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Stress Analysis of Guide Rails of Elevators

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Abstract

The mechanical design of elevator elements is always performed by international standards. The engineer selects the appropriate elements of elevator according to catalogues without knowing scientific details. Therefore, a theoretical analysis is achieved at two operating conditions for guide rails (1) safety gear operation, and (2) running condition with the loads unevenly distributed on the elevator car. The guide rail is considered a continuous beam with variable supports. Then the British code is listed showing the equations used in it.

The theoretical equations showed that guide rails are never subjected to stress in simultaneous combined buckling and bending in the plane, where the bending moment is exerted. It is always a combination of pressure and bending. Consequently, it is wrong to consider a simultaneous effect of buckling and bending. The equations in the catalogues oppose the theoretical results concerning buckling of guide rails. Therefore, a recommended calculation method for guide rails is presented to be an acceptable method for analysis of guide rail.

Keywords: *Guide rail, stress, deflection, safety gear, buckling, continuous beam, standard codes.*

1. Introduction

The functions of guide rails are as follows: (1) to guide the car and the counterweight in their vertical travel and to minimize their horizontal movement, (2) to prevent tilting of the car due to eccentric load, and (3) to stop and hold the car on the application of the safety gear. Fig.1 shows the components of the elevator and the cross-section of the guide-rail [1].

Both the car and the counterweight must be guided by at least two rigid steel guide rails, which are manufactured from a structural steel having a tensile strength of no less than 370 MPa and not greater than 520 MPa [2]. In the U.S., a suitable nonmetallic material may be used for guide rails where steel may present an accident hazard, as in chemical and explosive plants, provided the rated speed of the car does not exceed 0.76 m/s.

In recent years, round guide rails have been successfully used for hydraulic elevators and counterweights without safety gears [3]. Through investigating the studies on the elevator components, the calculations of guide rails depend mainly upon USA or European codes.

Utsunomiya et. al. [4] invented a guide device for an elevator in which a pair of corresponding actuators were controlled in accordance with information from acceleration sensors, and the force with which guide members were pressed against guide rails was adjusted.

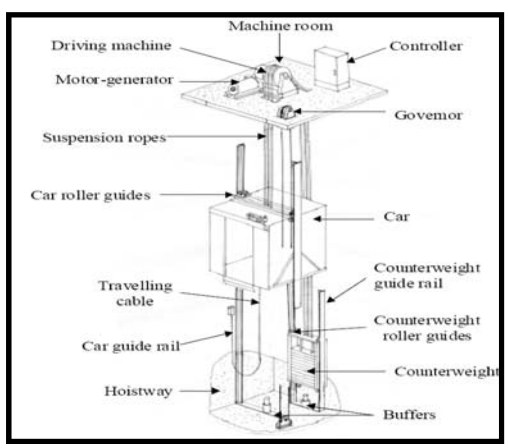

Fig. 1. Components of Elevators

Recep et. al.[5] performed the calculation and selection of guide rails according to international standard and compared the results with FEA. Penn et. al. [6] uses ANSYS to improve the manufacturing and design of a neoprene elevator roller guide. The study uses 2d and 3d hyperelastic and contact elements to model neoprene material tests and also highly deformed roller guide proof tests. The ANSYS analysis succeeded in modeling the neoprene material performance. Geometry changes in the shape of the neoprene were studied to reduce adhesion stresses between the neoprene and aluminum center hub, and yet to maintain the spring stiffness of the current roller guide design.

Clem Skalski and Barker Mohandas [7] performed a procedure to control the vibration of the guide rail. Zhu et. al. [8] analyzed the vibration of elevators depending on the present theory of dynamic analysis of elevator systems. The vibration models of elevator systems in the horizontal and vertical direction are established. The seismic responses of the building are used as excitation and input into the model. Differential equations of the system are set up and the time-history of the dynamic responses of the main parts are worked out. Finally, some earthquake protective measures for elevators are proposed. The current study proved that there are problems with the used codes, and a preferred method was invented quoting from the theoretical and international standard to calculate and select the best guide rails.

