Al- Khwarizmi Engineering Journal

Al- Khwarizmi Engineering Journal, Vol.3, No.1 , pp 48-62 (2007)

Image restoration using regularized inverse filtering and adaptive threshold wavelet denoising

Mr. Firas Ali

Mechatronics Engineering Department ,
Al- Khwarizmi College of Engineering ,
University of Baghdad

(Received 31 July 2006; accepted 19December 2006)

Abstract :-

Although the Wiener filtering is the optimal tradeoff of inverse filtering and noise smoothing, in the case when the blurring filter is singular, the Wiener filtering actually amplify the noise. This suggests that a denoising step is needed to remove the amplified noise .Wavelet-based denoising scheme provides a natural technique for this purpose .

In this paper a new image restoration scheme is proposed, the scheme contains two separate steps : Fourier-domain inverse filtering and wavelet-domain image denoising. The first stage is Wiener filtering of the input image , the filtered image is inputted to adaptive threshold wavelet denoising stage . The choice of the threshold estimation is carried out by analyzing the statistical parameters of the wavelet sub band coefficients like standard deviation, arithmetic mean and geometrical mean . The noisy image is first decomposed into many levels to obtain different frequency bands. Then soft thresholding method is used to remove the noisy coefficients, by fixing the optimum thresholding value by this method .

Experimental results on test image by using this method show that this method yields significantly superior image quality and better Peak Signal to Noise Ratio (PSNR). Here, to prove the efficiency of this method in image restoration , we have compared this with various restoration methods like Wiener filter alone and inverse filter.

Key words : Image Restoration , Generalized Inverse filter , Generalized Wiener filter , Wavelet Transform , Denoising using Discrete Wavelet Transform , Adaptive Threshold Wavelet Transform, Gaussian Noise.

1. Introduction

Image restoration are used to improve the appearance of an image by application of a restoration process that uses a mathematical model to remove image degradation . Examples of the types of degradation include blurring caused by motion of atmospheric disturbance , geometric distortion

caused by imperfect lenses , superimposed interference patterns caused by mechanical systems , noise from electronic sources , for example in the case of acquiring images with a CCD (Charge Coupling Device) camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image [1],[2].
The degradation process model consists of two parts , the degradation function and the noise function . The general model in the spatial domain follows :

$$d(x,y) = h(x,y) ** f(x,y) + n(x,y) \quad ...(1)$$

where the ** denotes two dimensional convolution process
d(x,y) = degraded image
h(x,y) = degradation function
f(x,y) = original image
n(x,y) =additive noise function

Because convolution in the spatial domain is equivalent to multiplication in the frequency domain , the frequency domain model is

$$D(u,v) = H(u,v)F(u,v) + N(u,v) \; ... \; (2)$$

where D(u,v) = Fourier transform of the degraded image
H(u,v) = Fourier transform of the degradation function
F(u,v) = Fourier transform of the original image
N(u,v) = Fourier transform of the additive noise function

In this model, the true image and noise are coupled linearly , therefore, the problem of recovering image from degraded one is referred to as the linear image restoration problem .

Number of techniques [l], [2] have been proposed in the literature to restore an image from degradations due to blurring and additive noise .

Inverse filtering ,Wiener filtering **,** parametric Wiener filtering **,** power spectrum filtering , and geometrical mean filtering are some of the examples .

The inverse filtering is a restoration technique for deconvolution, i.e., when the image is blurred by a known lowpass filter, it is possible to recover the image by inverse filtering or generalized inverse filtering. However, inverse filtering is very sensitive to additive noise. The Wiener filtering executes an optimal tradeoff between inverse filtering and noise smoothing. It removes the additive noise and inverts the blurring simultaneously. So the Wiener filtering is optimal in terms of the mean square error because it minimizes the overall mean square error in the process of inverse filtering and noise smoothing.

