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Thermo Elasto-Plastic Analysis of Rotating Axisymmetrical Bodies Using Modified Von-Mises Yield Criterion

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Abstract

In the present work, the behavior of thick-walled cylinder of elasto-plastic material (polymeric material) has been studied analytically. The study is based on modified Von-Mises yield criterion (for non metallic material). The equations of stress distribution are obtained for the cylinder under general cases of elastic expansion, plastic initiation and elastic-plastic expansion.

A computer program is developed for evaluating the stress distribution. The solution is carried out for worst boundary conditions when the cylinder is subjected to the combination of pressure load, inertia load, and temperature gradient.

The results are presented graphically in terms of dimensionless stresses and radius ratio. They indicate that the thermal and rotational loads are greatly influencing the stress distribution and the initiation of plastic zone, as well as the spreading out of the plastic zone. Moreover, it was found that the critical values of loads required for starting plastic deformation are determined by the amount of the applied load and the type of loading conditions, and it is found that the variation of stresses are greatly influenced by increasing of the temperature gradient at constant pressure and inertial loads than other increases in loading conditions.

Keywords **:** Thick wall cylinder, Elastic-Plastic, Axisymmetrical Bodies.

1. Introduction

There are many practical engineering problems, which may related to the determination and predicting of stress state in elastic-plastic problem of thick-walled cylinder that will cause a particular material to fail under a particular set of loading conditions, presents a major problem to the engineer.

The phenomenon of the elastic-plastic is appeared because of the structural material exhibited to load exceeding the required critical load to produce initial yielding condition in the material. This phenomenon plays an important role in diverse applications including pressure vessels, flywheels, driving shafts, shafts in turbines and generators, and solid propellant grain. The problems of elastic-plastic for thick-walled cylinders have been solved by most investigators based cylinders subjected to mechanical loadings such as internal and external pressures with either thermal loading or angular velocity.

Bland [1], obtained the solutions of the elastic and plastic stresses and strains and the displacements by using Tresca's yield criterion and its associated flow rule, when a thick-walled tube of linear work-hardening material is subjected to internal and external pressures and its surfaces are maintained at different temperature gradients. He found the equivalent plastic strain, which depends on the yield stress for a work-hardening material and determined the stresses and residual stresses for non-work-hardening material.

Derrington [2], derived the principal stresses in a long elastic cylinder subjected to pressure and temperature and calculated the greatest shear stress under combined loading. Also he calculated the largest pressure before first yield occurs at the inner radius by using Tresca's yield criterion. He presented with graphs the effects of load combinations, thickness ratios, and end conditions.

Mahdavian [3], developed the stress equations for a closed end, long, thick walled cylinder subjected to a logarithmic temperature gradient under an internal pressure and an external tensional moment. The solution of the derived equations is based on the shear strain energy yield criterion. The results indicated that the temperature effect plays an important role in the initiation of yielding.

Johnson and Mellor [4], and Chakrabarty [5], have been treated the theoretical solution of the elastic and elastic-plastic behavior of thick-walled cylinders and the rotating disks made of an isotropic non hardening material. They assumed that the mechanical and physical constants are all unaffected by the variation in the temperature. The solution of their equations is based on Tresca's yield criterion.

Hearn [6], has been studied theoretically the solution of thick-walled cylinders and rotating disks. He found the distribution of elastic and elastic-plastic stresses by using Tresca's yield criterion. in the following cases.

Hassan et al. [7] presented the theoretical solution for the elasto-plastic problem of solid propellant grain under the internal pressure, heat gradient during burning and linear acceleration of the rocket during flying course for the case of plane strain. They were also studied the thermal effect due to temperature gradient, and the effect of axial acceleration and failure due to yielding of material.

Arnold et al. [8] studied the problem of a rotating disk of a single or a number of concentric disks forming a unit. An analytical model capable of performing an elastic stress analysis for single/multiple, annular/solid, isotropic disk systems subjected to both pressure surface tractions and body forces (in the form of temperature-changes and rotation fields) is derived. He discussed the influence of applying pressure, the temperature gradient, and rotating conditions.

Meijeren [9] studied the deformation and failure of glassy polymers. The study is based on a compressible Leonov model which provides an adequate description for the rate and temperature depended yield, strain softening, and strain hardening. It is concluded that an increase in network density of the polymer results in a proportional increase in strain hardening modulus.

Many analytical or numerical investigations studied the problem of elastic-plastic for thick-walled cylinder was made of metallic material where it is subjected either to internal pressure only, temperature gradient only, angular velocity only or any combinations of these loads. Most of the pervious researches are using Tresca's yield criterion and little are using the modified Von-Mises yield criterion.

The elastic-plastic problem of thick-walled cylinders that made of a polymeric material had received a little attention in spite of its importance and variety of its applications. Thus in the present study an attempt was made to put into perspective the problem of the elastic-plastic state of rotating cylinder made of polymeric material. The analysis adopted in the present work is based on the modified Von-Mises yield criterion which takes into account the behavior of polymeric materials.

2. Mathematical Modeling:

The stress analysis of thick-walled cylinder under the internal pressure or/and external pressure, uniform temperature gradient, and angular velocity can be considered as plane-strain elasto-plastic problems. The formulation of elasto-plastic relation for a complex problem under multi axial stresses can be achieved by assuming a reasonable mathematical model to correlate between the uni-axial test results and the multi-axial case. The elastic-plastic behavior, for a real multi axial state of stress can be transformed from that of uni-axial state of stress by defining the following parameters [9].

