \int_{kT}^{(k+1)T} \phi((k+1)T, \tau) G \cdot 2b \sigma_m^2 G^T \phi^T((k+1)T, \tau) d\tau \quad \dots (14)$$

After substituting the matrices  $G, G^T, \phi$  and  $\phi^T$  in Eq. (14), the covariance matrix is simplified to:

$$Q(k) = 2b \cdot \sigma_m^2 \int_{kT}^{(k+1)T} \begin{bmatrix} n_1^2(\tau) & n_1(\tau)n_2(\tau) & n_1(\tau)n_3(\tau) \\ n_2(\tau)n_1(\tau) & n_2^2(\tau) & n_2(\tau)n_3(\tau) \\ n_3(\tau)n_1(\tau) & n_3(\tau)n_2(\tau) & n_3^2(\tau) \end{bmatrix} d\tau$$

$$= 2b \cdot \sigma_m^2 \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \quad \dots (15)$$

where:

$$q_{11} = \frac{1}{2b^5} \left[ 1 - e^{-2bT} + 2bT + \frac{2b^3 T^3}{3} - 2b^2 T^2 - 4bT \cdot e^{-bT} \right]$$

$$q_{12} = \frac{1}{2b^4} \left[ 1 + e^{-2bT} - 2e^{-bT} + 2bT e^{-bT} - 2bT + b^2 T^2 \right]$$

$$q_{13} = \frac{1}{2b^3} [1 - e^{-2bT} - 2bT \cdot e^{-bT}]$$

$$q_{22} = \frac{1}{2b^3} [4 \cdot e^{-bT} - 3 - e^{-2bT} + 2bT]$$

$$q_{23} = \frac{1}{2b^2} [1 + e^{-2bT} - 2 \cdot e^{-bT}]$$

$$q_{33} = \frac{1}{2b} [1 - e^{-2bT}]$$

$$q_{21} = q_{12}$$

$$q_{31} = q_{13}$$

$$q_{32} = q_{23}$$

When bT is sufficiently small then

$$\lim_{bT \rightarrow 0} Q(k) = 2b\sigma_m^2 \begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix}$$

Reflecting the fact that for sufficiently short time periods the physical target moves at essentially constant velocity. For a fixed sampling period T, as  $b \rightarrow \infty$

$$\lim_{b \rightarrow \infty} Q(k) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_m^2 \end{bmatrix}$$

Furthermore, to be able to apply the theory of optimal Kalman filtering, an output equation is needed to supply the desired information about the system. Along each independent coordinate (range, elevation or azimuth angle) being analyzed and processed, an observation or output model should be defined. This model describes the tracking sensor or measuring channel which is simply modeled as a sampled version of the observation disturbed by an additive white Gaussian noise corrupting the measured information. Again the range channel is considered here as follows:

$$y_r(k) = r(k) + n_r(k) \quad \dots (16)$$

where:  $y_r(k)$  is the measured range,  
 $r(k)$  is the exact range,  
 $n_r(k)$  is the additive white Gaussian noise uncorrelated with  $u(k)$  and have the following statistics

$$E\{n_r(k)\} = 0$$

$$E\{n_r(k) \cdot n_r(i)\} = \sigma_r^2 \cdot \delta(k-i)$$

$$E\{n_r(k) \cdot u(i)\} = 0 \text{ for all } k \text{ \& } i .$$

And  $\sigma_r^2$  is the variance of the observation channel noise.

Rewriting Eq.(16) in terms of the target state,

$$y_r(k) = H \cdot X(k) + n_r(k) \quad \dots (17)$$

where:  $H = [1 \ 0 \ 0]$  is the observation matrix.