Although the Wiener filtering is the optimal tradeoff of inverse filtering and noise smoothing , in the case when the blurring filter is singular, the Wiener filtering actually amplify the noise. This suggests that a denoising step is needed to remove the amplified noise . Image denoising techniques are necessary to remove the random additive noises while retaining as much as possible the important signal features. Statistical filters like Average filter [1] [2], can be used for removing such noises but the wavelet based denoising techniques proved better results than these filters[3] ,[6] and [7]. In general, image de-noising imposes a compromise between noise reduction and preserving significant image details. To achieve a good performance in this respect, a denoising algorithm has to adapt to image discontinuities.

The wavelet representation naturally facilitates the construction of such spatially adaptive algorithms. It compresses essential information in a signal into relatively few, large coefficients, which represent image details at different resolution scales. In recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal and image denoising [3] [4] [6] [7] [8] [10] , because wavelet provides an appropriate basis for separating noisy signal from image signal. Many of these wavelet based thresholding techniques have proved better efficiency in image denoising. We consider here an efficient thresholding technique used in [4] for image denoising by analyzing the statistical parameters of the wavelet coefficients.

This paper is organized as follows: A brief review of Generalized Inverse Filter for image restoration is provided in section 2. Generalized Wiener filter equations for image restoration are developed in section 3. Image Restoration Using Regularized Inverse Filtering and Wavelet Denoising is discussed in section 4. A brief review of Discrete Wavelet Transform (DWT) and wavelet filter banks are provided in section 5. The wavelet based adaptive thresholding technique is explained in Section 6. The proposed algorithm steps involved in this work are explained in Section 7. In Section 8 the experimental results of this proposed work and other restoration techniques are present and compared. Finally concluding remarks are given in Section 9.

2. Image restoration using generalized inverse filter

If we know or can create a good model of the blurring function that corrupted an image, the quickest and easiest way to restore that is by inverse filtering. Unfortunately, since the inverse filter is a form of high pass filer, inverse filtering responds very badly to any noise that is present in the image because noise tends to be high frequency. In this section , we explore a method of inverse filtering called a thresholding method .

We can model a blurred image by

$$d(x,y) = h(x,y) ** f(x,y) \qquad \text{... (3)}$$

where f is the original image, h is some kind of a low pass filter and d is our blurred image. So to get back the original image, we would just have to convolve our blurred function with some kind of a high pass filter r

$$f(x,y) = d(x,y) ** r(x,y) \qquad \text{... (4)}$$

In the ideal case, we would just invert all the elements of h to get a high pass filter. However, notice that a lot of the elements in h have values either at zero or very close to it. Inverting these elements would give us either infinities or some extremely high values. In order to avoid these values, we will need to set some sort of a threshold on the inverted element. So instead of making a full inverse out of H, we can an "almost" full inverse by

$$R_{inv}(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & if \quad \dfrac{1}{H(u,v)} < \gamma \\ \gamma & else \end{cases} \qquad \text{... (5)}$$

So the higher we set γ , the closer R_{inv} is to the full inverse filter. This is called regularized inverse filter that implemented in our work .

3. Image restoration using generalized wiener filter

The inverse filtering is a restoration technique for deconvolution. However, inverse filtering is very sensitive to additive noise as explained in the previous section .

Because Wiener can removes the additive noise and inverts the blurring simultaneously, it is usually used instead of inverse filter in these cases.

The Wiener filtering is optimal in terms of the mean square error. In other words, it minimizes the overall mean square error in the process of inverse filtering and noise smoothing. The approach is based on a stochastic framework. The Wiener filter in Fourier domain can be expressed as follows:

$$R_w(u,v) = \frac{H^*(u,v)\,S_{im}(u,v)}{|H(u,v)|^2\,S_{im}(u,v) + S_n(u,v)} \quad \dots (6)$$

where S_{im}, S_n are respectively power spectra of the original image and the additive noise, and H(u,v) is the blurring filter. It is easy to see that the Wiener filter has two separate part, an inverse filtering part and a noise smoothing part. It not only performs the deconvolution by inverse filtering (high pass filtering) but also removes the noise with a compression operation (low pass filtering).