- i- The elastic stress-strain relations.
- ii- An initial yield condition, specifying the state of stress for which plastic flow first sets in.
- iii- A flow rule which relates the plastic strain increments to the stresses and stress increments.
- iv- Hardening rule for establishing the condition for subsequent yield form a plastic state.

2.1. Modified Von-Mises Criterion:

The assumption of Von-Mises yield criterion quite reasonable for ductile materials [10], but it is of unacceptable assumption for polymers. For this reason a modified Von-Mises criteria was proposed to account for the differences in tensile and compressive yield stresses, and the influence of the mean stress or the "Hydrostatic component" of the applied stress on the yield surface.

The modified Von-Mises yield criterion can be deduced [11] by inserting the unequal Y_t and Y_c into Von-Mises yield criterion thus:-

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+C_{11}+C_{22}+C_{33}=2Y_tY_c \quad ...(1)$$

where C_{11}, C_{22} and C_{33} are constant in the principle directions and are added to keep Eq. (1) in balance. To evaluate these constants consecutively, uniaxial tension is assumed, where $\sigma_1=\sigma_x$ and $\sigma_2=\sigma_3=0$ then the Eq. (1) becomes:

$$2Y_tY_c=\sigma_x^2+\sigma_x^2+C_{11}$$

where $C_{11}=2(Y_tY_c-\sigma_x^2)$

At yielding $\sigma_x=Y_t$

Then $C_{11}=2Y_t(Y_c-Y_t)$ or $C_{11}=2\sigma_1(Y_c-Y_t)$

Similarly

$C_{22}=2\sigma_2(Y_c-Y_t)$ and $C_{33}=2\sigma_3(Y_c-Y_t)$

Substituting C_{11}, C_{22} and C_{33} into Eq. (1), then:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+2(Y_c-Y_t)(\sigma_1+\sigma_2+\sigma_3)=2Y_tY_c \quad ...(2)$$

2.2. Thermo Elastic-Plastic Solution of Thick-Walled Cylinder:

The basic assumptions introduced for the analysis are as follows: [7]

The thick-walled cylinder is a polymeric material and the yielding behavior is covered by modified Von-Mises yield criterion. The stress-strain curve in simple tension can be assumed elastic perfectly plastic (Fig (1)).

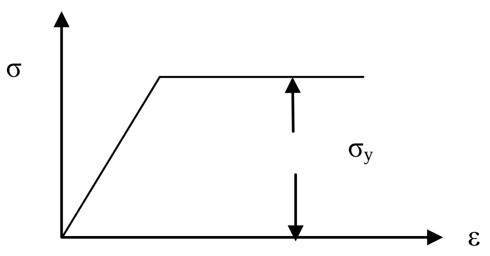


Fig.1. Behavior of Elastic- Perfectly Plastic Material.

1. The plane strain condition is prevailed i.e. εz =0.
2. The temperature is taken to be symmetrical about the axis of symmetry independent of the axial coordinate Z.
3. Material is homogeneous and isotropic.
4. Material properties such as yield surface, modulus of elasticity and Poisson's ratio are assumed to be temperature-independent for suitable temperature range [7].

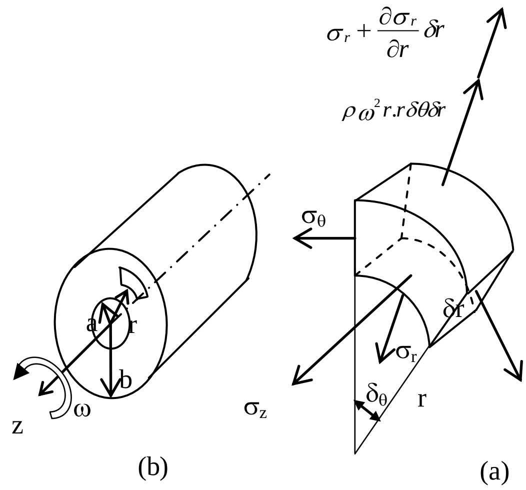


Fig. 2. Thick-Walled Cylinder

Fig.(2) shows a thick-walled cylinder subjected to pressure load, thermal gradient and rotating about longitudinal axis, with inner radius (a), and outer radius (b). The stress distribution through the wall cylinder are given by (see appendix A):

$$\sigma_r=\frac{E}{1+\nu}\left[\frac{C_1}{2(1-2\nu)}-\frac{C_2}{r^2}\right]-\frac{E\alpha}{(1-\nu)r^2}\int_a^r Trdr-\frac{(3-2\nu)}{(1-\nu)}\frac{\rho\omega^2r^2}{8} \quad ...(3.a)$$

$$\sigma_\theta=\frac{E}{1+\nu}\left[\frac{C_1}{2(1-2\nu)}+\frac{C_2}{r^2}\right]-\frac{E\alpha T}{(1-\nu)}+\frac{E\alpha}{(1-\nu)r^2}\int_a^r Trdr-\frac{(1+2\nu)}{(1-\nu)}\frac{\rho\omega^2r^2}{8} \quad ...(3.b)$$

Where C_1 and C_2 are constants of integration. These constants can be determined by the boundary conditions according to the type of loading conditions.