The target and observation model for elevation angle  $\alpha(k)$  and azimuth angle  $\beta(k)$  can be easily derived using exactly the same manipulations that are used to derive the range model. However, the final form of these models are given here and as follows :

### Elevation Angle

Target model :

$$X_e(k+1) = \phi(k+1, k) \cdot X_e(k) + u_e(k)$$

Observation model :  $y_e(k) = H \cdot X_e(k) + n_e(k)$

Prior statistics:  $E\{u_e(k)\} = 0$

$$E\{n_e(k)\} = 0$$

$$E\{u_e(k) \cdot u_e^T(i)\} = Q_e(k) \cdot \delta(k-i)$$

$$E\{n_e(k) \cdot n_e(i)\} = \sigma_e^2 \cdot \delta(k-i)$$

$$E\{u_e(k) \cdot n_e(i)\} = 0 \text{ for all } i \text{ \& } k$$

Matrices:

$$\phi(k+1, k) = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix},$$

$$H = [1 \ 0 \ 0],$$

and  $X_e(k) = [\alpha(k) \ \dot{\alpha}(k) \ \ddot{\alpha}(k)]^T$

### Azimuth Angle

Target model :

$$X_a(k+1) = \phi(k+1, k) \cdot X_a(k) + u_a(k)$$

Observation model :  $y_a(k) = H \cdot X_a(k) + n_a(k)$

Prior statistics:  $E\{u_a(k)\} = 0$

$$\begin{aligned}
E\{n_a(k)\} &= 0 \\
E\{u_a(k) \cdot u_a^T(i)\} &= Q_a(k) \cdot \delta(k-i) \\
E\{n_a(k) \cdot n_a(i)\} &= \sigma_a^2 \cdot \delta(k-i) \\
E\{u_a(k) \cdot n_a(i)\} &= 0 \text{ for all } i \& k
\end{aligned}$$

Matrices :

$$\phi(k+1, k) = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix},$$

$$H = [1 \ 0 \ 0],$$

$$\text{and } X_a(k) = [\beta(k) \ \dot{\beta}(k) \ \ddot{\beta}(k)]^T$$

where:

$\alpha(k), \dot{\alpha}(k), \ddot{\alpha}(k)$  are elevation angle, first and second derivative of elevation angle respectively,

$\beta(k), \dot{\beta}(k), \ddot{\beta}(k)$  are azimuth angle, first and second derivative of azimuth angle respectively,

$\sigma_e^2, \sigma_a^2$  are the error (variance of observation channel noise) in the measured elevation and the azimuth angles respectively,

$y_e(k), y_a(k)$  are the measured elevation and the azimuth angles respectively.

It is clear that the developed models for range, elevation and azimuth channels are decoupled, because there is no cross-coupling or dependence between any two associated items of any channel. Thus, these coordinates can be processed and estimated via implementing three independent tracking filters.

#### 4. Optimal Kalman Tracking Filter

The aforementioned target and observation models for range, elevation and azimuth coordinates have similar aspects and they are suitably to confirm the requirements of implementation Kalman filtering algorithm. It is recommended here to define Kalman tracking filter for one channel only (the range channel), while the others are exactly the same. The following equations summarize the recursive Kalman tracking filter for the range coordinate [7]:

Target model:

$$X(k+1) = \phi(k+1, k) \cdot X(k) + u(k)$$

Observation model :  $y_r(k) = H \cdot X(k) + n_r(k)$

Filtered estimate:

$$\hat{X}(k+1/k+1) = \phi \cdot \hat{X}(k/k) + K(k+1) \cdot [y_r(k+1) - H \cdot \phi \cdot \hat{X}(k/k)]$$

Predicted estimate:  $\hat{X}(k+1/k) = \phi \cdot \hat{X}(k/k)$

Kalman gain:

$$K(k+1) = P(k+1/k) \cdot H^T [H \cdot P(k+1/k) H^T + \sigma_r^2]^{-1}$$

Covariance matrix of predicted error:

$$P(k+1/k) = \phi \cdot P(k/k) \cdot \phi^T + Q(k)$$

Covariance matrix of filtered error:

$$P(k+1/k+1) = [I - K(k+1) \cdot H] P(k+1/k)$$

Estimate of maneuvering target range coordinate by these Kalman filter recursive equations require an initial estimates of  $\hat{X}(0/0)$  and  $P(0/0)$  to be inspired. The initialization is based on the first two observations as follows:

$$\hat{r}(0/0) = y_r(0)$$

$$\hat{v}(0/0) = \frac{(y_r(1) - y_r(0))}{T}$$

$$\hat{a}(0/0) = 0$$

$$\hat{X}(0/0) = [\hat{r}(0/0) \ \hat{v}(0/0) \ \hat{a}(0/0)]^T$$

where:  $y_r(0)$  and  $y_r(1)$  are, respectively, the first and second received sensor measurements. The corresponding covariance matrix of the filtered error estimated is defined as:

$$P_{11}(0/0) = \sigma_r^2$$

$$P_{12}(0/0) = P_{21}(0/0) = \sigma_r^2 / T$$

$$P_{22}(0/0) = 2 \sigma_r^2 / T^2$$

$$P_{13}(0/0) = P_{31}(0/0) = P_{23}(0/0) =$$

$$P_{32}(0/0) = P_{33}(0/0) = 0$$

#### 5. Computer Simulations

Computer simulation studies are used to verify, compare and evaluate the performance of the developed model. The tracking filter is exercised under different flight environments. Tracking

performance is evaluated by tracking an accurate figure given by:

$$\sigma_{est.}(r) = \sqrt{\frac{1}{N} \sum_{i=1}^N [r(i) - \hat{r}(i/i)]^2}$$

where it is interpreted as the Root-Mean-Square (RMS) of the range estimate error of N estimated points on the tracked trajectory.

However, computer simulation requires two additional subroutines. The first is used to generate a wide class of maneuvering target trajectories, from lightly to heavily maneuvered targets and with three different turns (90°, 180°, 270°). The second subroutine generates a white Gaussian noise with different strengths representing the additive observation channel corruptions  $n_r(\cdot)$ .

For different cases are examined as follows:

### Case One:

Different target aviations are simulated and unified to the datum of  $\sigma_r = 150m$ ,  $T = 0.1s$ , initial velocity  $v(0) = 500m/s$ . The target performs three independent turns of 90°, 180°, and 270° for each single flight and with different accelerations: 1, 10, 20, 30, 40, and 50 m/s<sup>2</sup>. These trajectories are generated and sampled at an interval  $T = 0.1$  s. Observations are formed using white Gaussian noise generator with the specified standard deviation ( $\sigma_r = 150m$ ). For each run, 1200 observations ( $N = 1200$ ) are constructed. These data are then filtered by standard Kalman filter based on the developed model assuming a moderate value for  $\sigma_m$  ( $\sigma_m = 2m/s^2$ ) and for all trajectories. The tracking accuracy is computed (MSE) using 1200 estimated points. The results are listed in table (1).

**Table 1,**  
**Range Tracking Accuracy for Different Target Maneuvers and Turns.**

Target acceleration (m/s <sup>2</sup> )	Range tracking accuracy $\sigma_{est.}(r)$ m		
	90° turn	180° turn	270° turn
1	38.67	38.73	39.32
10	38.89	40.62	40.93
20	40.53	43.40	44.76
30	46.65	48.17	48.52
40	49.84	53.31	54.16
50	55.26	58.07	58.78

### Case Two:

For the purpose of evaluating the tracking accuracy of the proposed tracker, the tracking performance of the proposed tracker is compared with the performance of other tracking filters such as  $\alpha - \beta$  filter [6] and Wiener filter [4] under different flight environments. It is assumed that  $\sigma_r = 150m$ ,  $T = 0.1s$ ,  $N = 1200$  and 90° turn. Computer results are shown in table (2).

**Table 2,**  
**Range Tracking Accuracy of Different Filters.**

Target acceleration (m/s <sup>2</sup> )	Tracking accuracy $\sigma_{est.}(r)$ m		
	Proposed filter	$\alpha - \beta$ filter	Wiener filter
1	38.67	30.71	28.26
10	38.89	36.13	34.88
20	40.53	72.62	78.04
30	46.65	divergent	divergent
40	49.84	divergent	divergent
50	55.26	divergent	divergent

It is clearly seen from these results that Wiener filter and  $\alpha - \beta$  filter are suitable only for tracking non-maneuvering or slowly fluctuating targets.

### Case Three:

All parameters in target and observation models can be specified with sufficient accuracy before processing the trajectory of enemy maneuvering target except for the variance of the target acceleration or maneuver since this parameter describes the target behavior or statistic of target maneuverability during its flight. Actually, the target usually behaves in undetermined aspects unknown to the tracking filter. This fact leads to incorrect choice of  $\sigma_m$  and hence degradation in filter tracking accuracy.