2. Guide Rails Calculations

In the calculation of the guide rails, two operating conditions should be taken into consideration (1) safety gear operation, (2) running conditions with the load unevenly distributed on the car floor.

In most national standards, the calculation of stress in guide rails is carried out for (2), while the calculation of deflection concerns quite different operating conditions, namely(1) [8].

Three stages of calculations are performed in the current paper: 1) theoretical analysis, 2) the British codes, and 3) an acceptable method for design of guide rails is introduced after completing the previous two stages.

2.1. Theoretical Analysis

2.1.1. During Safety Gear Operation (Without Taking Buckling Into Consideration)

The aim of the theoretical analysis is to find the nature of relation between buckling and bending moments during safety gear operation. The following will be studied:

- The bending moment distribution without the effect of buckling and the guide rail is considered as a continuous beam.
- The effect of buckling is considered with the bending moment at the section of guide rail that is subjected to compression force.

In the first stage, the guide rails will be analyzed by calculating the maximum bending moment produced by the braking force without taking buckling into consideration. We will assume a simultaneous effect of buckling and bending moments in the second stage.

The guide rails will be considered a continuous beam with a variable number of supports. The Theorem of Three Moments and the Finite Element Method may be used as methods of solution. The first method is used for the derivation of the related equations.

The guide rail is subjected to a combined effect of the braking F_b, acting parallel to the longitudinal axis of the guide rail, and the outer moment $F_b \times e$. The outer moment is induced due to the eccentric position of the braking force F_b, which is represented by the distance e, as shown in Fig. 2. The bending moment M (z) depends on the number of beam fields and on the outer moment $F_b \times e$(both value and location).

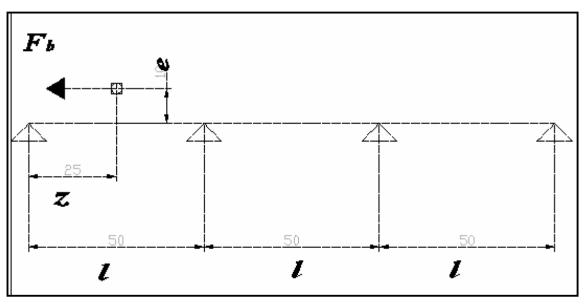

Fig. 2. Location of the Braking Force F_b

Equations for the maximum bending moment M(z) as a function of z, and values and locations of the extremes of individual functions are reviewed in Table 1 for $F_b \times e$ acting in field I of the beam and in Table 2 for $F_b \times e$ acting in field II. The maximum bending moment always occurs at the point of application of the outer moment $F_b \times e$. We will derive only two equations which are listed in the mentioned tables.

a) Assumptions

i. $F_b \times e$ in field I(span I).

ii. No. of *fields* is two, as shown in Fig 3.

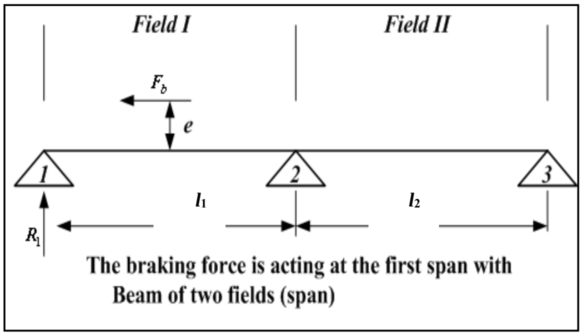

Fig. 3. The Location of the Braking Force at Field I

By using Theorem of three moments [6]**,

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 + \frac{6A_1\bar{a}_1}{l_1} + \frac{6A_2\bar{b}_2}{l_2} = (\frac{h_1}{l_1} + \frac{h_3}{l_2}) * 6EI \quad \dots (1)$$

where h_1, h_3 are the deflections at supports 1 and 3 respectively. There are no moments at the edge supports,