In the present work, the cylinder is assumed to be subjected to general loading conditions represented by pressure load (internal pressure or/and external pressure), steady state temperature gradient and inertia load (angular velocity), thus the boundary conditions are given by :

$$\sigma_r = -P_i\ ,\quad T = T_i \quad \text{at} \quad r = a$$
$$\sigma_r = -P_o\ ,\quad T = T_o \quad \text{at} \quad r = b \qquad \ldots(4)$$

2.3. Elastic Expansion:

The elastic solution of general loading condition can be deduced by substituting the boundary conditions of Eq. (4) into Eqs. (3) then:

$$\sigma_r = A_i\left[\frac{b^2}{r^2}(\beta-1)+\left(1-\lambda^2\beta\right)\right]+R_p\left[a^2+b^2-\frac{a^2b^2}{r^2}-r^2\right]$$
$$+T_p\left[\frac{\frac{b^2}{r^2}-1}{\lambda^2-1}\ln\lambda-\ln\frac{b}{r}\right] \qquad \ldots(5.a)$$

$$\sigma_\theta = A_i\left[\frac{b^2}{r^2}(1-\beta)+\left(1-\lambda^2\beta\right)\right]+R_p\left[a^2+b^2+\frac{a^2b^2}{r^2}-\frac{1+2\nu}{3-2\nu}r^2\right]$$
$$-T_p\left[\frac{\frac{b^2}{r^2}+1}{\lambda^2-1}\ln\lambda+\ln\frac{b}{r}-1\right] \qquad \ldots(5.b)$$

where a, b : Inner and outer radii.
Pi, Po : Internal and external pressures.
Ti, To : Inner and outer temperature.

$$\lambda = \frac{b}{a}\ ,\qquad A_i = \frac{P_i}{\lambda^2-1}\quad ,\ \beta = \frac{P_o}{P_i}$$

$$R_p = \frac{3-2\nu}{1-\nu}\frac{\rho\omega^2}{8}$$
$$T_P = \frac{E\alpha\Delta T}{2(1-\nu)\ln\lambda}\ ,\quad \Delta T = T_i - T_o$$

2.4. Yield Initiation and First Plastic Zone Formation:

When the pressure, temperature difference and angular velocity are sufficiently high, yielding will occur at the radius where the induced stresses are satisfying the modified Von-Mises yield criterion; i.e.: [7]

$$(\sigma_r-\sigma_\theta)^2+(\sigma_\theta-\sigma_z)^2+(\sigma_z-\sigma_r)^2+2Y_t(Y_r-1)(\sigma_r+\sigma_\theta+\sigma_z)$$
$$=2Y_rY_t^2 \qquad \ldots(6)$$

where
Yr : Yield ratio and is given by (Yr =Yc / Yt)

The general equation for thick-walled cylinder under general loading conditions is given by: (see appendix A) :

$$\left(A_iC_{11}+R_pC_{12}+T_pC_{13}\right)^2+\left(A_iC_{21}+R_pC_{22}+T_pC_{23}\right)^2$$
$$+\left(A_iC_{31}+R_pC_{32}+T_pC_{33}\right)^2$$
$$+2Y_t(Y_r-1)\left(A_iC_{41}+R_pC_{42}+T_pC_{43}\right)$$
$$-2Y_rY_t^2=0 \qquad \ldots(7)$$

It is clear that, in the case of general loading conditions there are three cases depending on which loading is firstly initiates yielding. Thus the stresses distributions in this case are similar to that given in Eq.(5) with taken into consideration that the terms corresponding to each type of loads are given in terms of its critical values that first causing yield initiation. It is worthily mentioning that the yielding is beginning at the inner radius (a) in all cases.

2.5 Elastic-Plastic Expansion:

When the applied load exceeds the critical load (required to causes the initial yielding) a plastic zone spreads from the inner radius to the outer radius.

The radius of the elastic-plastic boundary at any stage may be named "c", as shown in Fig (3). Where c is represents the radius of intersection between the elastic and plastic zones.

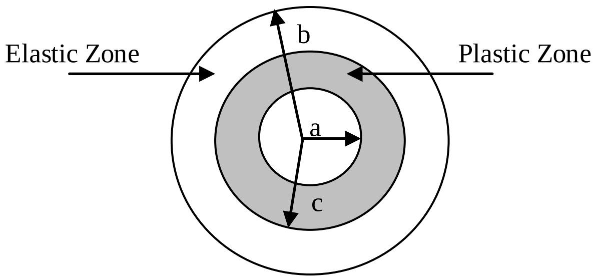


Fig.3. Elastic - Plastic Zones of Thick-Walled Cylinder.

* Plastic Zone (a ≤ r ≤ c):

Analytical solution (Exact Solution) can not be derived to evaluate the stresses distribution in the plastic region (i.e. at (a ≤ r ≤ c)) in the above mentioned cases. This attributed to the fact that the modified Von-Mises yield criterion added a term that take into consideration the yielding for polymeric material. Whereas, this term resulting in a complex form for the yielding criterion. Thus, to evaluate the stresses distribution an alternative approach using numerical method is adopted. Whereas, the radial stress component in the plastic zone can be evaluated from the equilibrium equation, by means of Runge-Kutta method, which is given by :

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \rho\,\omega^2 r = 0 \qquad a \le r \le c \qquad ...(8)$$

While, the hoop stress component can be obtained for corresponding value of radial stress, by using Newton-Raphson algorithm on the bases of the modified Von-Mises yield criterion which defined by :

$$\begin{aligned}&(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 \\ &+ (\sigma_z - \sigma_r)^2 + 2Y_t(Y_r - 1)(\sigma_r + \sigma_\theta + \sigma_z) \\ &= 2Y_r Y_t^2 \end{aligned} \qquad ...(9)$$

