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Flutter Speed Limits of Cantilever Rectangular and Tapered Plates

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Abstract:

The aerodynamic and elastic forces may cause an oscillation of the structure such as the high frequency of the airfoil surfaces and the dynamic instability occurring in an aircraft in flight and failure may occur at a speed called flutter speed. In this work, analytical and numerical investigations of flutter limits of thin plates have been carried out. The flutter speed of rectangular plates were obtained and compared with some published results. Different design parameters were investigated such as aspect ratio, thickness and their effects on flutter velocity. It was found that the structural mode shape plays an important role in the determination of the flutter speed and the coupling between the bending and torsional mode is the main cause of flutter.

Key words:Flutter, Speed, Plates.

Introduction:

In mechanics, there are three types of forces aerodynamic, elastic and inertia. The interaction between aeroelasticity and elastic forces is called static aerodynamic such as the divergence problem and the interaction between elastic, aerodynamic and the inertiaforces called the dynamic aero elasticity and the flutter is an example for this phenomena in which the amplitude of the oscillations may diverge causing failure. Therefore, this research is concerned with the determination of flutter speed for thin plates which are found in many aeronoutical structures.

R.S Srinivasan and B.J. Baba [2] studied the flutter analysis of cantilevered quadrilateral plates and the problem is solved by using a numerical method involving an integral equation based upon finding the strain and kinetic energies of the plate.

E. Nissim and I. Lottat [3] suggested an optimization method for the determination of the important flutter modes. The method is based on the minimization of the quadratic values derived from the equation of motion.

R.S. Srinivasan and B.J. Baba [4] described the free vibration and flutter of laminated quadrilateral plates. They derived the differential equation required to obtain the flutter speed of simple rectangular plates.

M.W. Kehoe [5] reviewed the test techniques developed over the last several decades for flight flutter testing of aircraft practical experiences and example test programs were presented to test the effectiveness of the various approaches used.

Brain Danowsky [6] studied the development of an aircraft structural variation model, accounting for variations in structural mode shape as well as structural frequency, which has many advantages for the design of aircraft and aircraft flight control systems.

J.Hoas, etal [7] described the flutter of circulation control wings and explained the flutter stability of high aspect ratio circulation control wing using lumped approach on conjunction with a modified unsteady aerodynamic strip analysis method.

J.E. Sedaghat et al [8] developed to predict the speed and frequency at which flutter occur based upon the use of symbolic programming. The approach performs the computation in a single step and does not require the repeated calculations at various speeds required when using the classical V.G. method.

T.S Talib [9] determined analytically using the damping method and calculated the natural frequencies and mode shapes using the finite element technique for subsonic wing structure. He showed that the flutter speed change with changing the skin thickness, material properties and the altitude. His calculations indicate that structural mode shape variation plays a significant role in the determination of wing flutter limits.

In this work, the V.G. method together with developed finite element package will be adopted and the flutter speed will be defined for thin plates with different aspect ratios and thickness for both rectangular and tapered plates.

Theory:

To determine the equation of motion, consider the section shown in Fig.1 with two degrees of freedom α and h. [9]

The downward displacement of any other point on the airfoil is: [8]

$Z=h\pm x\alpha$

The strain energy is composed of two components linear and rotational parts

$$U=\frac{1}{2}K_a a^2+\frac{1}{2}K_h.h^2 \quad \ldots\ldots(1)$$

And the kinetic energy is:

$$T=\frac{1}{2}[h^2\int rdx+2ha\int rxdx+a^2\int rx^2dx] \quad \ldots(2)$$

Using:

$$I_a=\int rx^2dx=mr_a^2$$

$$S_a=\int rxdx=mr_a$$

Therefore,

$$T=\frac{1}{2}mh^2+mx_a ha+\frac{1}{2}I_a.a^2$$

Using Lagrange equation in the form:

$$\frac{d}{dt}=[\frac{\partial(T-U)}{\partial \dot{q}}]-\frac{\partial(T-U)}{\partial q}=Q_q \quad \ldots\ldots (3)$$

Where q=q (h,α)

Inserting equations (1) and (2) into (3) yield the following characteristic equation,

$$\begin{bmatrix}1 & x_a\\ x_a & r_a\end{bmatrix}\begin{Bmatrix}\frac{\ddot{h}}{b}\\ \ddot{a}\end{Bmatrix}+\begin{bmatrix}W_h^2 & 0\\ 0 & W_a^2 r_a^2\end{bmatrix}\begin{Bmatrix}h/b\\ a\end{Bmatrix}=\begin{Bmatrix}F/mb\\ M/mb^2\end{Bmatrix} \quad \ldots (4)$$