$$M_1 = M_3 = 0, \quad l_1 = l_2 = l \quad \dots (1a)$$

$$\frac{6A_1\bar{a}_1}{l_1} = -\frac{F_b e}{l}(3z^2 - l^2) \quad \dots (1b)$$

$$\frac{6A_2\bar{b}_2}{l_2} = 0 \quad \dots (1c)$$

By substituting eqns.(1a,1b and 1c) into eqn.(1), it results in:

$$2M_2(l + l) = \frac{F_b e}{l}(3z^2 - l^2) \quad \dots (1d)$$

$$M_2 = \frac{F_b e}{4l^2}(3z^2 - l^2) \quad \dots (1e)$$

By taking moments about support (2),

$$R_1(l) + \frac{F_b e}{4l^2}(3z^2 - l^2) - F_b e = 0 \Rightarrow$$

$$R_1(l) = -\frac{F_b e}{4l^2}(3z^2 - l^2 - 4l^2) = -\frac{F_b e}{4l^2}(3z^2 - 5l^2)$$

$$\Rightarrow R_1 = -\frac{F_b e}{4l^2}(3z^2 - 5l^2) \quad \dots (1f)$$

$$M(z) = R_1 z \quad \dots (1j)$$

To find the maximum moment that produces through the guide rail,

$$\frac{dM(z)}{dz} = 0 = -\frac{F_b e}{4l^3}(5l^2 - 9z^2) \Rightarrow$$
$$z = 0.7454\, l \quad \dots (1i)$$

By substituting eqn.(1i) into eqn.(1j) to find the maximum bending moment, it results in:

$$\therefore M(z)_{\max} = -0.6211 F_b e \quad \dots (2)$$

This value is listed in Table 1.

b) Assumption

i. $F_b \times e$ in field II(span II).

ii. No. of fields is two.

iii. z will be taken from the left support of field II, as shown in Fig. 4.

The equations are the same as (case a), by replacing z with (*l-z*), therefore, from eqn.(1j),

$$M(z) = \frac{F_b e}{4l^3}(5l^2 - 3(l - z)^2)\ (l\text{-}z)$$

$$M(z) = \frac{F_b e}{4l^3}(3z^3 - 9lz^2 + 4l^2 z + 2l^3) \quad \dots (3)$$

This value is listed in Table 2.

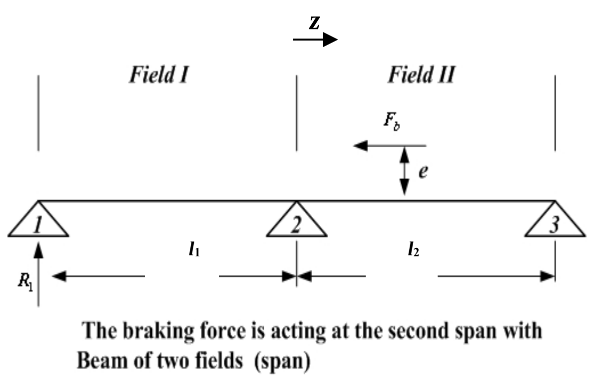

Fig. 4. The Location of the Braking Force at FieldII

Table 1,
Bending Moment M(*z*) and its Maximum Values M_{max} ($F_b \times e$ in field I) (spanI)

Number of fields (spans)	M(z)	Maximum value $M(z)_{max}$	Location of the extreme z_m	Equation No.
2	$-\frac{F_b \times e}{4l^3}(5l^2 \times z - 3z^3)$	$-0.6211F_b \times e$	0.7454 l	1&2
3	$-\frac{F_b \times e}{15l^3}(19l^2 \times z - 12z^3)$	$-0.6135F_b \times e$	0.7265 l	
4	$-\frac{F_b \times e}{56l^3}(71l^2 \times z - 45z^3)$	$-0.6130F_b \times e$	0.7252 l	
5	$-\frac{F_b \times e}{209l^3}(265l^2 \times z - 168z^3)$	$-0.6129F_b \times e$	0.7251 l	