** Elastic Zone (c ≤ r ≤ b):

The stress distribution in this case can be obtained by determining the constants of Eq. (5) in terms of the radius (c) which is satisfying the elastic-plastic interfacing. Thus Eq. (5) are given by :

$$\begin{aligned}\sigma_r = {} & A_c\left[\frac{b^2}{r^2}(\beta_c - 1) + \left(1 - \lambda_c^2 \beta_c\right)\right] + R_p\left[c^2 + b^2 - \frac{c^2 b^2}{r^2} - r^2\right] \\ & + T_p\left[\frac{\frac{b^2}{r^2} - 1}{\lambda_c^2 - 1}\ln\lambda_c - \ln\frac{b}{r}\right] \qquad c \le r \le b \end{aligned} \qquad ...(10.a)$$

$$\begin{aligned}\sigma_\theta = {} & A_c\left[\frac{b^2}{r^2}(1 - \beta_c) + \left(1 - \lambda_c^2 \beta_c\right)\right] + R_p\left[c^2 + b^2 + \frac{c^2 b^2}{r^2} - \frac{1+2v}{3-2v} r^2\right] \\ & - T_p\left[\frac{\frac{b^2}{r^2} + 1}{\lambda_c^2 - 1}\ln\lambda_c + \ln\frac{b}{r} - 1\right] \end{aligned} \qquad ...(10 \cdot b)$$

where:

$$A_c = \frac{P_c}{\lambda_c^2 - 1}, \quad \lambda_c = \frac{b}{c}, \quad \beta_c = \frac{P_o}{P_c}$$

$$R_p = \frac{3-2v}{1-v}\frac{\rho\,\omega^2}{8}$$

$$T_p = \frac{E\alpha\Delta T}{2(1-v)\ln\lambda}, \quad \Delta T = T_i - T_o$$

3. Results & Discussion:

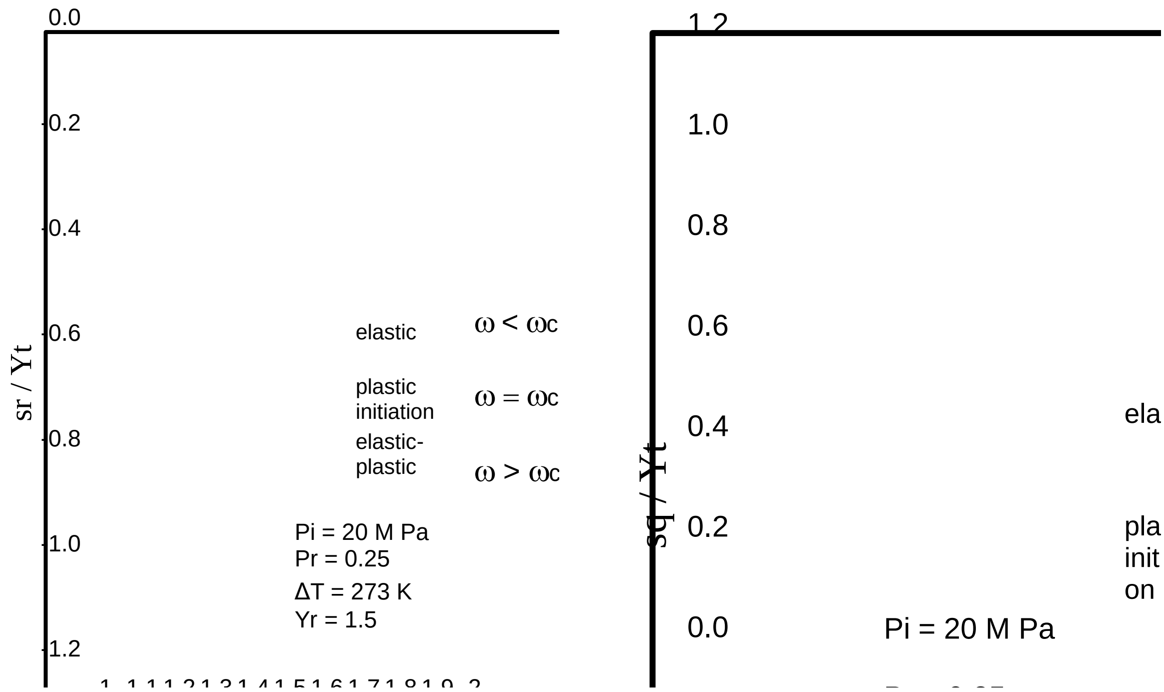
A numerical technique was used to solve the derived equations of stress distribution throughout the cylinder wall. A cylinder made of polyvinylchloride (P.V.C.) of inner radius (a) of 100 mm and outer radius (b) of 200 mm (i.e.a/b=0.5), that assumes elastic-perfectly plastic behavior, was used as a polymeric material. The properties of the material are shown below in table (1). The properties of the material were assumed to be independent of the temperature for appropriate temperature range [7].