To implement the Wiener filter in practice we have to estimate the power spectra of the original image and the additive noise. For white additive noise the power spectrum is equal to the variance of the noise. To estimate the power spectrum of the original image many methods can be used. A direct estimate is the periodogram estimate of the power spectrum computed from the observation:

$$S_{yy}^{per} = \frac{1}{N^2}\left[D(u,v)\,D(u,v)^*\right] \quad \dots (7)$$

where D(u,v) is the DFT of the observation. The advantage of the estimate is that it can be implemented very easily without worrying about the singularity of the inverse filtering. Another estimate which leads to a cascade implementation of the inverse filtering and the noise smoothing is

$$S_{im} = \frac{S_{yy} - S_n}{|H|^2} \quad \dots (8)$$

which is a straightforward result of the fact $: S_{yy} = S_n + S_{im}|H|^2$ The power spectrum S_{yy} can be estimated directly from the observation using the periodogram estimate. This estimate results in a cascade implementation of inverse filtering and noise smoothing:

$$R_w(u,v) = \frac{1}{H}\,\frac{S_{yy}^{per} - S_n}{S_{yy}^{per}} \quad \dots (9)$$

4. Image restoration using regularized inverse filtering and wavelet denoising

Although the Wiener filtering is the optimal tradeoff of inverse filtering and noise smoothing, in the case when the blurring filter is singular, the Wiener filtering actually amplify the noise. This suggests that a denoising step is needed to remove the amplified noise. Wavelet-based denoising scheme, a successful approach provides a natural technique for this purpose. Therefore, the image restoration contains two separate steps: Fourier-domain inverse filtering and wavelet-domain image denoising. The diagram is shown as follows.

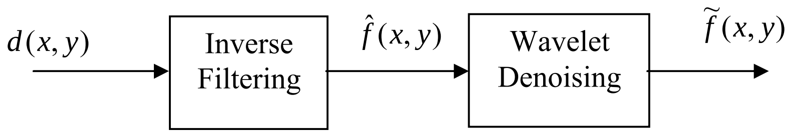


Fig.(1) Block diagram of Image Restoration Using Inverse Filtering and Wavelet Denoising

The approach that shown in fig.(1) for image restoration improves the performance, however, in the case when the blurring function is not invertible, the method is not applicable. Furthermore, since the two steps are separate, there is no control over the overall performance of the restoration. So a wavelet-based deconvolution technique for ill-conditioned systems can be proposed. The idea is simple: employ both Fourier-domain Wiener-like and wavelet-domain regularization. The regularized inverse filter is introduced by modifying the Wiener filter given in equation (6) with a new-introduced parameter α :

$$G_\alpha = \frac{H * S_{im}}{|H|^2 S_{im} + \alpha S_n} \qquad \text{... (10)}$$

The parameter α can be optimally selected to minimize the overall mean-square error. The diagram of the algorithm is displayed as follows:.

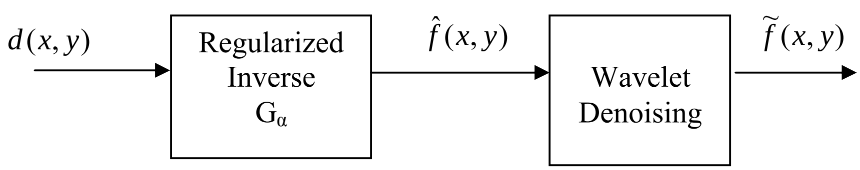
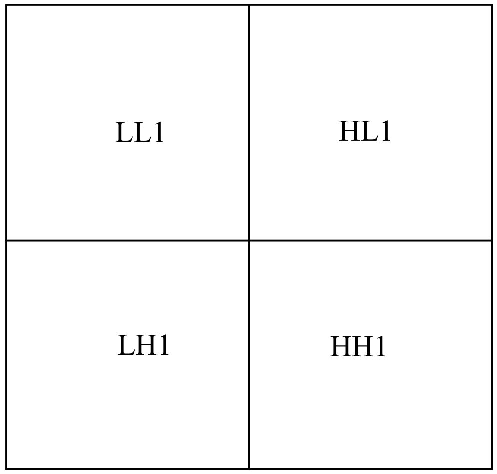


Fig.(2) Block diagram of Image Restoration Using Regularized Inverse Filtering and Wavelet Denoising