The solution may be assumed as follows:

$h=h_0e^{iwt}$

And

$$a=a_0e^{iwt} \quad \ldots\ldots(5)$$

equation (4) is reduced to

$$w^2\begin{bmatrix}1 & x_a\\ x_a & x_a^2\end{bmatrix}\begin{Bmatrix}h/b\\ a\end{Bmatrix}+\begin{bmatrix}w_h^2 & 0\\ 0 & w_a^2 r_a^2\end{bmatrix}\begin{Bmatrix}h/b\\ a\end{Bmatrix}=\begin{Bmatrix}F/mb\\ M/mb^2\end{Bmatrix} \quad \ldots (6)$$

The aerodynamic force and moment are given by [9]:

$$F = prb^3w^2[\frac{h}{b}L_n + a\,(L_a - (\frac{1}{2}+a)L_n)]$$

Where

$$L_h = 1 - i2c\frac{1}{K}$$

$$L_a = \frac{1}{2} - i\frac{1+2C}{K} - \frac{2C}{K^2}$$

$$M = prb^4w^2[\left\{M_h - (\frac{1}{2}+a)L_h\right\}\frac{h}{b}$$

$$+\{M_a - (\frac{1}{2}+a)(L_a + M_h) + (\frac{1}{2+a})^2 L_h\}]$$

$$M_h = \frac{1}{2}, M_a = \frac{3}{8} - i\frac{1}{k}$$

Then the equation of motion becomes:

$$L_h = -w^2\begin{bmatrix}1 & x_a\\ x_a & r_a^2\end{bmatrix}\begin{Bmatrix}h/b\\ a\end{Bmatrix}$$

$$+\begin{bmatrix}w_h^2 & 0\\ 0 & w_a^2 r_a^2\end{bmatrix}\begin{Bmatrix}h/b\\ a\end{Bmatrix}$$

Simplifying and using

$h^2 = \frac{w^2}{w_a^2}$ and $R^2 = \frac{w_h^2}{w_a^2}$

with $m = \frac{m}{prb^2}$ gives:

$$L_h = \frac{\Omega^2}{m}\begin{bmatrix}L_h & L_a - (\frac{1}{2}+a)L_h\\ M_h - (\frac{1}{2}+a) & M_a - (\frac{1}{2}+a)(L_a + M_h) + (\frac{1}{2}+a)^2 L_h\end{bmatrix}\begin{Bmatrix}h/b\\ a\end{Bmatrix} \quad \ldots (7)$$

which can be expressed as a flutter equation in the form:

$$[K_{ij}]\begin{Bmatrix}h/b\\ a\end{Bmatrix} = \Omega^2[A_{ij} + M_{ij}]\begin{Bmatrix}h/b\\ a\end{Bmatrix}$$

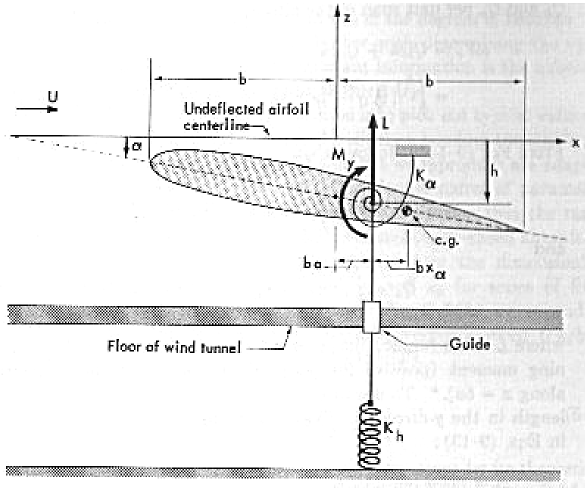

Fig. 1 Airfoil restraind from bending and torsional motion in an airstreem by spring K_h and K_α, acting a distance ba aft of midehord. Also shown are lift L and pithing moment M_y about the axis twist.[9]

Where

$[K_{ij}]$ is the stiffness matrix

$[M_{ij}]$ is the mass matrix and

$[A_{ij}]$ is the aerodynamic matrix

Finite Element Method

The plate is analyzed by using a suitable element. The element have 8 degrees of freedom with 5 degrees of freedom of each node (u, v, w, θ_x and θ_y in the x, y and z directions) and the rotations about x and y axes respectively.