Table 1
Properties of P.V.C. Material

Material properties	value
Young's modulus (GPa)	3
Yield stress (MPa)	55
Coefficient of linear thermal expansion (K-1)	60x10-6
Specific gravity	1.4
Poisson's ratio	0.4
Material density (kg/mm3)	1400x10-9
Yield ratio (Yr)	1.5

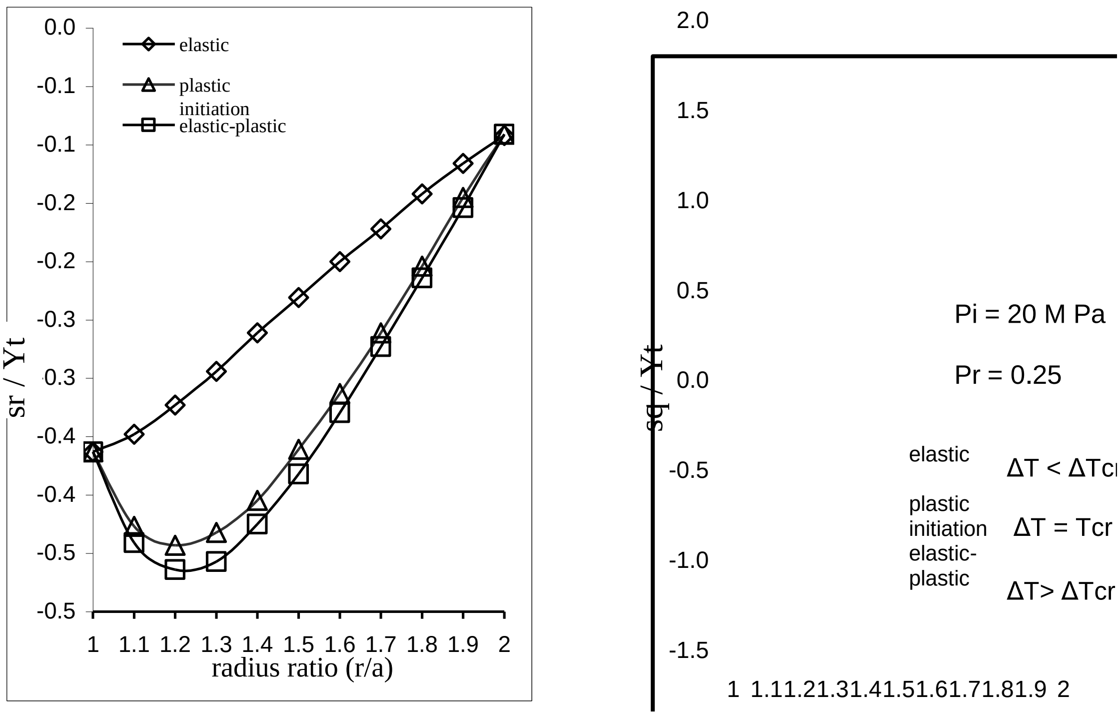
The numerical solution is achieved by developing of a computer program based on a Runge-Kutta method and Netwon-Raphson algorithm. In the present study, a cylinder of inner radius of (100 mm) and outer radius of (200mm) subjected to general loading condition (pressure load, inertia load , and temperature gradient) was investigated.

The results of program runs are shown graphically in Figs.(4), (5), and (6) respectively, which show the variation of stress ratio with radius ratio for rotational speed, temperature gradient, and pressure loads. The results reflect the effect of the rotational speed, temperature gradient, and pressure load on the variation of the radial and hoop stresses throughout the cylinder wall respectively. The results in Fig.(4) and (5) of the stress variation indicate that the values of stresses in the elastic case are greater than those values of the plastic initiation and elastic-plastic cases. This attributed to the fact that when the elastic limit is exceeded, the plastic flow is occurred and law deformation forces are experienced which reflects the assumed elastic – perfectly plastic behavior of the cylinder material.



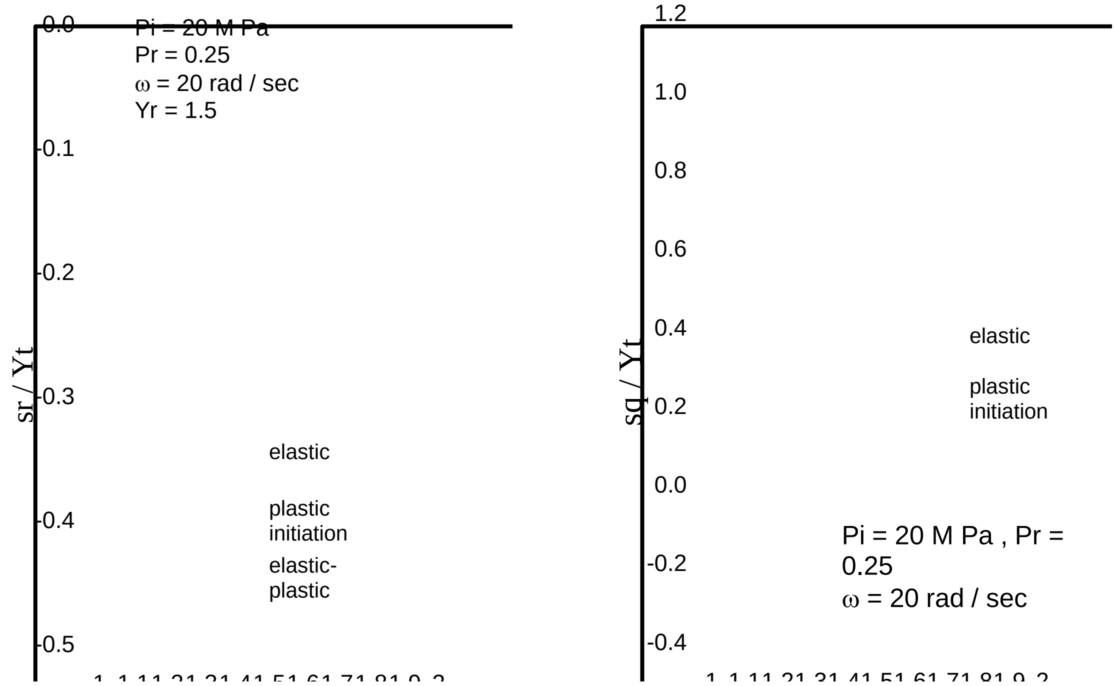
(a) **(b)**

Fig.4. Variation of Stresses With Respect to the Radius Ratio for Cylinder Subjected to Variable Inertia Load (Angular Speed), Constant Pressure and Thermal Loads.