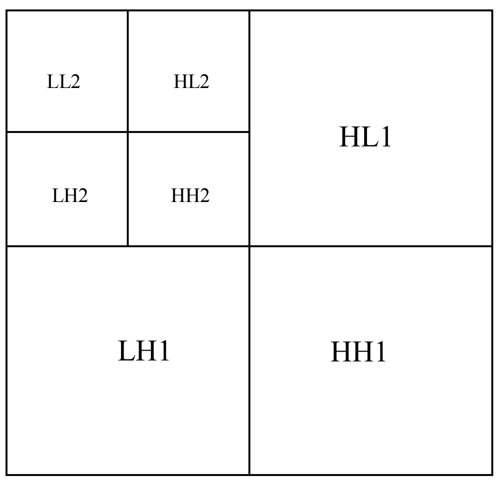
5. Discrete wavelet transform

The DWT is identical to a hierarchical subband system where the subbands are logarithmically spaced in frequency and represent octave-band decomposition. Due to the decomposition of an image using the DWT [1] the original image is transformed into four pieces which is normally labeled as LL, LH, HL and HH as in the schematic depicted in Fig. (3 a.).

The LL subband can be further decomposed into four subbands labeled as LL2, LH2, HL2 and HH2 as shown in Fig.3 b.



(a) One-Level



(b) Two -Level

Fig. 3 Image decomposition by using DWT

The LL piece comes from low pass filtering in both directions and it is the most like original picture and so is called the approximation. The remaining pieces are called detailed components. The HL comes from low pass filtering in the vertical direction and high pass filtering in the horizontal direction and so has the label HL. The visible detail in the sub-image, such as edges, have an overall vertical orientation since their alignment is perpendicular to the direction, of the high pass filtering and they are called vertical details. The remaining components have analogous explanations. The filters LD and HD shown in Fig. 4 are one-dimensional Low Pass Filter (LPF) and High Pass Filter (HPF) respectively for image decomposition. To obtain the next level of decomposition, sub band LL1 alone is further decomposed. This process continues until some final scale is reached. The decomposed image can be reconstructed using a reconstruction filter as shown in Fig. 5. Here, the filters LR and HR represent low pass and high pass reconstruction filters respectively. Here, since the image size is not changed after decomposition this DWT is called critically sampled transform without having any redundancy.