Table 2,
Bending Moment M(*z*) and its Maximum Values M_{max} ($F_b \times e$ in field II) (span II)

Number of fields	M(z)	Maximum value $M(z)_{max}$	location of the extreme z_m	Equation No.
2	$-\frac{F_b \times e}{4l^3}(2l^3 + 4l^2 \times z - 9l \times z^2 + 3z^3)$	$-0.6210F_b \times e$	0.2546 l	3
3	$-\frac{F_b \times e}{15l^3}(7l^3 - 14l^2 \times z + 45l \times z^2 - 30z^3)$	$-0.6161F_b \times e$ $+0.6161F_b \times e$	0.1927 l 0.8073 l	
4	$-\frac{F_b \times e}{56l^3}(26l^3 - 52l^2 \times z + 171l \times z^2 - 117z^3)$	$-0.6162F_b \times e$ $+0.6060F_b \times e$	0.1885 l 0.7858 l	
5	$-\frac{F_b \times e}{209l^3}(97l^3 - 194l^2 \times z + 693l \times z^2 - 438z^3)$	$-0.6162F_b \times e$ $+0.6057F_b \times e$	0.1882 l 0.7844 l	

2.1.2. During Safety Gear Operation (Taking Buckling Into Consideration)

Safety gear location is of prime significance. When the safety gear is located under the car floor, the gripping of the gide rails may take place in field I. If the safety gear is mounted above the car roof the guide rails may be gripped in field II only, as shown in Fig.5. The calculation will be carried out in case of the combined bending and buckling (simultaneous bending and buckling).

(A) Field I

Fig. 5. shows the guide rail (as a beam) with the braking force. The derivation of the equation of the bending moment at any point on the left side of $F_b \times e$ is listed below,
From the fundamentals of statics,

$$R_1 = \frac{F_b e + M_1}{l} \quad \text{... (4a)}$$

$$M(z) = \frac{(F_b e + M_2)}{l} z + F_b y \quad \text{... (4b)}$$

$$EI\frac{d^2y}{dz^2} = -M(z) = -F_b y - \frac{(F_b e + M_2)}{l} z$$

$$\frac{d^2y}{dz^2} + \alpha^2 y = -\frac{(F_b e + M_2)}{EIl} z$$

… (4C1)

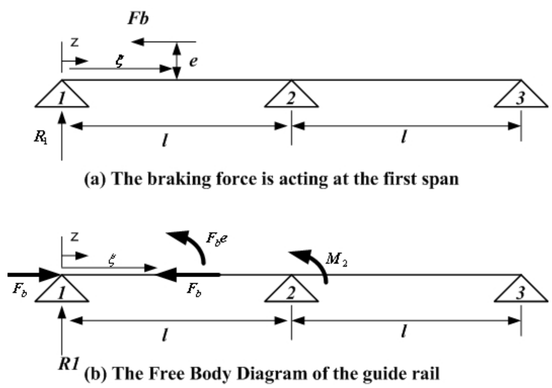

Fig. 5. F.B.D. of Guide Rail (at Field I)

The solution of this non-homogeneous second order D.E. has a combined particular and homogeneous solution shown as follows:

$y = y_h + y_p$, where y_h and y_p are the homogeneous and particular solutions respectively.

For homogeneous solution,

$$\frac{d^2y}{dz^2} + \alpha^2 y = 0$$

$$\therefore y_h = c_1 \cos\alpha z + c_2 \sin\alpha z$$

While for particular solution, we will assume $y_p = Az + B$

$$y' = A \Rightarrow y'' = 0 \qquad \ldots (4C_2)$$

By substituting eqn. ($4C_2$) into eqn.($4C_1$), it results in:

$$0 + \alpha^2 (Az + B) = -\frac{(F_b e + M_2)}{EIl} z \,,$$

where $\alpha^2 = \frac{F_b}{EI}$ by comparing the factors of z , it results in,

$$A = -\frac{(F_b + M_2)}{F_b l}$$

$$\therefore y_p = -\frac{(F_b + M_2)}{F_b l} z$$

$$y = C_1 \cos\alpha z + C_2 \sin\alpha z - (F_b + M_2)\frac{z}{F_b l} \qquad \ldots(4cc)$$