The generic displacements at any point of the middle surface are [10]:

$\{d\} = \{u, v, w\}$

$$\text{and } u = Zq_y = 2\sum_{i=1}^{9} N_i q_{yi} \quad \ldots (8)$$

$$v = -Zq_x = -2\sum_{i=1}^{9} N_i q_{xi} \quad \ldots (9)$$

$$w = \sum_{i=1}^{9} N_i w_i \quad \ldots (10)$$

Strain–Displacement Relationship :

The strain matrix $\{\in\}$ is given by :

$$\{\in\}=\begin{Bmatrix}\in_x\\ \in_y\\ g_{xy}\\ g_{yz}\\ g_{zx}\end{Bmatrix}=\begin{bmatrix}u_{,x}\\ v_{,y}\\ u_{,y}+v_{,x}\\ v_{,z}+w_{,y}\\ w_{,x}+u_{,z}\end{bmatrix} \quad ....(11)$$

$$= [B]\{d\}$$

And the strain displacement matrix [B] may be written as:

$$B_i\begin{bmatrix}0 & 0 & z\frac{h}{2}a_i\\ 0 & -z\frac{h}{2}b_i & 0\\ 0 & -z\frac{h}{2}a_i & z\frac{h}{2}b_i\\ b_i & N_i & 0\\ a_i & 0 & N_i\end{bmatrix}(i=1,2,...9) \quad ....(12)$$

We can isolate terms in b_i that multiply by $z\frac{h}{2}$, then

$$B_i = B_{Ai} + z\,\frac{h}{2}B_{Bi}......$$

$$B_{Ai}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\b_i&-N_i&0\\a_i&0&-N_i\end{bmatrix}, B_{Bi}=\begin{bmatrix}0&0&0\\0&-b_i&0\\0&-a_i&-b_i\\0&0&0\\0&0&-N_i\end{bmatrix}$$

$$a_i = J*11N,x + *12N,h$$

$$b_i = J*12N,x + *22N,h$$

And J* is Jacobian inverse matrix.

$$[D]=\begin{bmatrix}X,_x & Y,_h & 0\\ X,_x & Y,_h & 0\\ 0 & 0 & Z,_x\end{bmatrix} \quad ....(13)$$

Formulation of Stiffness Matrix :

The element stiffness matrix $[K]^e$ for the element, using B matrix in eq. (12) as follows :

$$[K]^e = \int\int\int [B]^T[D][B]|J|dxdhdz \quad ......(14)$$

Taking integration through thickness; eq . (14) becomes

$$[K]^e = \int\int 2[B_A]^T[D][B_A] + \frac{h^2}{6}[B_B]^T[D][B_B]|J|dxdh$$

Where {D} is the stress-strain matrix for isotropic material

$$[D]=\begin{bmatrix}D_{bending} & 0\\ 0 & D_{shear}\end{bmatrix} \quad ....(15)$$

$$D_{bending}\,\frac{Eh^3}{12(1-v^2)}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix};$$

$$D_{shear} = \frac{Eh}{2(1.2)(1+v)}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

The first part of eq (15) is due to transverse shearing deformation where as the second part is associated with flexural deformation.

Formulation of Mass Matrix :

The formulation of consistent mass matrix for element QPB9 becomes

$$[M]^e = r\int\int (2N_A^T N_A + \frac{h^2}{6}N_B{}^T N_B)|J|dxdh$$

(17)

Where

$$N_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} Ni, N_B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} Ni$$

The first part of eq. (17) consists of translation inertias and the second part gives rotational (or rotary) inertias.

