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Vibration Analysis of Cross-Ply Plates Under Initial Stress Using Refined Theory

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Abstract

Natural frequency under initial stresses for simply supported cross-ply composite laminated plates (E glass- fiber) are obtained using Refind theory (RPT). This theory accounts for parabolic distribution of the transverse shear strain through the plate thickness and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. The governing equations for Eigen value problem under initial stress are derived using Hamilton's principle and solved using Navier solution for simply supported cross-ply symmetric and antisymmetric laminated plates. The effect of many design factors such as modulus ratio, thickness ratio and number of laminates on the Natural frequency and buckling stresses of orthotropic plates are studied. The results are compared with other researcher.

Keyword: *Composite laminated plate, buckling analysis, free vibration analysis, Refined plate theory.*

1. Introduction

Laminated composite plates have very importance in the engineering applications because of their useful features so a many variety of laminated theories for static and dynamic behavior have been developed such as approximate, experimental and exact methods.

[1] presented static analysis using higher-order refined theory of angle ply plate and sandewich plates hitherto. No requirement to use shear correction factors (SCF), because the transverse-shear strains vary parabolicaly from side to side which lead to vanish the shear-stresses on the upper and bottom surface of the plate. From principle of potential energy, the equations of equilibrium are derived and solved by using Navier-type method. Correctness of the theoretical preparations and the solution method confirmed by comparing the results with other theory described in the literature. [2] Presented buckling analysis of SS plate exposed to in-plane loading using refined plate theory of orthotropic and isotropic plates. The governing equations G.E which derivative from the principle of virtul-displacements, and solved by using the Navier method. This theory is simple, comparable to the(FSDT) theory and there no exists a need for using SCF. **[3]** Studied a two-variable Refind theory (RPT) of lamineted composite plates. The theory contents the zero traction B.C on the upper and bottom faces of the plate without wanting to use SCF. The equations of motion are derivative using Hamilton's principle (H.P) and solved using Naveir method of angle-ply and cross-ply antisymmetric laminate. This theory is simple and accurate in solving the buckling behaviors and static bending of laminated composite plates. [4] Studied free vibration of laminated composite plates using two variable Refined plate theory (RPT) and using Hamilton's principle to derive the equations of motion, and these equations solved using Navier solutions of cross-ply and angle-ply antisymmetric laminates. This theory is accurate and effective in obtain the natural frequencies N.F of laminated composite plates. [5] Studied the buckling analysis using Refind theory for orthotropic plates. No requirement to use SCF in this theory and the Governing equations solved

using Levy-type method. It considering the effect of some design limitations such as boundary conditions, orthotropy ratio, thickness ratio and loading condition on the critical-buckling load. **[6]** Presented free-vibration investigation of functionaly arranged material (FGM) sandwich rectangular plates by the four variable refined theory (RPT) which not requirement to use SCF. the equation of motion achieved using Hamilton's principle for the (FGM) sandwich plates and these equations solved by using the Navier type. This theory simple and accurate in resolving the free-vibration behavior of the functionaly arranged material sandwich plates when its results comparing with other theories such as classical laminated theory(CLP), first order theory (FSDT). **[7]** Presented free vibration analysis of simply supported plate which made of functionaly arranged materials using four variable Refind theory. No requirement to use shear correction factors, because the transverse-shear strains vary parabolicaly from side to side the thickness which lead to disappear the shear stresses on the upper and bottom faces of the plate. From the principle of virtual displacements, the governing equations for the (FGM) rectangular plates are derived and solved by using Navier-type method. The natural frequencies are found using the Ritz method in the case of FG clamped plates. The strength of this present theory which gave accurate free vibration of FG plate shown by comparing the present results with others theories and also the influence of vying rises, aspect ratios, and thick ratio on the free-vibration of the FG plates is showed. [8] Presented free vibration analysis of rectangular plate with two opposite edges simply supported (SS) and the other two edges having arbitrary boundary conditions using 'refined plate theory'. From the principle of virtual displacements, the governing equations are derived and solved by using the Levy-type method. No need to use shear correction factors in this theory, it considering the effect of some design parameters such as boundary conditions, modulus ratio, and aspect ratio on the natural frequency.

In present work, the equation of motion of Refined plate theory are programming to find the Critical buckling and fundamental natural frequency for cross-ply plate for different thickness ratio, symmetric and antisymmetric and orthotropy ratio, while to obtain vibration characteristic of plate under initial stress, we derive equation of motion depending on Refined plate theory for simply supported plates using Navier solution.

2. Theoretical Analysis

2.1 Displacement Field

In present work, a rectangular plate of total thickness (h) of (n) orthotropic layers with the coordinate system as shown in Fig (1) are considered the displacement of Refined plate theory (RPT) which satisfies equilibrium conditions at the top and bottom faces of the plate without using shear correction factor is developed. The transverse displacement W contains three components; bending w_{be} , extension w_a and shear w_{sh} which these components are functions of coordinates x, y, and time t only. Similarly, the displacements u in x-direction and v in y-direction have bending, extension and shear components [3].