The boundary conditions from Fig.5,

$$\text{at } z = 0, y = 0 \Rightarrow C_1 = 0 \qquad \ldots(4d)$$

$$z = \xi, y = 0 \Rightarrow C_2 = \frac{(F_b e + M_2)}{F_b \sin\alpha\xi l}\xi \qquad \ldots (4e)$$

Substituting eqns.(4d&4e) into eqn.(4cc), it results in,

$$y = \frac{(M_2 + F_b e)}{F_b}\left[\frac{\sin\alpha z}{\sin\alpha\xi}\cdot\frac{\xi}{l} - \frac{z}{l}\right] \qquad \ldots (4f)$$

Substituting eqn.(4f) into eqn.(4b), it results in,

$$M(z) = \frac{(F_b e + M_2)}{l} z + F_b\left(\frac{M_2 + F_b e}{F_b}\right) * \left(\frac{\sin\alpha z}{\sin\alpha\xi}\frac{\xi}{l} - \frac{z}{l}\right)$$

$$\therefore M(z) = \frac{F_b e + M_2}{l \sin\alpha\xi}\xi \times \sin\alpha z \qquad \ldots (5)$$

The extreme is located at $z_m = \pi / 2\alpha$ and its value is given by the following formula

$$M(z) = \frac{F_b \times e + M_2}{l \times \sin\alpha\xi} \times \xi \qquad \ldots (6)$$

The extreme moment is located at $z_m = \pi/2\alpha$ and its value is given by the following formula

$$M(z) = \frac{F_b e + M_2}{l \sin\alpha\xi} \times \xi \qquad \ldots (6a)$$

The graphical illustration of M(z)will be shown in the results.

(B) Field II

The formulae are obtained from the same initial equations by means of the same mathematical methods which are more complicated than those in field I. Fig. 6 shows this case; the moment at any section through the beam(guide rail),

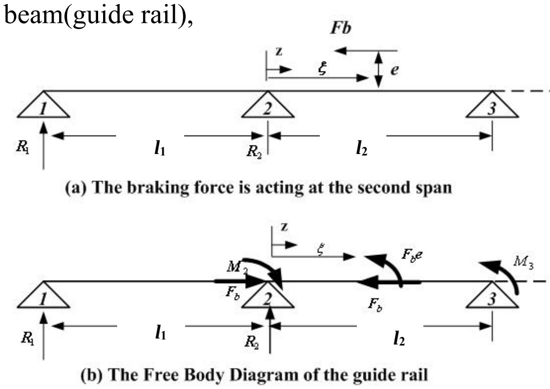

Fig. 6. F.B.D. of Guide Rail (at Field II)

$$M = R_2 z + M_2 + F_b y \qquad ... (7)$$

$$\frac{dM}{dz} = R_2 + F_b \frac{dy}{dz}$$

$$\frac{d^2M}{dz^2} = F_b \frac{d^2y}{dz^2} = -F_b \frac{M}{EI} = -\alpha^2 M$$

$$\frac{d^2M}{dz^2} + \alpha^2 M = 0 \qquad \dots (8)$$

And the solution of this D.E.,

$$M = C_1 \sin \alpha z + C_2 \cos \alpha z \qquad \dots (9)$$

The boundary conditions,

at $z=0$, $M=M_2 \Rightarrow C_2=M_2$ …(10)

at $z = \xi \Rightarrow M = M_2 - F_b e + R_2 \xi$

$$M = M_2 - F_b e + \left(\frac{M_3 + F_b e - M_2}{l} \right)$$

$$C_2 = \frac{(F_b e + M_3)\xi + M_2 (l - \xi - l \cos \alpha\xi)}{l \sin \alpha\xi} \qquad \dots (11)$$

M_3 is moment at the right support of field II (support 3).