(a) **(b)**

Fig.5. Variation of Stresses With Respect to the Radius Ratio for Cylinder Subjected to Variable Thermal Load, Constant Pressure and Inertia Loads.



(a) **(b)**

Fig.6. Variation of Stresses With Respect to The Radius Ratio for Cylinder Subjected to Variable Pressure Load, Constant Inertia and Thermal Loads.

While, the results in Fig.(6) shows different behavior because the pressure is clearly a compressive load.

Finally, the results show that, the increase of the temperature gradient influences the stress distribution with presence of inertial and pressure loads more than other cases for the same values of inertial and pressure loads. This attributed to the fact that the inertial load results in additional deformation load in form of centrifugal of thick wall cylinder subjected to general loading conditions has been investigated. forces.

4. Conclusions:

The elasto-plastic behavior It has been clarified that the type and amount of loads are greatly influence the stress distribution, the plastic initiation as well as the spreading out of the plastic zone throughout the cylinder wall. The inertial load results in producing an additional deformation force in form of centrifugal force which in leads to the fact that the increase in temperature gradient in the presence of the inertia and pressure loads is greatly affecting the stress distribution than other cases. Also, it is concluded that when the elastic limit of the material is exceeded, a plastic flow is occurred and law deformation forces is required, which reflects the elastic-perfectly plastic assumed behavior of the cylinder material.

5. Notation:

a, b	Inner and outer radii of cylinder
c	Radius of elastic-plastic interface
r	Variable radius
E	Modulus of elasticity
G	Modulus of rigidity
P_i , P_o	Internal and external pressures respectively.
P_r	Pressure ratio (P_i /P_o)
T_i , T_o	Inner and outer temperatures respectively
u	Displacement
U_s	Shear strain energy
Y_t , Y_c	Tensile and compressive yield stresses respectively.

6. Greek letters:

σ_x , σ_y , σ_z	Stress components in x, y and z axes
σ_1 , σ_2 , σ_3	Principal stresses
σ_r , σ_θ , σ_z	Radial, Hoop, and Axial stresses respectively.
ε_x , ε_y , ε_z	Strain components in x, y and z axes.
ε_r , ε_θ , ε_z	Radial, Hoop, and Axial strains respectively.
ρ	Density of material
ν	Poisson's ratio
ω	Angular velocity
α	Coefficient of thermal expansion
β	Outer pressure to inner pressure ratio

7. APPENDIX (A):
7.1. General Solution:

Referring to Fig. (2) which shows a thick-walled cylinder subjected to pressure load, thermal gradient and rotating about longitudinal axis, with inner radius (a), and outer radius (b). By considering the equilibrium of forces acting on the element, at radius (r), in the radial direction then:

$$2\sigma_\theta \delta r \sin\frac{\delta\theta}{2} + \sigma_r r\delta\theta - (\sigma_r + \delta\sigma_r)(r+\delta r)\delta\theta = \rho r^2 \omega^2 \delta\theta\delta r \quad \ldots(A.1)$$

For δθ is small then:

$$\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$$

If $\delta r \to 0$ then $\delta\sigma_r \to 0$, therefore, the above equation simplifies to

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \rho\omega^2 r = 0 \quad \ldots(A.2)$$

where

σ_r : radial stress

σ_θ: hoop stress

ρ : material density

ω: angular velocity

The elastic stress-strain relationships at a point in the cylinder wall are given by [3].

$$\begin{aligned} E\varepsilon_r &= \sigma_r - \nu(\sigma_\theta + \sigma_z) + E\alpha T \\ E\varepsilon_\theta &= \sigma_\theta - \nu(\sigma_r + \sigma_z) + E\alpha T \\ E\varepsilon_z &= \sigma_z - \nu(\sigma_r + \sigma_\theta) + E\alpha T \end{aligned} \quad \ldots(A.3)$$

where

E : modulus of elasticity

ε_r, ε_θ, and ε_z,: Radial, hoop and axial strains respectively.

σ_z: longitudinal stress

ν : Poisson's ratio

α : thermal coefficient of expansion

T: temperature equation [4],

$$T = \Delta T \frac{\ln\left(\frac{b}{r}\right)}{\ln\left(\frac{b}{a}\right)}$$

where

ΔT : temperature gradient

The plane strain condition can be assumed, $\varepsilon_z = 0$, and the above equations can be reduced to:

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) - E\alpha T \quad \ldots(A \cdot 4)$$

The strain-displacement relations [4]:

$$\varepsilon_r = \frac{du}{dr}$$
$$\varepsilon_\theta = \frac{u}{r} \quad \ldots(A \cdot 5)$$

where u is the displacement along the radial direction.