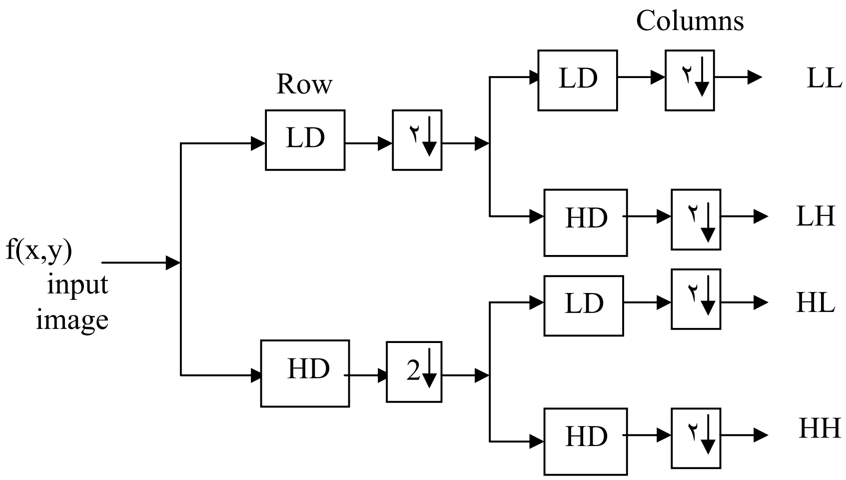


Fig.4 Wavelet Filter bank for one-level image decomposition

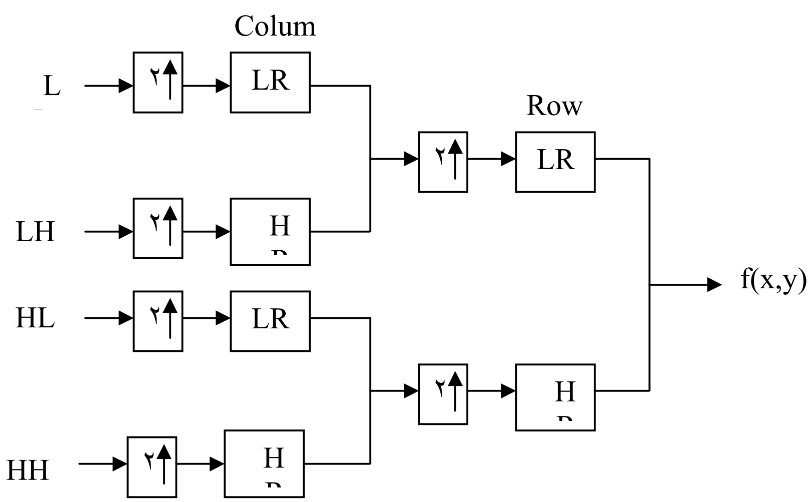


Fig.5 Wavelet Filter bank for one-level image reconstruction

6. Wavelet thresholding

Let *f={fij,i,j=1,2,....M}* denotes a M x M matrix of original image to be recovered and M is some integer power of 2. During the transmission, the signal *f* is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian noise *nij* with standard deviation σ ,and at the receiver end, the noisy observation dij=fij+nij is obtained. The goal is to estimate the signal *f* from the noisy observations dij such that the Mean Square Error (MSE) is minimum. To achieve this the dij is transformed into wavelet domain, which decomposes the dij into many subbands as explained in section 5, which separates the signal into so many frequency bands. The small coefficients in the subbands are dominated by noise, while coefficients with large absolute value carry more signal information than noise. Replacing noisy coefficients (small coefficients below certain value) by zero and an inverse wavelet transform may lead to reconstruction that has lesser noise. Normally Hard Thresholding and Soft Thresholding techniques are used for such denoising process. Hard and Soft thresholding [4] with threshold λ are defined as follows:

The hard thresholding operator is defined as

$$D(U,\lambda)=\begin{cases} U \ \textit{for all} \ |U|>\lambda \\ 0 \ \textit{otherwise} \end{cases} \quad \text{... (11)}$$

The soft thresholding operator on the other hand is defined as

$$D(U,\lambda)=\mathrm{sgn}(U)*\max(0,|U|-\lambda) \quad \text{...(12)}$$

Hard thresholding is "keep or kill" procedure and is more intuitively appealing and also it introduces artifacts in the recovered images. Soft thresholding , on the other hand , have proved to be more efficient and it is used for the entire algorithm for the following reasons : Soft thresholding has been shown to achieve near minmax rate [3],[4] and [8]. Moreover, it is also found to yield visually more pleasing images. The above factors motivate us to use this Soft thresholding in our denoising method.

6.1. Estimation of Parameters for Threshold Value

Finding an optimum threshold value (λ) for soft thresholding is not an easy task. A small threshold value will pass all the noisy coefficients and hence the resultant denoised signal may still be noisy. A large threshold value on the other hand, makes more number of coefficients as zero which leads to smooth signal and destroys details and may cause blur and artifacts. So, optimum threshold value should be found out, which is adaptive to different subband characteristics. Here, we describe an efficient method for fixing the threshold value for denoising by analyzing the statistical parameters of the wavelet coefficients. The threshold value (λ) that used for soft thresholding technique is given by:

$$\lambda = C\,\sigma - (AM - GM) \quad \text{...(13)}$$

Where σ is the noise variance of the corrupted image , AM and GM are the abbreviates for Arithmetic Mean and Geometric Mean of the subband coefficients respectively and the term C is included to make the threshold value depend on decomposition level. In some applications of image denoising, the value of the input noise variance is known, or can be measured based on the information other than the corrupted data. If this is not the case, one has to estimate it from the input data. For this, wavelet based method commonly used the highest frequency subband of the decomposition. In the DWT of the image, the HH1 subband contains mainly noise. For estimating the noise level we use the relation used in [3],[4] and [8] , which is denoted as

$$\sigma = \frac{median[Y_{ij}]}{0.6745} \quad , \quad Y_{ij} \in subband \ HH_1 \quad \text{... (14)}$$