$U= u + u_{be} + u_{sh}$

$V= v + v_{be} + v_{sh}$

$W(x,y,z,t) = w_a(x,y,t) + w_{be}(x,y,t) + w_{sh}(x,y,t)$

The shear components u_{sh} and v_{sh} , w_{sh} lead to the parabolic variations of shear strains γ_{xz}, γ_{yz} and to shear stresses σ_{xz}, σ_{yz} through the thickness of the plate in such a way that shear stresses σ_{xz}, σ_{yz} are zero at the bottom and top surfaces of the plate.

$$u_{sh}= z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_{sh}}{\partial x}$$

$$v_{sh}= z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_{sh}}{\partial y}$$

The following displacement field assumptions [3]:

$$U(x,y,z,t) = u(x,y,t) - z\left(\frac{\partial w_{be}}{\partial x}\right) + z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_{sh}}{\partial x}$$

$$V(x,y,z,t) = v(x,y,t) - z\left(\frac{\partial w_{be}}{\partial y}\right) + z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_{sh}}{\partial y}$$

$$W(x,y,z,t) = w_a(x,y,t) + w_{be}(x,y,t) + w_{sh}(x,y,t) \quad \ldots(1)$$

For small strain, the strain-displacement relations take the form:

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) = \frac{1}{2}\gamma_{xy}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) = \frac{1}{2}\gamma_{xz}$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{yz} \quad \ldots(2)$$

By substituting eq. (1) into eq. (2) to give:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_{be}}{\partial x^2} + z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_{sh}}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w_{be}}{\partial y^2} + z\left[\frac{1}{4}-\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_{sh}}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w_{be}}{\partial x \partial y} + 2z\,[\tfrac{1}{4} - \tfrac{5}{3}\left(\tfrac{z}{h}\right)^2]\,\frac{\partial^2 w_{sh}}{\partial x \partial y}$$

$$\gamma_{yz} = \frac{\partial w_a}{\partial y} + [\tfrac{5}{4} - 5\,\tfrac{z^2}{h^2}]\,\frac{\partial w_{sh}}{\partial y}$$

$$\gamma_{xz} = \frac{\partial w_a}{\partial x} + [\tfrac{5}{4} - 5\,\tfrac{z^2}{h^2}]\,\frac{\partial w_{sh}}{\partial y} \qquad \ldots(3)$$

The strain field is:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}$$

$$\begin{Bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} = \begin{Bmatrix}\gamma_{xz}^a\\ \gamma_{yz}^a\end{Bmatrix} + g\begin{Bmatrix}\gamma_{xz}^s\\ \gamma_{yz}^s\end{Bmatrix} \qquad \ldots (4)$$

Where:

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{Bmatrix}$$

$$\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\frac{\partial^2 w_{be}}{\partial x^2}\\ -\frac{\partial^2 w_{be}}{\partial y^2}\\ -2\frac{\partial^2 w_{be}}{\partial x \partial y}\end{Bmatrix}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}-\frac{\partial^2 w_{sh}}{\partial x^2}\\ -\frac{\partial^2 w_{sh}}{\partial y^2}\\ -2\frac{\partial^2 w_{sh}}{\partial x \partial y}\end{Bmatrix},$$

$$\begin{Bmatrix}\gamma_{xz}^a\\ \gamma_{yz}^a\end{Bmatrix} = \begin{Bmatrix}\frac{\partial w_a}{\partial x}\\ \frac{\partial w_a}{\partial y}\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{xz}^s\\ \gamma_{yz}^s\end{Bmatrix} = \begin{Bmatrix}\frac{\partial w_{sh}}{\partial x}\\ \frac{\partial w_{sh}}{\partial y}\end{Bmatrix}$$

$$f= -\tfrac{1}{4}z + \tfrac{5}{3}z\left(\tfrac{z}{h}\right)^2, \quad g = \tfrac{5}{4} - 5\left(\tfrac{z}{h}\right)^2 \qquad \ldots (5)$$

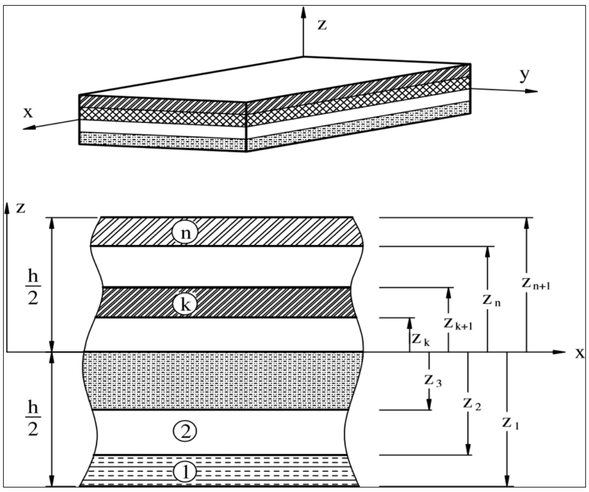

Fig. 1. coordinate system of laminated plates.

2.2 Principle of Virtual Work

Using Hamilton's principles, the equations of motion of the refined plate theory will be derived. Reddy, 2004

$$0 = \int_0^t (\delta U + \delta V - \delta T)\ dt \qquad \ldots (6)$$

- The virtual strain energy δU is:

$$\delta U = [\int_{\frac{h}{2}}^{-\frac{h}{2}} \left(\int_\Omega^k [\sigma_x \delta\varepsilon_x^k + \sigma_y \delta\varepsilon_y^k + \sigma_{xy}\delta\gamma_{xy}^k + \sigma_{yz}\gamma_{yz}^k + \sigma_{xz}\gamma_{xz}^k\,]\,\partial x\,\partial y\right)\partial z] = 0 \qquad \ldots (7)$$