The location of the extreme is

$$z_m = \frac{1}{\alpha} \times \tan^{-1} \frac{C_2}{C_1} \qquad ... (12)$$

Constants of integration C_1 and C_2 are dependent upon the moments at supports, i.e., on the location of the moment $F_b \times e$. Consequently, in contrast to field I, the location of the maximum bending moment (z_m) is a function of the location of $F_b \times e$ (ξ) in this case. The maximum value of the bending moment,

$$M_{\max} = M_2 \times \cos \alpha zm + \frac{(F_b \times e + M_3) \times \xi}{l \times \sin \alpha\xi} \times \sin \alpha z_m + \frac{M_2 \times (l - \xi - l \times \cos \alpha\xi)}{l \times \sin \alpha\xi} \times \sin \alpha\xi \qquad \dots (13)$$

$F_b \times e$ is applied at the right support (2) of field II. This case is decisive when the safety gear is mounted above the car roof.

2.1.3. During Normal Operation

Under normal operating conditions, the load may be unevenly distributed in two perpendicular directions. In Fig.7, a pictorial diagram of guide rails and all forces exerted upon them due to uneven car loading are shown. Forces F_y are exerted in the plane of guide rails $(y - y)$, in which Fx_1 and Fx_2 are acting in $x - x$ planes at right angles to the $y - y$ plane. Each guide rail is subjected to bending due to Fy and combined bending and torsion.

By taking moments about the axes x,y and z respectively, as shown in Fig. 7.

$$F_y = \frac{Q \times g \times e_y}{h} \qquad ... (14)$$

$$F_{x1} = \frac{Q \times g \times e_x \times (b + 2e_y)}{2h \times b} \qquad ... (15)$$

$$F_{x1} = \frac{Q \times g \times e_x \times (b - 2e_y)}{2h \times b} \qquad ... (16)$$

where Q is rated load(kg), g is standard acceleration of free fall (m/s^2), e_y and e_x are eccentricity of the load in the car(mm), b is width of the car (mm), c is depth of the car (mm), h is vertical distance between the centerlines of car guide shoe(mm).

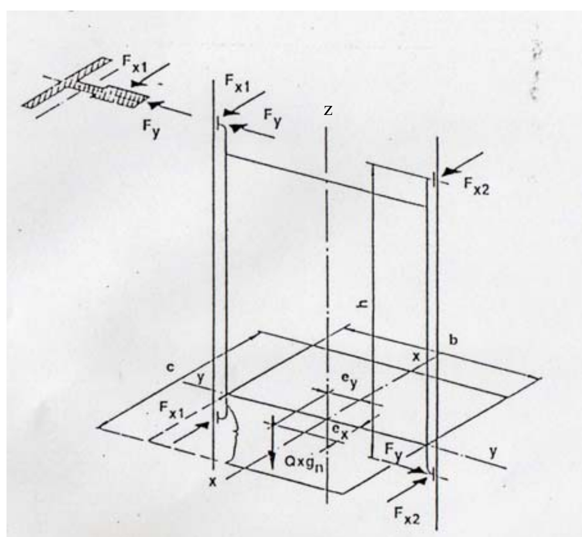

Fig. 7. Forces on guide rails during normal operation

2.2. Code Calculations (British Standard BS 5655: Part 9) [9]

The stress in guide rail during the safety gear operation σ is given by the equation,

$$\sigma = \frac{F_b}{A} + \frac{F_b \times e}{2Z_x}\left[\cos^{-1}\left(\frac{l_k}{2}\sqrt{\frac{F_b}{E \times I_x}}\right) + 1\right] \quad \dots (17)$$

We should note that the second term of the above equation is combined of bending and buckling moment,i.e., there is a silmultaneous effect of buckling and bending moments.