Normally in wavelet subbands, as the level increases the coefficients of the subband becomes smoother. For example the subband HL2 is smoother than the corresponding subband in the first level (HL1) and so the threshold value of HL2 should be smaller than that for HL1. So , the term C is included for this purpose to make the threshold value as decomposition level dependent and is given by,

$$C = 2^{(L-k)} \quad \text{... (15)}$$

Where, L is the no. of wavelet decomposition level k is the level at which the subband is available (for HL2, k=2). The term $|AM - GM|$ is the absolute value of difference between Arithmetic Mean and Geometric Mean of the subband coefficients. This term is the measure of the smoothness of the image to be denoised. If an image is having more edges, then the noisy wavelet coefficients of the image should be removed by fixing slightly lower threshold value so as to keep the image edges undisturbed. This can be achieved by this term, since small value of this term indicates that the subband is having smooth edges and vice versa. This term makes the threshold value as image dependent and helps to preserve the image edges to some extent, which results in better image quality. The Arithmetic Mean and Geometric Mean of the subband matrix X(i,j) are denoted as,

$$Arithmetic\,Mean\ (AM) = \frac{1}{M^2}\sum_{i=1}^{M}\sum_{j=1}^{M} X(i,j) \qquad \dots (16)$$

$$Geometric\ Mean\ (GM) = \left[\prod_{i=1}^{M}\prod_{j=1}^{M} X(i,j)\right]^{\frac{1}{M^2}} \qquad \dots (17)$$

Thus each term in equation (13) has its' own importance as explained above to make the threshold value more adaptive to subband coefficients as well as level of decomposition.

7. Image restoration algorithm

The Complete algorithm of proposed Wiener - wavelet based restoration technique is explained in the following steps :

1. Restore the degraded image without denoising using the regularized inverse filter with $\alpha = 1$.

2. Perform the DWT of the restored noisy image coming from step 1 up to 2 levels (L=2)to obtain seven subbands which are named as HH1, LH1,HL1, HH2, LH2, HL2 and LL2.

3. Compute the threshold value for each subband, except theLL2 band using equation (13) , after finding out its following terms.

(i) Obtain the noise variance using the equation (14).

(ii) Find the term C for each subband using the relation given in equation (15).

(iii) Calculate the term |AM-GM| using the equations (16) and (17).

5. Threshold the all subband coefficients (except LL2) using Soft Thresholding technique given in equation (12), by substituting the threshold value obtained in the step 3.

6. Perform the inverse DWT to reconstruct the degraded image.

8. Experimental results and discussion

A test image Zoza of size 256x 256 is blurred by 4x4 average low pass filter have a coefficients values of 1/16 , then the proposed algorithm has been applied on different added Gaussian noise of levels: (Standard Deviation) $\sigma = 1$ to 100 . We used 'Daubechies' (Db4)[13] at two levels of decomposition. To evaluate the performance of the proposed method, it is compared with inverse filter , and Wiener filter alone without denoising step using Peak Signal to Noise Ratio (PSNR) [4], which is defined as :

$$PSNR = 10\log_{10}\left[\frac{(d(x,y)_{max} - d(x,y)_{min})^2}{MSE}\right] \qquad \dots (18)$$

Where, MSE denotes the Mean Square Error between the original and the restored images, and is given as:

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f(i,j)-\tilde{f}(i,j))^2 \quad \ldots (19)$$

Where,
M - Width of image
N - Height of Image
$\tilde{f}$ (i,j) - Restored Image or processed Image
f (i,j) - Original Image

Figure(6) shows the PSNR against white Gaussian noise standard deviation for three filters , inverse filter , Wiener filter alone and our proposed method the involve additional denoising method after Wiener filter. It is obvious from fig.(6) that our proposed method outperforms the inverse and the Wiener filters . It removes noise significantly and hence restore the image better than traditional Wiener filter.