Substituting Eq. (4) into Eq. (7)

$$\delta U = \int \{N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + Q_{yz}^a\delta\gamma_{yz}^a + Q_{xz}^a\delta\gamma_{xz}^a + Q_{yz}^s\delta\gamma_{yz}^s + Q_{xz}^s\delta\gamma_{xz}^s\}\ \partial x\,\partial y = 0 \qquad \ldots (8)$$

Where:

$$(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2}(\sigma_x, \sigma_y, \sigma_{xy})dz = \sum_{k=1}^N \int_{z_k}^{z_{k+1}}(\sigma_x, \sigma_y, \sigma_{xy})dz$$

$$(M_x^b, M_y^b, M_{xy}^b) = \int_{-h/2}^{h/2}(\sigma_x, \sigma_y, \sigma_{xy})z\,dz = \sum_{k=1}^N \int_{z_k}^{z_{k+1}}(\sigma_x, \sigma_y, \sigma_{xy})z\,dz$$

$$(M_x^s, M_y^s, M_{xy}^s) = \int_{-h/2}^{h/2}(\sigma_x, \sigma_y, \sigma_{xy})f\,dz = \sum_{k=1}^N \int_{z_k}^{z_{k+1}}(\sigma_x, \sigma_y, \sigma_{xy})f\,dz$$

$$(Q_{xz}^a, Q_{yz}^a, Q_{yz}^s, Q_{xz}^s) = \int_{-h/2}^{h/2}(\sigma_{xz}, \sigma_{yz}, g\sigma_{xz}, g\sigma_{yz})\,dz = \sum_{k=1}^N \int_{z_k}^{z_{k+1}} (\sigma_{xz}, \sigma_{yz}, g\sigma_{xz}, g\sigma_{yz})\,dz \qquad \ldots (9)$$

The virtual strains are known in terms of virtual displacement eq.(4) and then Substituting the virtual strain into Eq. (8) and integrating by parts to relative virtual displacement $(\delta u, \delta v, \delta w_a, \delta w_{be}, \delta w_{sh})$ in range of any differentiation, then we get:

$$0 = \int[-\delta u\,\frac{\partial N_x}{\partial x} - \delta v\,\frac{\partial N_y}{\partial y} - \delta u\,\frac{\partial N_{xy}}{\partial y} - \delta v\frac{\partial N_{xy}}{\partial x} - \delta w_{be}\,\frac{\partial^2 M_x^b}{\partial x^2} - \delta w_{be}\,\frac{\partial^2 M_y^b}{\partial y^2} - 2\,\delta w_{be}\,\frac{\partial^2 M_{xy}^b}{\partial x\partial y} - \delta w_{sh}\,\frac{\partial^2 M_x^s}{\partial x^2} - \delta w_{sh}\frac{\partial^2 M_y^s}{\partial y^2} - 2\,\delta w_{sh}\,\frac{\partial^2 M_{xy}^s}{\partial x\partial y} - \delta w_a\,\frac{\partial Q_{yz}^a}{\partial y} - \delta w_a\,\frac{\partial Q_{xz}^a}{\partial x} - \delta w_{sh}\,\frac{\partial Q_{yz}^s}{\partial y} - \delta w_{sh}\,\frac{\partial Q_{xz}^s}{\partial x}]\ \partial x\,\partial y \qquad \ldots (10)$$

The virtual work done δV is:

$$\delta V = -\int_A \left[N_x^0\delta\,\frac{\partial^2(w_a+w_{be}+w_{sh})}{\partial x^2}\right]\partial x\,\partial y = 0 \qquad \ldots (11)$$

$$\delta T = \int\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\,\{\left[\dot{u} - z\frac{\partial \dot{w}_{be}}{\partial x} + f\,\frac{\partial \dot{w}_{sh}}{\partial x}\right]\left[\delta\dot{u} - z\frac{\delta\partial\dot{w}_{be}}{\partial x} + f\,\frac{\delta\partial\dot{w}_{sh}}{\partial x}\right] + \left[\dot{v} - z\,\frac{\partial\dot{w}_{be}}{\partial y} + f\,\frac{\partial\dot{w}_{sh}}{\partial y}\right]\left[\delta\dot{v} - \right.$$

$z\frac{\delta\partial\dot{w}_{be}}{\partial y}+f\frac{\delta\partial\dot{w}_{sh}}{\partial y}\Big]+[\dot{w}_a+\dot{w}_{be}+\dot{w}_{sh}][\delta\dot{w}_a+\delta\dot{w}_{be}+\delta\dot{w}_{sh}]\}\,dv$