Formulation of Aerodynamic Matrix

The Formulation of aerodynamic Matrix for element QPB9 becomes.

$$[A]^e = \int\int\int [N]^T [\partial N][\partial x]|J| dxdhdz$$

$$[A]^e = \int\int\int ([N_A] + x\frac{h}{2}(N_B)^T)(\partial N_A / \partial x + x\frac{h}{2}[\partial N_B / \partial h])|J| dxdhdz \quad \text{……….(18)}$$

For simplicity it is assumed that the air flowing above the panles is parallel to the X-axis as shown in fig. 2 and the effect of any air entrapped below may be neglected . By integrating Eq . (18) through the thickness produces.

$$[A]^e = \int\int (2[N_A]^T [\partial N_A / \partial x] + \frac{h^2}{2}[N_B]^T [\partial N_B / \partial h] |J| dxdh \quad \text{………..(19)}$$

Where

$$Aij \neq Aji$$

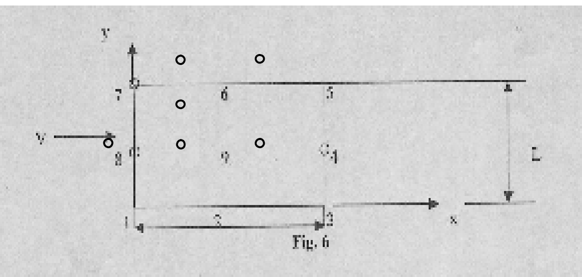

Fig.2 panal under airfolow.

Flutter Equation and Solution Method :

The flutter equation can be derived as:

$$-K[M] + [K] - l[A])\{Q\} = 0 \quad \text{…………….(20)}$$

Where K, M and A are matrices of size (m*m) degrees of freedom. Here

l Non dimentional dynamic parameter and it is equal to

$$l = \frac{1}{2} r_o V^2 \frac{b_r^3}{D} \quad \text{……………………(21)}$$

Where p_a is air density, V is the free stream velocity, b_r is the half length of wing at root, D is the flexural rigidity of the plate and, k^2 is the Eigen value and it is equal to [8]

$$k^2 = w^2 br^4 (r_s t_s / D) \quad \text{…………..…(22)}$$

Where w is the natural frequency, ρ_s is the material density and t_s is the plate thickness. Equation (20) is a standard eigen value problem. The parameters and k are non-dimensional quantities. Note that for zero flow velocity and k then represent the square of the non-dimensional natural frequency, so that in this case the eigenvalues are real. As the flow velocity increases from zero, two eigenvalues will usually approach each other and coalesce (become complex conjugates) to k_{cr} at a value of 2 which is the critical value of dynamic pressure. For a large system of equation, it is a relatively difficult task to find the eigen values of equation (12), since in general the matrices involved are complex and asymmetric. Therefore, the problem is separated into two parts. First, set zero flow veiocity (w=0) in equation (12) and determine a finite number, say n,of normal modes of the structure ,where n<m and m is the order of the matrix K, M. At this point, the finite element method is used to calculate the modes as accurately as desired by selecting the proper mesh size, and a suitable

eigen value technique. Use the subspace technique to find the first five modes. Second, after finding the eigen value and eigen vectors then use the normal mode method to represent the eigen vectors (mode shapes) in normal coordinates [4].

Results and Discussions

The results from the finite element program together with the relations between the frequency of the plate and the eigen value at the point of intersection between bending and twisting modes which represents the critical case from which the flutter speed is obtained according to the analytical solution presented in this work.

Validity of the work

The plate shown in Fig. 2 is solved by using different F.E meshes
(2x2) 128 d.o.f , (3x3) 264 d.o.f , (4x4) 550 d.o.f , (5x5) 488 d.o.f
The geometry and material properties are as follows:
Modulus of elasticity E = 40 Gpa
Poisson's ratio υ =0.25
Density of the material = 1500 kg/ m^3
Thickness t = 16 mm
Width b = 0.5m
Air density = 0.46 kg / m^3
It is found that the first fundamental frequency was 1.683 rad/S. The convergence study is shown in Fig. 3.
Therefore, this mesh is used to analyze the flutter speed. To obtain the flutter speed, two cases are investigated

1) Aspect ratio AR = $\frac{L}{2br}$

Where L= 4m , br = 0.5m

$$AR = \frac{4}{2x0.5} = 4$$

2) AR= 1.5

The results of the finite element program together with results of the used analytical method, are shown in table 1.

Table 1: F.E Results – Natural Frequencies of the plate with aspect ratio=1.5

Mode	Freq. (Hz)		Freq. (Hz)	
	FEM	Analytical	FEM	Analytical
1	1.68	1.75	12.03	13.3
2	10.6	11.01	14.48	15.2
3	13.85	14.5	75.4	76.02
4	32.12	33.05	140.6	142.31
5	43.12	44.30	189.25	190.25
	AR = 4		AR = 1.5	

The eigan value l = 252
From equn (21) and by using V_f =33.2 m/s
the flutter speed becomes:
w_f = 5.7 rad/s

Compared to the results obtained from [3], in which ω_f =6.8 rad/s and V_f = 34 m/s, therefore, the percentage of discrepancy will be 2.35% which gives a confidence in using the developed finite element program and analytical investigations.