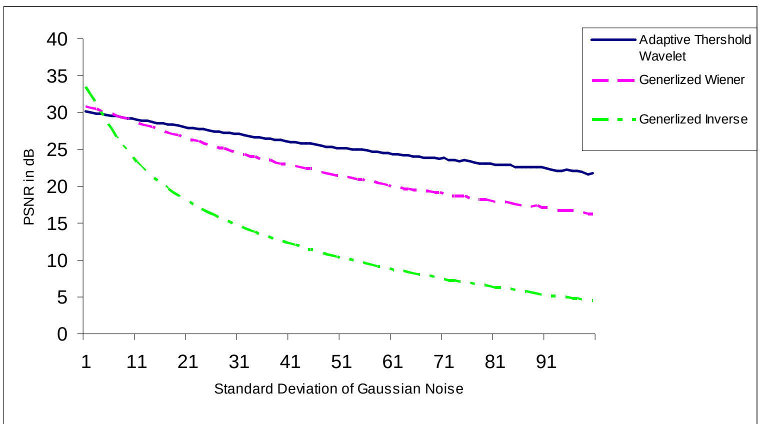


Fig.(6) Comparison Chart of PSNR for Different Methods

Fig. (7) shows the original Zoza image , the degraded image for sigma = 25 , 50 and 75 and resulting images of inverse filter , wiener filter, and proposed method of Zoza image. It is clear that the proposed method outperforms the other methods in visual perception especially in high noise level.

(a) Zoza Image

(b) Degraded image
with sigma = 25

(c) Resorted image
using inverse filtering

(d) Resorted image
using wiener filtering

(e) Resorted image using
our proposed method

(f) Degraded image
with sigma = 50

(g) Resorted image
using inverse filtering

(h) Resorted image
using wiener filtering

(i) Resorted image using
our proposed method

(j) Degraded image
with sigma = 75

(k) Resorted image
using inverse filtering

(l) Resorted image
using wiener filtering

(m) Resorted image
using our proposed
method

Fig.(7) Restoration of 'Zoza' image blurred by 4x4 average filter and corrupted by Gaussian Noise of Standard Deviation of 25 , 50 and 75 using different methods

9. Conclusion

A method for image restoration have been proposed here. The proposed method added to Wiener filter a denoising step to remove the amplified noise. Wavelet-based denoising scheme with adaptive threshold is the denoising step that used in our work.

Experiments are conducted on image corrupted by various noise levels to access the performance of thresholding method in comparison with filters like Wiener and inverse filters. The obtained experimental results proved that our proposed method better than inverse and Wiener filters because it is removed noise significantly and restored the image better than the other methods , and this is very clear in visual perception of results especially in high noise level . From the results , its clear that our proposed restoration technique has possessed better PSNR than inverse and than traditional Wiener filter and this is because of the additional noise removal step, and because this denoising step is more subband adaptive and is based on the analysis of statistical parameters like arithmetic mean, geometrical mean and standard deviation of the subband coefficients, so at higher values of sigma our proposed method overcame the other two methods. The method also overcame the Wiener filtering in the case when the blurring filter is singular where Wiener filtering actually amplified the noise.On the other hand, and at high values of sigma, some artifacts have been added to the restored image due to wavelet denoising, and this can be left for future work to suggest a suitable additional filter or method to remove these artifacts.

Finally, and from applications point of view, since the proposed method proved to have better PSNR than other methods , it can be used in restoration applications where the images are degraded during transmission , which adds random noise normally in nature.

References:-

[1] Rafael C.Gonzalez & Richard E.Wodds, “Digital Image Processing”, Prentice-Hall, Inc., 2002.

[2] Scott E Umbang., "Computer Vision and Image Processing", A Practical Approach Using CVIP Tools, Prentice Hall, Inc., Upper Saddle River, 1998.

[3] Lakhwinder Kaur , Savita Gupta , R.C. Chauhan , "Image Denoising using Wavelet Thresholding", Dept. of CSE SLIET, Longowal Punjab (148106), India, E-mail: mahal2k8@yahoo.com, savita2k8,@yahoo.com, rcchauhan@hotmail.com

[4] D.Gnanadurai, and V.Sadasivam, “An Efficient Adaptive Thresholding Technique for Wavelet Based Image Denoising", International Journal of Signal Processing Volume 2 Number 2 2005 ISSN 1304-4494.