$$\delta T=\int\Big[\Big(\mathrm{I}_1\dot{u}-\mathrm{I}_2\frac{\partial\dot{w}_{be}}{\partial x}+\mathrm{I}_4\frac{\partial\dot{w}_{sh}}{\partial x}\Big)\delta\dot{u}+\Big(-\mathrm{I}_2\dot{u}+\mathrm{I}_3\frac{\partial\dot{w}_{be}}{\partial x}-\mathrm{I}_5\frac{\partial\dot{w}_{sh}}{\partial x}\Big)\frac{\delta\,\partial\dot{w}_{be}}{\partial x}+\Big(\mathrm{I}_4\dot{u}-\mathrm{I}_5\frac{\partial\dot{w}_{be}}{\partial x}+\mathrm{I}_6\frac{\partial\dot{w}_{sh}}{\partial x}\Big)\frac{\delta\,\partial\dot{w}_{sh}}{\partial x}+\Big(\mathrm{I}_1\dot{v}-\mathrm{I}_2\frac{\partial\dot{w}_{be}}{\partial y}+\mathrm{I}_4\frac{\partial\dot{w}_{sh}}{\partial y}\Big)\delta\dot{v}+\Big(-\mathrm{I}_2\dot{v}+\mathrm{I}_3\frac{\partial\dot{w}_{be}}{\partial y}-\mathrm{I}_5\frac{\partial\dot{w}_{sh}}{\partial y}\Big)\frac{\delta\,\partial\dot{w}_{be}}{\partial y}+\Big(\mathrm{I}_4\dot{v}-\mathrm{I}_5\frac{\partial\dot{w}_{be}}{\partial y}+\mathrm{I}_6\frac{\partial\dot{w}_{sh}}{\partial y}\Big)\frac{\delta\,\partial\dot{w}_{sh}}{\partial y}+(\dot{w}_a+\dot{w}_{be}+\dot{w}_{sh})\delta\dot{w}_a+(\dot{w}_a+\dot{w}_{be}+\dot{w}_{sh})\delta\dot{w}_b+(\dot{w}_a+\dot{w}_{be}+\dot{w}_{sh})\delta\dot{w}_{sh}\Big]\,dx\,dy \quad \ldots(12)$$

Where:

$$(I_1, I_2, I_3, I_4, I_5, I_6)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\,(1,z,z^2,f(z),zf(z),[f(z)]^2)\;dz$$

2.3 Equation of Motion

The Euler-Lagrange is obtained by substituting equation(8 - 12) into equation (6), then setting the coefficient of (δu, δv, δw_a,δw_{be}, δw_{sh}) of Eq.(6) to zero separately, this give five equations of motion as follows:

$$\delta u:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\mathrm{I}_1\ddot{u}$$

$$\delta v:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=\mathrm{I}_1\ddot{v}$$

$$\delta w_{be}:\ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\,\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}+N(w)=\mathrm{I}_1(\ddot{w}_a+\ddot{w}_{be}+\ddot{w}_{sh})-\mathrm{I}_3\frac{\partial^2}{\partial t^2}\Big(\frac{\partial^2 w_{be}}{\partial x^2}+\frac{\partial^2 w_{be}}{\partial y^2}\Big)$$

$$\delta w_{sh}:\ \frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial x\,\partial y}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial Q_{xz}^s}{\partial x}+\frac{\partial Q_{yz}^s}{\partial y}+N(w)=\mathrm{I}_1(\ddot{w}_a+\ddot{w}_{be}+\ddot{w}_{sh})-\mathrm{I}_6\frac{\partial^2}{\partial t^2}\Big(\frac{\partial^2 w_{sh}}{\partial x^2}+\frac{\partial^2 w_{sh}}{\partial y^2}\Big)$$

$$\delta w_a:\ \frac{\partial Q_{xz}^a}{\partial x}+\frac{\partial Q_{yz}^a}{\partial y}+N(w)=\mathrm{I}_1(\ddot{w}_a+\ddot{w}_{be}+\ddot{w}_{sh}) \quad \ldots(13)$$

Where:

$$N(w)=N_x^0\frac{\partial^2(w_a+w_{be}+w_{sh})}{\partial x^2}+N_y^0\frac{\partial^2(w_a+w_{be}+w_{sh})}{\partial y^2}+2N_{xy}^0\frac{\partial^2(w_a+w_{be}+w_{sh})}{\partial x\,\partial y}$$

The result forces are given by: Reddy [9].

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{Bmatrix}dz$$

$$\begin{Bmatrix}M_x^b\\M_y^b\\M_{xy}^b\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{Bmatrix}z\,dz$$

$$\begin{Bmatrix}M_x^s\\M_y^s\\M_{xy}^s\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\end{Bmatrix}f\,dz$$

$$\begin{Bmatrix}Q_{xz}^a\\Q_{yz}^a\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_{xz}\\\sigma_{yz}\end{Bmatrix}dz$$

$$\begin{Bmatrix}Q_{xz}^s\\Q_{yz}^s\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_{xz}\\\sigma_{yz}\end{Bmatrix}g\,dz \quad \ldots(14)$$

The plane stress reduced stiffness Q_{ij} is:

$Q_{11}=\frac{E_1}{1-v_{12}v_{21}}$, $Q_{21}=\frac{v_{12}E_2}{1-v_{12}v_{21}}$

$Q_{22}=\frac{E_2}{1-v_{12}v_{21}}$, $Q_{66}=G_{12}$, $Q_{44}=G_{23}$, $Q_{55}=G_{13}$

From the constitutive relation of k^{th} layer lamina, the transformed stress-strain relation are:

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_{xy}\\\sigma_{yz}\\\sigma_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\Q_{21}&Q_{22}&0&0&0\\0&0&Q_{66}&0&0\\0&0&0&Q_{44}&0\\0&0&0&0&Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\\varepsilon_y\\\gamma_{xy}\\\gamma_{yz}\\\gamma_{xz}\end{Bmatrix} \quad \ldots(15)$$

The force results are related to the strains by relations:

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11}&A_{12}&A_{16}\\A_{12}&A_{22}&A_{26}\\A_{16}&A_{26}&A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{Bmatrix}k_x^b\\k_y^b\\k_{xy}^b\end{Bmatrix}+\begin{bmatrix}B_{11}^s&B_{12}^s&B_{16}^s\\B_{12}^s&B_{22}^s&B_{26}^s\\B_{16}^s&B_{26}^s&B_{66}^s\end{bmatrix}\begin{Bmatrix}k_x^s\\k_y^s\\k_{xy}^s\end{Bmatrix}$$

$$\begin{Bmatrix}M_x^b\\M_y^b\\M_{xy}^b\end{Bmatrix}=\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\\varepsilon_y^0\\\gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}D_{11}&D_{12}&D_{16}\\D_{12}&D_{22}&D_{26}\\D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{Bmatrix}k_x^b\\k_y^b\\k_{xy}^b\end{Bmatrix}+\begin{bmatrix}D_{11}^s&D_{12}^s&D_{16}^s\\D_{12}^s&D_{22}^s&D_{26}^s\\D_{16}^s&D_{26}^s&D_{66}^s\end{bmatrix}\begin{Bmatrix}k_x^s\\k_y^s\\k_{xy}^s\end{Bmatrix}$$

$$\begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + \begin{bmatrix} H_{11}^s & H_{12}^s & H_{16}^s \\ H_{12}^s & H_{22}^s & H_{26}^s \\ H_{16}^s & H_{26}^s & H_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{yz}^a \\ Q_{xz}^a \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^a \\ \gamma_{xz}^a \end{Bmatrix} + \begin{bmatrix} A_{44}^a & A_{45}^a \\ A_{45}^a & A_{55}^a \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{yz}^s \\ Q_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^a & A_{45}^a \\ A_{45}^a & A_{55}^a \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^a \\ \gamma_{xz}^a \end{Bmatrix} + \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \quad \dots(16)$$

Where:

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}\,(1,\ z,\ z^2, f(z), z\,f(z), [f(z)]^2)\ dz$$

2.4 Navier's Solution

To solve equations of motion (15-16), Navier's generalized displacements are used which satisfy the boundary conditions of the problem as shown in Fig.2, therefore Simply supported boundary conditions are satisfied by assuming the following form of displacements: Reddy [9]

$$U = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}\ \cos\alpha x \sin\beta y$$
$$V = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}\ \sin\alpha x\ \cos\beta y$$
$$W_{be} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{bemn}\ \sin\alpha x\ \sin\beta y$$
$$W_{sh} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{shmn}\ \sin\alpha x\ \sin\beta y$$
$$W_a = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{amn}\ \sin\alpha x\ \sin\beta y \quad \dots(17)$$

Where: $\alpha = \frac{m\ \pi}{a}$, $\beta = \frac{n\ \pi}{b}$ and $(U_{mn}\, V_{mn}\, W_{bemn}\, W_{shmn}\, W_{amn})$ are arbitrary constants.

The following stiffnesses are zero if the Navier solution exists,

$$A_{16} = A_{26} = D_{16} = D_{26} = D_{16}^s = H_{16}^s = H_{26}^s = 0$$

$$B_{12} = B_{16} = B_{26} = B_{66} = B_{12}^s = B_{16}^s = B_{26}^s = B_{66}^s = 0$$

$$A_{45} = A_{45}^a = A_{45}^s = 0$$

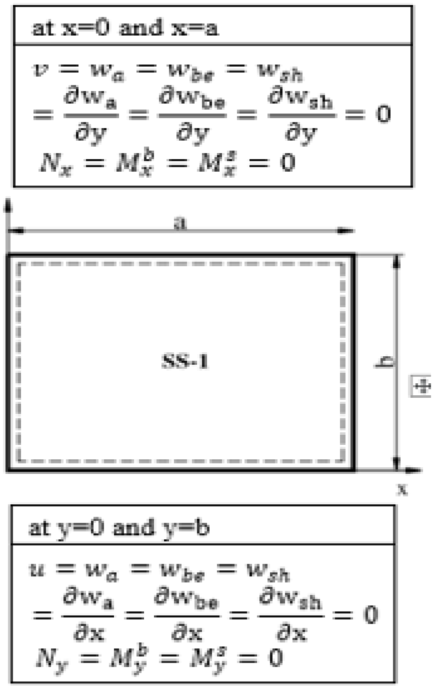

Fig. 2. Boundary condition for simply supported plate.

2.5 Vibration Analysis

Developing mass matrix and stiffness matrix from solution of homogeneous equations, when mechanical loading is equal to zero for free vibration, then eigenvalue equation is derived and the natural frequencies of vibration for simply supported plate are obtained.

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & 0 \\ s_{12} & s_{22} & s_{23} & s_{24} & 0 \\ s_{13} & s_{23} & s_{33} & s_{34} & 0 \\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45} \\ 0 & 0 & 0 & s_{45} & s_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \\ W_{amn} \end{Bmatrix} + \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} & m_{35} \\ 0 & 0 & m_{34} & m_{44} & m_{45} \\ 0 & 0 & m_{11} & m_{11} & m_{55} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{bemn} \\ \ddot{W}_{shmn} \\ \ddot{W}_{amn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$|[S] - w^2[M]| = 0$$

Where $[S_{ij}]$ = stiffness matrix elements and $[M_{ij}]$ = mass matrix.