Tapered plate

The Tapered plate shown in Fig.4 with the following geometry and material properties.

Steel of modulus of elasticity E= 207 GPa poisson's ratio ν= 0.3

Case 1

Aspect ratio =

$$\text{AR} = \frac{LZ}{A} = \frac{(0.18)^2}{0.027} = 1.2$$

And the Air properties are:

Density = 1.2256 Kg/m^3 at sea leves with sonic speed = 340m/s at 20^{o}C, therefore a = 20.04$\sqrt{T}$

$$l_{\text{critical}} = \frac{1}{2} r_a v^2$$

With Mach no. $= \frac{v}{a}$

The finite element mesh used for the previous problem was employed her. Different thicknesses was used and the corresponding modes were inserted in table 2, as obtained from the finite element program. The corresponding l_{cr} were obtained from the corresponding mode shapes for each case for both , bending and torsional modes and their point of coupling defines l_{cr} from which the flutter speed and the flutter frequency.

Case 2

l = 1.671.9, ω_f =28.628 rad/s and the flutter speed V_f =108 rad/s are obtained.

Four thicknesses were tried 0.5, 1.0, 1.5 and 2 mm as shown in figures 4,5, 6 and 7.

It is seen that as the thicknesses is increased l or is decreased and the flutter velocity is decreased and the flutter frequency is decreased. This is again due to the increasing in the flexibility in reducing the thickness and decreasing the stiffness when the thickness is increased, the stiffness and the mass, both are increased and the flutter speed is decreased. Therefore the limits of the flutter speed depend upon material properties and geometry.

It can be concluded that increasing the aspect ratio, the flutter speed is increased. This is because the flexibility of the plate is increased when the aspect ratio is increased.

Table 2: F.E Results of the Tapered plate

Thickness (mm)	Modes	l_{cr}	Flutter velocity (m/s) FEM	V-G Method	Discrc-pancy %	Mach No.	Flutter frequency (Hz) FEM	V-G Method	Discrc-pancy %
0.5	14.2 42.8 84.8 137.00	27382.4	668.5	690.6	3.200	1.966	28.2	30.5	7.541
1.0	28.4 85.5 169.7 274.9	135851	472.6	488	3.155	1.39	56.4	58.2	3.092
1.5	42.62 128.24 254.42 410.6	91167.7	385.7	405.6	4.906	1.13	84.2	88.4	4.751
2.0	56.8 170.92 399.1 547.0	68306.4	333.87	345.6	3.394	0.98	112.8	115.3	2.168
		Case 1				Case 2			

Conclusions

1. For flat plates, the flutter speed is decreased, when the thickness is increased. This is because the stiffness is increased and weight penalty is introduced and a compromise between the required flutter speed limitation and the thickness of the plate.

2. For tapered plate similar conclusion is obtained i.e about 50% flutter speed reduction when using thickness = 2mm instead of using sheet thickness = 0.5 mm
3. The interaction between the finite element and the flutter characteristic is a powerful technique to predict the flutter of the plates.

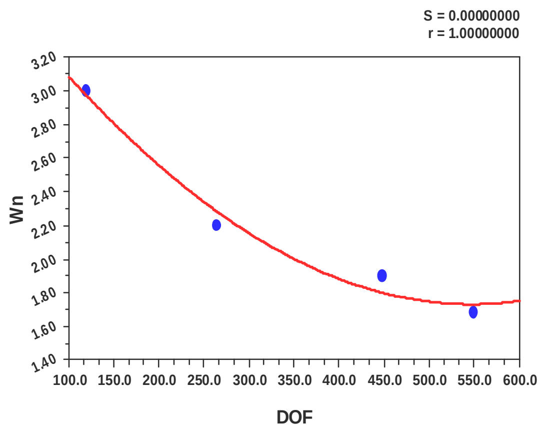

Fig.3 convergences study of the finite element results
Sinusoidal Fit: W_n=a+b X cos(cx+d)

Coefficient Data:
a = 9.2770748
b = 7.5451525
c = 0.0013603357
d = 2.397878

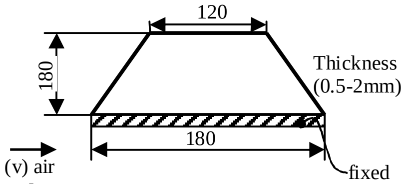

Fig.4 Tapered cantilever plate.