[5] Zeljko Devcic Sven Loncaric “Blur Identification Using Averaged Spectra of Degraded Image Singular Vectors “, Institute for Defence Studies, Research and Development, University of Zagreb, Bijenicka 46, 10000 Zagreb, Croatia,
E-mail: zdevcic@zvonimir.morh.tel.hr

[6] Byung-Jun Yoon and P. P. Vaidyanathan, " Wavelet-Based Denoising by Customized Thresholding", Dept. of Electrical Engineering California Institute of

Technology, Pasadena, CA 91125, USA,
E-mail:
bjyoon@caltech.edu,
ppvnath@systems.caltech.edu

[7] Xiaoyan Xu, “Image Denoising in Wavelet Domain”, School of Engineering, University of Guelph,
E-mail: xux@uoguelph.

[8] Carl Taswell , " Experiments in Wavelet Shrinkage Denoising " C. Taswell (ctaswell@toolsmiths.com) , Computational Toolsmiths , POB 18925, Stanford, CA 94309-8925.

[9] Yi Wan and Robert D. Nowak ," Wavelet-Based Statistical Model For Image Restoration " , Rice University , Department of Electrical Engineering, 6100 South Main Street, Houston, TX 77005, USA.

[10] Javier Portilla and Eero P. Simoncelli ," Image Denoising Via Adjustment of Wavelete Coefficient Magnitude Correlation", Center for Neural Science, and Courant Institute of Mathematical Sciences New York University, NY 10003,
E-mail: fjavier,eerog@cns.nyu.edu.

[11] Jae S**.** Lim, " Image Restoration by Short Space Spectral Subtraction”, IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-28, No. 2, April, 1980.

[12] A. Buades, B. Coll, And J. M. Morel, “A review of Image Denoising Algorithms with a New One”, Multiscale Model. Simul. 2005 Society for Industrial and Applied Mathematics, Vol. 4, No. 2, pp. 490–530.

[13] MATLAB Help, Version 7.

استعادة الصور باستخدام الترشيح المنظم المعكوس و إزالة الضوضاء بطريقة التحويل المويجي ذو العتبة العتبة المتكيفة

فراس علي جواد
قسم هندسة الميكاترونيكس / كلية هندسة الميكاترونيكس
جامعة بغداد

الخلاصة :

يعتبر مرشح وينر كحل مثالي بين الترشيح المعكوس وأزاله الضوضاء لكن في حالة كون مرشح التشويه منفرد سيؤدي مرشح الوينر الى تضخيم الضوضاء لذلك يجب إضافة خطوة بعد الوينر لإزالة الضوضاء . سيكون نظام ازالة الضوضاء المبني على تحويلات المويجة مفضلا" لهذا الغرض.

في هذا البحث تم اقتراح نظام استعادة الصورة يتضمن خطوتين منفصلتين : معكوس الترشيح في مجال فورير وازالة الضوضاء باستخدام التحويل المويجي . المرحلة الاولى هي ترشيح وينر للصورة الداخلة , الصورة المرشحة يتم ادخالها الى مرحلة ازالة الضوضاء بطريقة التحويل المويجي ذات العتبة المتكيفة . اختيار تخمين العتبة يتم عن طريق تحليل العوامل الاحصائية لمعاملات نصف الحزمة المويجية والتي هي الانحراف المعياري , المعدل الحسابي والمعدل الهندسي . الصورة المشوشة تجزأ الى مستويات عدة للحصول على عدة حزم ترددية. ثم يتم استخدام طريقة العتبة الناعمة لازالة الضوضاء عن طريق تثبيت قيمة العتبة المثلى بهذه الطريقة .

النتائج التجريبية على صورة اختبار بأستخدام هذه الطريقة تبين انه هذه الطريقة تؤدي الى الحصول على نوعية صور افضل مقارنة مع مرشحات وينر ومرشح المعكوس وايضا" تؤدي الى الحصول على قمة نسبة اشارة الى ضوضاء افضل من باقي الطرق .