2.6 Buckling

The applied loads for buckling analysis, are supposed to be in-plan forces

$$N_x^0 = -N_0\ ,\ N_y^0 = \gamma \mathrm{N}_0\ ,\ \gamma = \frac{\mathrm{N}_\mathrm{x}^0}{N_y^0}\ ,\ N_{xy}^0 = 0$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & 0 \\ s_{12} & s_{22} & s_{23} & s_{24} & 0 \\ s_{13} & s_{23} & s_{33} - N_0(\alpha^2+\gamma\beta^2) & s_{34} - N_0(\alpha^2+\gamma\beta^2) & -N_0(\alpha^2+\gamma\beta^2) \\ s_{14} & s_{24} & s_{34} - N_0(\alpha^2+\gamma\beta^2) & s_{44} - N_0(\alpha^2+\gamma\beta^2) & s_{45} - N_0(\alpha^2+\gamma\beta^2) \\ 0 & 0 & -N_0(\alpha^2+\gamma\beta^2) & s_{45} - N_0(\alpha^2+\gamma\beta^2) & s_{55} - N_0(\alpha^2+\gamma\beta^2) \end{bmatrix} * \begin{Bmatrix} \mathrm{U_{mn}} \\ \mathrm{V_{mn}} \\ \mathrm{W_{bemn}} \\ \mathrm{W_{shmn}} \\ \mathrm{W_{amn}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

2.7 Free Vibration Analysis Under Initial Stress

The natural frequency is investigated with action of buckling, a ratio critical load (d) is applied. The effect of the load ratio (d) is studied to present the behavior of the plate and its frequency.

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & 0 \\ s_{12} & s_{22} & s_{23} & s_{24} & 0 \\ s_{13} & s_{23} & s_{33} - N_0(\alpha^2+\gamma\beta^2) & s_{34} - N_0(\alpha^2+\gamma\beta^2) & -N_0(\alpha^2+\gamma\beta^2) \\ s_{14} & s_{24} & s_{34} - N_0(\alpha^2+\gamma\beta^2) & s_{44} - N_0(\alpha^2+\gamma\beta^2) & s_{45} - N_0(\alpha^2+\gamma\beta^2) \\ 0 & 0 & -N_0(\alpha^2+\gamma\beta^2) & s_{45} - N_0(\alpha^2+\gamma\beta^2) & s_{55} - N_0(\alpha^2+\gamma\beta^2) \end{bmatrix}$$

$$\begin{Bmatrix} \mathrm{U_{mn}} \\ \mathrm{V_{mn}} \\ \mathrm{W_{bemn}} \\ \mathrm{W_{shmn}} \\ \mathrm{W_{amn}} \end{Bmatrix} + \begin{bmatrix} \mathrm{m_{11}} & 0 & 0 & 0 & 0 \\ 0 & \mathrm{m_{22}} & 0 & 0 & 0 \\ 0 & 0 & \mathrm{m_{33}} & \mathrm{m_{34}} & \mathrm{m_{35}} \\ 0 & 0 & \mathrm{m_{34}} & \mathrm{m_{44}} & \mathrm{m_{45}} \\ 0 & 0 & \mathrm{m_{11}} & \mathrm{m_{11}} & \mathrm{m_{55}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathrm{U}}_{\mathrm{mn}} \\ \ddot{\mathrm{V}}_{\mathrm{mn}} \\ \ddot{\mathrm{W}}_{\mathrm{bemn}} \\ \ddot{\mathrm{W}}_{\mathrm{shmn}} \\ \ddot{\mathrm{W}}_{\mathrm{amn}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

3. Results and Discussion

3.1 Vibration and Buckling Results

The fundamental natural frequency and critical buckling for cross-ply plate with different design parameters for simply supported boundary condition, is analyzed and solved used MATLAB programming. We derive equation of motion depending on Refined plate theory using Navier solution to obtain vibration characteristic of plate under initial stress. To examine the validity of the derived equation and performance of computer programming for vibration and buckling stress of cross-ply laminated simply supported plate, a comparison with others researchers for different layers, thickness ratio (a/h) and orthotropy ratio (E1/E2). The non-dimensional natural frequency of antisymmetric cross-ply two, four, six and ten layer of thick plate (a/h=5) as a function of orthotropy ratio (E1/E2) shown in table (1) for the mechanical properties $[G_{12} = G_{13} = 0.6\,\mathrm{E}_2\ ,\ G_{23} = 0.5\,\mathrm{E}_2\ ,\ v_{12} = 0.25]$ while Table (2) shows the Non-dimensional fundamental frequencies of antisymmetric square laminated plate for various values of thickness ratio and modulus ratio (E1/E2=40). The natural frequency shows good agreement with other researchers. The present theory is also close agreement with other theory for critical buckling as shown in table (3) which compared with [2] for Non-dimensional uniaxial buckling load of simply supported antisymmetric layer for (a/h=10), while Table (4) and Table (5) are compared with [9] which show the Non-dimensional uniaxial and biaxial buckling load of simply supported antisymmetric cross-ply for two and eight layers for various thickness ratio (a/h) and as a function of modulus ratios (E1/E2).

3.2 Vibration of Plate Under Initial Stress Results

Vibration analysis of present work is used but adding initial in-plane stress to investigate the validity of Refined theory for such case. Table (6) shows Non-dimensional natural frequency under various uniaxial loads ratio(d) for simply supported cross-ply [0/90/0] square plate with orthotropy ratio (E1/E2=10). Table (7) shows Non-dimensional natural frequency under various uniaxial loads ratio with various thickness ratio (a/h) for simply supported cross-ply [0/90/0] square plate with orthotropy ratio (E1/E2=40). The fundamental frequency decrease when increasing the value of compressive stress until the lowest natural frequency vanished when inplane stress reaches the critical buckling stress, which proved by other researchers, as shown in Fig (3) and Fig (4).