The guide rail deflection during the safety gear is limited to a maximum of 0.25 $\times$length of the machine face of the guide rail in order to avoid the risk of guide shoe disengagement from the guide rail. For this condition, the maximum permissible braking force is given by the equation,

$$F_b = \frac{4E \times I_{xx}}{l_k}\cos^{-2}\left(\frac{e}{2y_{\max} + e}\right) \quad \dots (18)$$

In general, the braking force in the event of two guide rails being employed is given by the formula

$$F_b = \frac{Q + K}{2} \times (a + g) \quad \dots (19)$$

where K is the car mass, (kg). Stress in guide rails calculated from eqn(17) must not exceed the values: 140 Mpa for steel of 370 Mpa tensile grade, 170 Mpa for steel of 430 Mpa tensile grade and 210 Mpa for steel of 520 Mpa tensile grade. The Young's modulus of elasticity is specified E=2.07 (10^5) Mpa.

Performance criteria based on stress and deflection in guide rails during normal operation are as follows: the guide rail is considered a simple beam with a certain degree of constraints on the fixing points and the lateral force is assumed to be imposed midway between the guide rail fixings.

Then the maximum stress in bending is given by

$$\sigma_y = \frac{F_y(l_k)}{6Z_x} \quad \dots (20)$$

$$\sigma_x = \frac{F_x(l_k)}{6Z_y} \quad \dots (21)$$

The constant factor in denominators of the above equations would be 4 for pin-jointed supports and 8 for fixed ends. Horizontal deflections at the midpoints of the beam in two perpendicular deflection are given by the formula

$$y_y = \frac{F_y(l_k^3)}{96EI_x}, y_x = \frac{F_x(l_k)^3}{96EI_y} \quad \dots (22), (23)$$

The constant factors in the above equations would be 48 for pin-jointed supports and 129 for fixed ends. The maximum permissible deflection in compliance with eqn.(21) is 3mm in the pane of guide rails (yy) and 6mm in the perpendicular directions (y_x).

The problems of standard codes is shown in the results.

2.3. Recommended Calculation Method for Guide Rails

From the theoretical analysis, it can be concluded that there is no simultaneous buckling and bending (as we will see in the results), therefore the current procedure of design of guide rails includes this note. Later a case study is performed to achieve the current procedure.

A. Safety Gear Operation

(1) Stress in combined bending and pressure (axial stress) is [4]

$$\sigma = F_b\left(\frac{1}{A} + C_1\frac{e}{Z_x}\right) \quad \dots (24)$$

Bending moment is induced by the eccentrically located braking force F_b; the outer moment is $F_b \times e$. The calculation is carried out for a continuous beam. Coefficient C_1 is given in Table.3 depending upon the number of fields of continuous beam C_1 is concluded from Table.(1,2). The braking force for all cases is calculated from eqn.(19) .

Table 3,
Coefficient C_1

Number of field	C_1
2	*0.621*
3 or more	*0.616*

(2) Stress in buckling

The guide rail is assumed as a simple beam with two pinn-jointed supports, subjected to the braking force F_b in its longitudinal axis. The procedure of buckling analysis is as follows [10,11,12,13,14]:

A) Determine the critical slenderness ratio,

$$S_r)_D = \frac{\pi\sqrt{2E}}{\sigma_y} \quad \text{... (25a)}$$

B) Determine the slenderness ratio of the guide rail, $S_r = \frac{l}{k}$, $k = \sqrt{\frac{I_{\min}}{A}}$ ...(25b)

C) If $S_r)_D \leq S_r$,then use Euler' s equation to find the critical force at which the frame will fail if it exceed this force,

$$P_{cr} = \frac{\pi^2 EA}{S_r^{\,2}} \quad \text{... (26)}$$

D) *And* if the oppose case existed, Johnson' s equation is used,

$$P_{cr} = A[\sigma_y - \frac{1}{E}(\frac{\sigma_y S_r}{2\pi})^2] \quad \text{... (27)}$$