The effect of uncertainty in  $\sigma_m$  on the range tracking accuracy is investigated by simulating various trajectories with the following parameters:

$T = 0.1s$ ,  $\sigma_r = 150m$ , 90° turn and  $a = 1, 10, 20, 30, 40,$  and  $50$  m/s<sup>2</sup> and processing each trajectory using three different values of  $\sigma_m$  as :

$0.5m/s^2$ ,  $2m/s^2$  and  $5m/s^2$ . The results are listed in table (3).

The symbol \* in each row of table (3) denotes the highest tracking accuracy achieved for the considered target maneuver or acceleration. It is

well evident from these results that for lightly maneuvered target (  $a \leq 10 \text{ m/s}^2$  )  $\sigma_m = 0.5 \text{ m/s}^2$  is more suitable while for heavily maneuvered targets (  $a \geq 30 \text{ m/s}^2$  )  $\sigma_m = 5 \text{ m/s}^2$  is more suitable than  $\sigma_m = 0.5$  or  $2 \text{ m/s}^2$ .

**Table 3,**  
**Range Tracking Accuracy for Different Standard Deviation of Target Acceleration.**

Target acceleration (m/s <sup>2</sup> )	Range tracking accuracy		
	$\sigma_{est.}(r) \text{ m}$		
	$\sigma_m = 0.5 \text{ m/s}^2$	$\sigma_m = 2 \text{ m/s}^2$	$\sigma_m = 5 \text{ m/s}^2$
1	24.13*	38.67	63.22
10	25.30*	38.89	54.87
20	49.06	40.53*	48.45
30	78.82	46.65	42.13*
40	106.55	49.84	44.28*
50	129.28	55.26	51.76*

#### Case Four:

Although  $\sigma_m = 0.5 \text{ m/s}^2$  and  $\sigma_m = 5 \text{ m/s}^2$  provide high tracking accuracy for processing trajectories of lightly and heavily maneuvered targets respectively; however, these values are not the proper or optimum  $\sigma_m$ . In this run, optimal value of  $\sigma_m$ , that yields the highest tracking accuracy, is searched for the simulation trajectories of case three. These attributes are shown in table (4).

**Table 4,**  
**Range Tracking Accuracy at Optimum Standard Deviation of Target Acceleration for Different Target Maneuvers.**

Target acceleration (m/s <sup>2</sup> )	Range tracking accuracy	Optimum
	$\sigma_{est.}(r) \text{ m}$	$\sigma_m (m/s^2)$
1	19.37	0.02
10	23.68	0.8
20	28.76	3.2
30	29.53	5.9
40	28.13	6.7
50	30.49	8.3

These results show that optimal variance of target acceleration varies in wide extents and have great influence on the tracking performance of the filter.

## 6. Conclusion

Using a simple target model that accounts statistically for the magnitude and duration of target maneuver has shown how a Kalman filter can be constructed to track maneuvering targets. The important features of the presented target model are : firstly, it is simple to be implemented, secondly, it is able to describe wide class of maneuvering target trajectories from lightly to heavily maneuvered targets, and thirdly, It is derived in a decoupled form for the range, elevation and azimuth angles. Thus, these coordinates can be processed and estimated via implementing three independent tracking filters. This advantage facilitates the tracker activity in two ways. First, the computational efforts are greatly reduced since the overall system dimension is reduced from 9 x 9 to three separate models of 3 x 3 dimensional subsystems for each coordinate. Secondly, the system reliability is further enhanced when applied for on-line tactical combat conditions.

The tracking performance of the proposed Kalman filter has been analyzed and tested by using different computer simulation studies. It is shown that using the proposed filter, the error in sensor range measurement is reduced from 150m ( $\sigma_r = 150m$ ) to (40 – 60) meters depending on the target maneuverability as shown in table (1).