Table 1,
Non-dimensional natural frequencies of square laminate with(a/h=5), $G_{12} = G_{13} = 0.6\,E_2$, $G_{23} = 0.5\,E_2$, $v_{12} = 0.25$

No.of layers	source	E1/E2		
		20	30	40
$(0/90)_2$	3D[10]	9.4055	10.165	10.6789
	TSDT[11]	9.6265	10.5348	11.1716
	FSDT[12]	9.6885	10.6198	11.2708
	PPT [4]	9.6252	10.5334	11.1705
	Present	9.632	10.538	11.173
	ANSYS	9.301	10.07	10.57
$(0/90)_3$	3D [10]	9.8398	10.6958	11.2728
	TSDT[11]	9.9181	10.8547	11.5012
	FSDT[12]	9.9427	10.8828	11.5264
	PPT[4]	9.9181	10.8547	11.5009
	Present	9.925	10.859	11.504
	ANSYS	9.69	10.58	11.16
$(0/90)_5$	3D[10]	10.0843	11.0027	11.6245
	TSDT[11]	10.0674	11.0197	11.673
	FSDT[12]	10.0638	11.0058	11.6444
	PPT[4]	10.0671	11.0186	11.6705
	Present	10.074	11.023	11.673
	ANSYS	10.905	10.84	11.47

Table 2,
Non-dimensional natural frequencies of cross-ply square laminate $\frac{E_1}{E_2}$=40.

No of layer	method	a/h			
		10	20	50	100
$(0/90)_1$	TSDT [11]	10.56	11.10	11.27	11.30
	FSDT [12]	10.47	11.07	11.27	11.29
	RPT [4]	10.56	11.10	11.27	11.30
	Present	10.55	11.10	11.27	11.30
$(0/90)_2$	TSDT [11]	14.84	16.57	17.18	17.27
	FSDT [12]	14.92	16.60	17.18	17.27
	RPT [4]	14.84	16.57	17.18	17.27
	Present	14.85	16.58	17.19	17.29
$(0/90)_3$	TSDT [11]	15.46	17.37	18.06	18.16
	FSDT [12]	15.50	17.39	18.06	18.17
	RPT [4]	15.46	17.37	18.06	18.16
	Present	15.47	17.38	18.07	18.18

Table 3,
Non-dimensional uniaxial buckling load of simply supported square laminates,(a/h=10), $G_{12} = G_{13} = 0.6\,E_2$, $G_{23} = 0.5\,E_2$, $v_{12} = 0.25$

No of layer	method	$\bar{N}$	Diff%
4	FSDT [12]	22.806	-
	RPT [3]	22.57	1.03
	Present	22.593	0.93
	ANSYS	22.134	2.94
6	FSDT [12]	24.5777	-
	RPT [3]	24.4581	0.48
	Present	24.483	0.38
	ANSYS	23.78	3.2
10	FSDT [12]	25.45	-
	RPT [3]	25.4225	0.10
	Present	25.44	0.03
	ANSYS	24.357	4.29

Table 4,
Non-dimensional uniaxial buckling load of antisymmetric square laminates $G_{12} = G_{13} = 0.5\,E_2$, $G_{23} = 0.2\,E_2$, $v_{12} = 0.25$

	Method	E1/E2=10 (0/90)	E1/E2=10 $(0/90)_4$	E1/E2=25 (0/90)	E1/E2=25 $(0/90)_4$	E1/E2=40 (0/90)	E1/E2=40 $(0/90)_4$
10	**[9]**	5.746	9.158	8.189	16.301	10.381	21.631
	present	5.792	9.186	8.317	16.292	10.615	21.601
20	**[9]**	6.205	10.380	9.153	20.623	11.980	29.965
	present	6.228	10.424	9.201	20.644	12.065	29.959
100	**[9]**	6.367	10.843	9.511	22.535	12.601	34.179
	present	6.382	10.895	9.525	22.583	12.617	34.225

Table 5,
Non-dimensional biaxial buckling load of antisymmetric cross-ply laminates

a/h	Method	E1/E2=10 (0/90)	E1/E2=10 $(0/90)_4$	E1/E2=25 (0/90)	E1/E2=25 $(0/90)_4$	E1/E2=40 (0/90)	E1/E2=40 $(0/90)_4$
10	**[9]**	2.873	4.579	4.094	8.150	5.190	10.816
	present	2.896	4.593	4.158	8.146	5.307	10.800
20	**[9]**	3.102	5.190	4.576	10.311	5.990	14.983
	present	3.114	5.212	4.600	10.322	6.032	14.979
100	**[9]**	3.184	5.422	4.755	11.267	6.300	17.090
	present	3.191	5.447	4.76	11.291	6.308	17.112

Table 6,
Dimensionless natural frequency of a laminated plate under buckling different ratio (d).

d	Method	Fundamental frequency ($\bar{\omega}$)
0	[13]	10.649
	ANSYS	10.645
	present	10.665
0.25	[13]	9.231
	ANSYS	9.231
	present	9.236
0.5	[13]	7.544
	ANSYS	7.544
	present	7.541
0.75	[13]	5.379
	ANSYS	5.379
	present	5.332

Table7,
Non-dimensional natural frequency under various uniaxial loads ratio with various thickness ratio (a/h) for simply supported cross-ply [0/90/0] square plate and orthotropy ratio (E1/E2=40).