Substituting Eqs (A.4) and (A.5) into Eq (A.3), then:

$$\varepsilon_r = \frac{du}{dr} = \frac{1+\nu}{E}\left[(1-\nu)\sigma_r - \nu\sigma_\theta + E\alpha T\right] \quad \ldots(A.6.a)$$

$$\varepsilon_\theta = \frac{u}{r} = \frac{1+\nu}{E}\left[(1-\nu)\sigma_\theta - \nu\sigma_r + E\alpha T\right] \quad \ldots(A.6.b)$$

Substituting Eqs. (A.6.a) and (A.6.b) into Eq. (A.2) and integrating with respect to r then, the general stress equations are given by:

$$\sigma_r = \frac{E}{1+v}\left[\frac{C_1}{2(1-2v)} - \frac{C_2}{r^2}\right] - \frac{E\alpha}{(1-v)r^2}\int_a^r Trdr - \frac{(3-2v)}{(1-v)}\frac{\rho\omega^2 r^2}{8} \quad ...(A\cdot 7\cdot a)$$

$$\sigma_\theta = \frac{E}{1+v}\left[\frac{C_1}{2(1-2v)} + \frac{C_2}{r^2}\right] - \frac{E\alpha T}{(1-v)} + \frac{E\alpha}{(1-v)r^2}\int_a^r Trdr - \frac{(1+2v)}{(1-v)}\frac{\rho\omega^2 r^2}{8} \quad ...(A\cdot 7\cdot b)$$

$$\sigma_z = \frac{vEC_1}{(1+v)(1-2v)} - \frac{E\alpha T}{(1-v)} - \frac{v}{(1-v)}\frac{\rho\omega^2 r^2}{2} \quad ...(A.7.c)$$

7.2. Derivation of the General Equation of Yield Initiation:

The boundary conditions when thick-walled cylinder is subjected to internal pressure or/and external pressure, steady state temperature gradient are:

σ r = -Pi ,　T= Ti　at　r = a
σ r = -Po ,　T= To　at　r = b

The elastic solution of general loading condition can be deduced by substituting the boundary conditions for each case into Eqs. (3.10) then:

$$\sigma_r = A_i\left[\frac{b^2}{r^2}(\beta-1) + (1-\lambda^2\beta)\right] + R_p\left[a^2+b^2-\frac{a^2b^2}{r^2}-r^2\right] + T_p\left[\frac{\frac{b^2}{r^2}-1}{\lambda^2-1}\ln\lambda - \ln\frac{b}{r}\right] \quad ...(B\cdot 1\cdot a)$$

$$\sigma_\theta = A_i\left[\frac{b^2}{r^2}(1-\beta) + (1-\lambda^2\beta)\right] + R_p\left[a^2+b^2+\frac{a^2b^2}{r^2}-\frac{1+2v}{3-2v}r^2\right] - T_p\left[\frac{\frac{b^2}{r^2}+1}{\lambda^2-1}\ln\lambda + \ln\frac{b}{r} - 1\right] \quad ...(B\cdot 1\cdot b)$$

$$\sigma_z = 2vA_i(1-\lambda^2\beta) + 2v R_p\left[a^2+b^2-\frac{2r^2}{3-2v}\right] + T_p\left[v - 2\ln\frac{b}{r} - \frac{2v}{\lambda^2-1}\ln\lambda\right] \quad ...(B\cdot 1\cdot c)$$

where

$$\beta = \frac{P_o}{P_i}, \quad A_i = \frac{P_i}{\lambda^2-1}, \quad \lambda = \frac{b}{a}$$

$$R_p = \frac{3-2v}{1-v}\frac{\rho\omega^2}{8}$$

$$T_p = \frac{E\alpha\Delta T}{2(1-v)\ln\lambda}, \quad \Delta T = T_i - T_o$$

When the pressure, temperature difference and angular velocity are sufficiently high, yielding will occur at the radius where the induced stresses are satisfying the modified Von-Mises yield criterion; i.e.:

$$(\sigma_r-\sigma_\theta)^2 + (\sigma_\theta-\sigma_z)^2 + (\sigma_z-\sigma_r)^2 + 2Y_t(Y_r-1)(\sigma_r+\sigma_\theta+\sigma_z) = 2Y_rY_t^2 \quad ...(B\cdot 2)$$

where

$$Y_r = \frac{Y_c}{Y_t}$$

From Eqs. (B.1) it is found that:

$$(\sigma_r-\sigma_\theta) = A_i\left[2\frac{b^2}{r^2}(\beta-1)\right] + R_p\left[-2\left(\frac{a^2b^2}{r^2}+\frac{1-2v}{3-2v}r^2\right)\right] + T_p\left[\frac{2\frac{b^2}{r^2}}{\lambda^2-1}\ln\lambda - 1\right] = (A_iC_{11} + R_pC_{12} + T_pC_{13}) \quad ...(B\cdot 3)$$

$$(\sigma_\theta-\sigma_z) = A_i\left[\frac{b^2}{r^2}(1-\beta) + (1-\lambda^2\beta)(1-2v)\right] + R_p\left[\begin{matrix}(a^2+b^2)(1-2v)+\\ \frac{a^2b^2}{r^2}-\frac{1-2v}{3-2v}r^2\end{matrix}\right] + T_p\left[\begin{matrix}\frac{2v-1-\frac{b^2}{r^2}}{\lambda^2-1}\ln\lambda + \\ 1-v+\ln\frac{b}{r}\end{matrix}\right] = (A_iC_{21} + R_pC_{22} + T_pC_{23}) \quad ...(B\cdot 4)$$