The tracking performance of the filter is also compared with the  $-\beta$  tracking filter [5, 6] and Wiener filter [4] under various flight environments. These two filters exhibit higher tracking accuracy than the suggested Kalman filter. In case of applying these two filters in real world, the filter may lose the track or diverge when the target performs moderate or heavy maneuver ( $a \geq 30 \text{ m/s}^2$  table (2)) while the target presented here will never diverge.

The main problem addressed by computer simulation studies is the degradation in tracking performance due to uncertainty in model parameters especially the variance of target acceleration  $\sigma_m^2$ . A comparison between table (4) and table (3) illustrates how the tracking accuracy is significantly improved when  $\sigma_m$  is properly

selected to fit the target maneuver. In all computer simulation studies, it is assumed that  $\sigma_m$  is constant for the whole trajectory; however, this assumption is not always correct. Thus, the demand for on-line adaptation of  $\sigma_m$  is greatly highlighted to enhance the filter performance in front of any sudden changes encountered during target flight trajectory.

## 7. References

- [1] Eli Brooker, "Tracking and Kalman Filtering Made Easy", John Wiley & Sons, Inc., 2001.
- [2] Lennert Ljung, "System Identification Theory for the User", Prentice Hall, Inc., 1999.
- [3] C. Schell, S.P. Linder and J.R. Zeidler, "Tracking Highly Maneuverable Target with Unknown Behaviour", Proceedings of the IEEE, Vol. 92, No.3, pp. 558-574, March 2004.
- [4] X.R. Li and V.P. Jilkov, "Survey of Maneuverable Target Tracking-Part I: Dynamic Models", IEEE Trans. on Aerospace and Electronic systems, Vol. AES-39, Issue4, pp. 1333-1364, Oct. 2003.
- [5] Jae-Chem Yoo and Young-Soo Kim, "Alpha-Beta Tracking Index ( $\alpha$ - $\beta$ - $\Lambda$ ) Filter", Signal Processing, Elsevier, Vol.83, Issue 1, pp. 169-180, Jan. 2003.
- [6] T.L. Ogle and W.D. Blair, "Fixed-Lag  $\alpha$ - $\beta$  Filter for Target Trajectory Smoothing, IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-40, Issue 4, pp. 1417-1421 Oct. 2004.
- [7] Dan Simon, "Optimal State Estimation: Kalman H-infinity and Nonlinear Approaches", John Wiley & Sons, Inc., 2006.
- [8] Y. Yu and Q. Cheng, "Particle Filters for Maneuvering Target Tracking Problems", Signal Processing, Elsevier, Vol. 86, Issue 1, pp. 195-203, Jan. 2006.
- [9] P. Gurfil and N.J. Kasdin, "Two Step Optimal Estimator for Three Dimensional Target Tracking", IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-41, Issue 3, pp. 780-793, July 2005.
- [10] A. Sinha, T.K. Barajan and Y. Bar-Shalom, "Application of the Kalman-Levy Filter for Tracking Maneuvering Targets", IEEE Trans. on Aerospace and Electronic Systems, Vol. AES-43, Issue 3, pp. 1097-1107, July 2007.
- [11] C.K. Chui, "Kalman Filtering: with Real-time Applications", Second Edition, New York, Springer-Verlag, 1999.



## النمآة والترشآ لمتابعة الأهآاف المناورة

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### الآلاصة :

آآضمن هآا البآآ بناء أنموآ رياضية آآلآ لتمآل حركة الأهآاف المناورة. آآمآز هآا الأنموآ الرياضي بسهولة الآنفآذ وآة تمآل حركة الأهآاف المناورة. أن مناورة الهآف أو آعآله آكون مترابآ مع الزمن. لآلآ تم أسآعمال مرشح كالمان المآلآ (Kalman filter) لآكون مرشح المتابعة (tracking) الآى آقق متابعة مؤآرة بآآ لا مآل لفآآان متابعة الهآف أو الأنآراف عنه كما آآآلآ عند أسآآام مرشآات المتابعة الآقلاآبة عنآما آقوم الهآف بمناورة معآلة أو شآآبة. تم مآكاة هآة الطرآفة بأسآآام الآسوب وآآآ ظروف متابعة مآآآفة وآآآلآ النآآآ الآهآآبة آآة آة مقبولة.