Ratio of N_{cr}	a/h=5	a/h=10	a/h=50	a/h=100
0	10.176	14.78	18.585	18.759
0.1	9.654	14.02	17.632	17.79
0.2	9.102	13.22	16.623	16.77
0.3	8.514	12.36	15.550	15.96
0.4	7.882	11.44	14.39	14.35
0.5	7.195	10.45	13.142	13.26
0.6	6.436	9.34	11.754	11.86
0.7	5.574	8.09	10.179	10.27
0.8	4.551	6.61	8.311	8.38
0.9	3.218	4.67	5.877	5.93
1	0	0	0	0

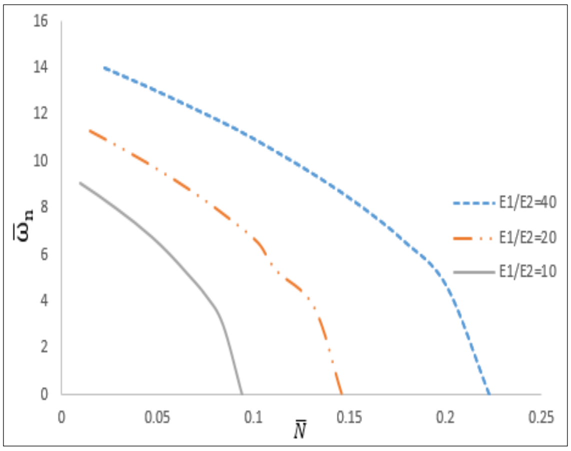

Fig. 3. Natural frequency with ratio of buckling load for [0/90/0] simply supported square plate [a/h=10] for different orthotrpy ratio.

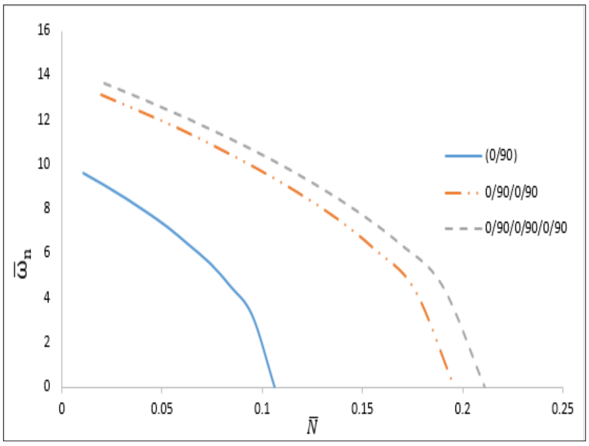

Fig. 4. Natural frequency with ratio of buckling load for different layers of simply supported square plate (a/h=10), (E1/E2=40).

4. Conclusion

Natural frequency and buckling stress of simply supported cross-ply square plate subject to initial axial stress have been obtained by using Refind plate theory. It is observed good results for natural frequency and critical buckling for uniaxial and biaxial load as compared with other researchers. The following conclusions may be drawn from the present analysis:

1. Refined plate theory for analyzing natural frequency and buckling stresses of cross-ply square plate has been presented. It is observed that the natural frequency and buckling load increasing as the number of layer and thickness ratio increases.
2. The buckling stresses can be calculated through the stability equation as Eigen value problems. Another method to obtain the critical stress of cross-ply plate subject to axial and uniaxial in-plane stresses is to compute natural frequency by increasing the absolute value of compressive stress until the lowest natural frequency vanishes.

Nomenclature

Symbol	Discretion	Units
a	Plate dimension in x-direction	m
b	Plate dimension in y-direction	m
h	Plate thickness	m
A_{ij} , B_{ij} , D_{ij} , B	Extension, bending extension coupling	N/m
E1, E2, E3	Elastic modulus components	GP
G_{23} , G_{12} G_{13}	Shear modulus components	GP

n	Total number of plate layers	-
N_x, N_y ,N_{xy}	In-plan force result	N/m
M_x^b , M_y^b , M_{xy}^b	Moment result per unit length	N.m/m
M_x^s , M_y^s , M_{xy}^s	Result force per unit length	N/m
Q_{xz}^a, Q_{yz}^a	Transvers shear force result	N
Q_{yz}^s,Q_{xz}^s	Transfers shear force result	N
X, y, z	Cartesian Coordinate system	m
ε_y , $\varepsilon_z \varepsilon_x$	Strain components	m/m
$\gamma_{yz}\gamma_{xz}$	Transvers shear strain	m/m
σ_y σ_{xy} σ_x σ_{yz} σ_{xz}	Stress components	Gpa
v_{12} v_{21}	Poisson's ratio	-

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تحليل الاهتزازات تحت تاثير الاجهاد الاولي باستخدام نظرية اللو□ة المكررة

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الخلاصة

يتم الحصول على التردد الطبيعي تحت الضغوط الأولية للحصول على صفائح الرقائق المتقاطعة لـ (simply supported) باستخدام نظرية (Refind plate)هذه النظرية مسؤولة عن التوزيع المكافئ لسلالة القص المستعرض عن طريق سماكة اللوح وتلبي الشروط الحدودية للجر صفر على أسطح الصفيحة دون استخدام عوامل تصحيح القص. يتم اشتقاق المعادلات الحاكمة لمشكلة قيمة Eigen تحت الإجهاد الأولي باستخدام مبدأ هاملتون ويتم حلها باستخدام حل (Navier) للألواح الطبقية المتناظرة المتماثلة وغير المتماثلة لـ (Simply supported). تم دراسة تأثير العديد من عوامل التصميم مثل نسبة المعامل (E1 / E2) ونسبة السمك (a / h) وعدد الصفائح (N) على التردد الطبيعي والضغط على الألواح المحبوسة. تتم مقارنة النتائج مع باحثين اخرين.