$$(\sigma_z-\sigma_r)=A_i\left[\begin{matrix}\frac{b^2}{r^2}(1-\beta)-\\(1-\lambda^2\beta)(1-2v)\end{matrix}\right]+R_p\left[\begin{matrix}\frac{a^2b^2}{r^2}-(a^2+b^2)(1-2v)\\+3r^2\frac{1-2v}{3-2v}\end{matrix}\right]$$

$$+T_p\left[\begin{matrix}\frac{1-2v-\frac{b^2}{r^2}}{\lambda^2-1}\ln\lambda+\\v-\ln\frac{b}{r}\end{matrix}\right]=(A_iC_{31}+R_pC_{32}+T_pC_{33})\quad\ldots(B.5)$$

$$(\sigma_r+\sigma_\theta+\sigma_z)=A_i\left[2(1-\lambda^2\beta)(1+v)\right]+R_p\left[2(1+v)\begin{pmatrix}a^2+b^2-\\\frac{2r^2}{3-2v}\end{pmatrix}\right]$$

$$+T_p\left[\begin{matrix}1+v-4\ln\frac{b}{r}-\\\frac{2(1+v)}{\lambda^2-1}\ln\lambda\end{matrix}\right]=(A_iC_{41}+R_pC_{42}+T_pC_{43})\quad\ldots(B.6)$$

The general equation for thick-walled cylinder when it is subjected to internal pressure and/or external pressure, temperature gradient and angular velocity can be found by Substituting Eqs (B.3), (B.4), (B.5) and (B.6) into Eq (B.2), thus:

$$\begin{aligned}&(A_iC_{11}+R_pC_{12}+T_pC_{13})^2\\&+(A_iC_{21}+R_pC_{22}+T_pC_{23})^2\\&+(A_iC_{31}+R_pC_{32}+T_pC_{33})^2\\&+2Y_t(Y_r-1)(A_iC_{41}+R_pC_{42}+T_pC_{43})\\&-2Y_rY_t^2=0\end{aligned}\quad\ldots(B.7)$$

where the constants are given by:

$$C_{11}=2\frac{b^2}{r^2}(\beta-1)\ ,C_{12}=-2\left(\frac{a^2b^2}{r^2}+\frac{1-2v}{3-2v}r^2\right)$$

$$C_{13}=\frac{2\frac{b^2}{r^2}}{\lambda^2-1}\ln\lambda-1\ ,C_{21}=\frac{b^2}{r^2}(1-\beta)+(1-\lambda^2\beta)(1-2v)$$

$$C_{22}=(a^2+b^2)(1-2v)+\frac{a^2b^2}{r^2}-\frac{1-2v}{3-2v}r^2$$

$$C_{23}=\frac{2v-1-\frac{b^2}{r^2}}{\lambda^2-1}\ln\lambda+1-v+\ln\frac{b}{r}$$

$$C_{31}=\frac{b^2}{r^2}(1-\beta)-(1-\lambda^2\beta)(1-2v)\ ,$$

$$C_{32}=\frac{a^2b^2}{r^2}-(a^2+b^2)(1-2v)+3r^2\frac{1-2v}{3-2v}$$

$$C_{33}=\frac{1-2v-\frac{b^2}{r^2}}{\lambda^2-1}\ln\lambda+v-\ln\frac{b}{r}$$

$$C_{41}=2(1-\lambda^2\beta)(1+v)$$

$$C_{42}=2(1+v)\left(a^2+b^2-\frac{2r^2}{3-2v}\right)$$

$$C_{43}=1+v-4\ln\frac{b}{r}-\frac{2(1+v)}{\lambda^2-1}\ln\lambda$$

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التحليل الحراري المرن- اللدن للأجسام المدورة المتناظرةباستخدام معيارية فون- ميسيز المطورة

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الخلاصة

عرضت النتائج بدلالة الاجهادات اللابعدي و نصف القطر اللابعدي . إشارت النتائج الى ان حالة الأحمال الحرارية و التدويرية لها تأثير كبير على طبيعة توزيع الاجهادات و على حدوث التشويه اللدن وسرعة انتشار منطقة التشويه اللدن. وكذلك و جد ان القيم الحرجة للأحمال المسلطة و التي تسبب التشويه اللدن تتأثر بقيمة وطبيعة التحميل على الاسطوانة، وقد بينت النتائج ان التغيير الحاصل في توزيع الاجهادات يتاثر بصورة كبيرة في حال زيادة الاحمال الحرارية وثبوت الاحمال التدويرية والضغط اكثر من الزيادة في حالات التحميل الاخرى.

في هذا البحث، تمت دراسة التصرف المرن– اللدن للاسطوانات السميكة و المصنوعة من المواد البوليمرية بصورة نظرية. بنيت الدراسة بالاعتماد على معيارية فون- ميسيز المطورة للمواد غير المعدنية .وجدت المعادلات الخاصة بتوزيع الاجهادات تحت أقسى ظروف الأحمال المسلطة على الاسطوانة ولحالات التشويه المرنة، المرنة-اللدنة، اللدنة. وجدت معادلات توزيع الاجهادات للاسطوانة تحت حالات التمدد المرن والنشوء اللدن و التمدد المرن- اللدن.

تم ايجاد توزيع الاجهادات ببناء برامج حاسوبية مطورة ووجدت الحلول للظروف الحدية الحرجة و الناتجة من الأحمال المشتركة من ضغط و درجة حرارة وأحمال تدويرية.