Material: steel, ρ=7800 kg/m^3, E=210GPa, ν=0.3, AR=180/180=1

Air property: ρ_a=1.2256 kg/m^3, sonic speed=340m/s at 20 ^{o}C, λ_{cr} =1/2 ρ_a.v^2

The results shown in table 2 have an acceptable agreement between the finite element results and the V-C Method, with maximum discrepancy Increasing thickness from 0.5mm to 2mm gives a reduction in the flutter speed. This is because that increasing the thicknesses given as increasing in the stiffness which decreases the flutter speed. The results are shown in Fig. 9.

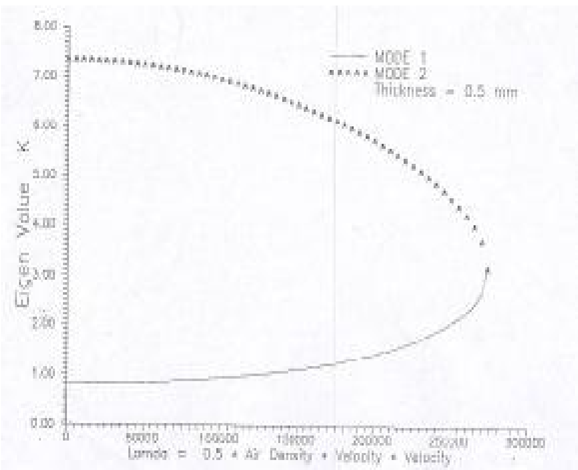

Fig.5 The bending and torsional modes for the cantilever tapered plate with thickness 0.5mm.

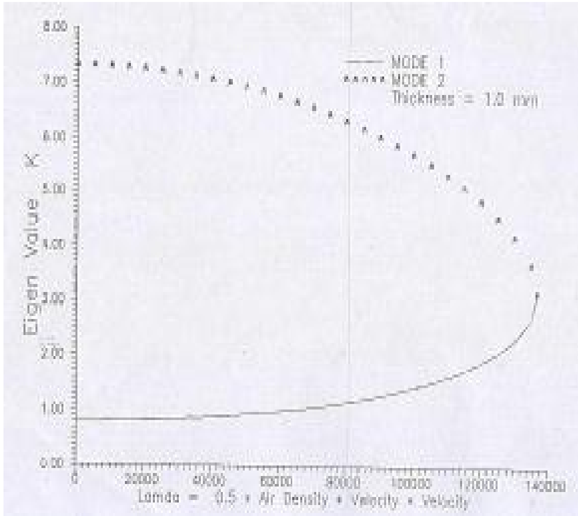

Fig.6 The bending and torsional modes for the cantilever tapered plate with thickness 1mm.

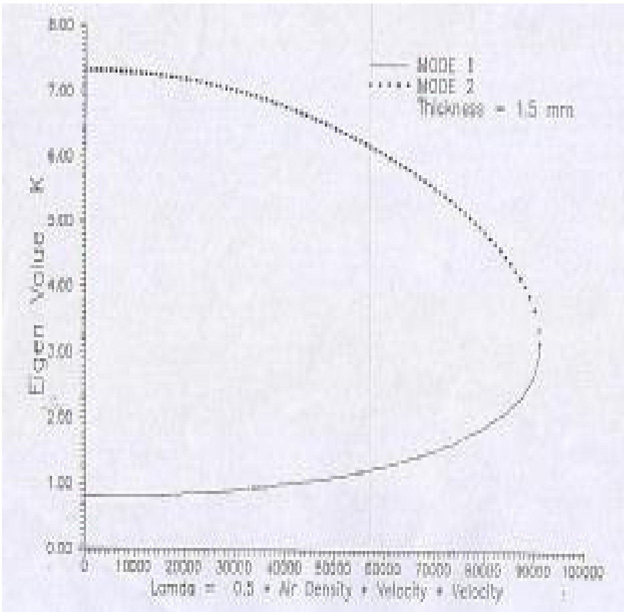

Fig.7 The bending and torsional modes for the cantilever tapered plate with thickness 1.5mm.

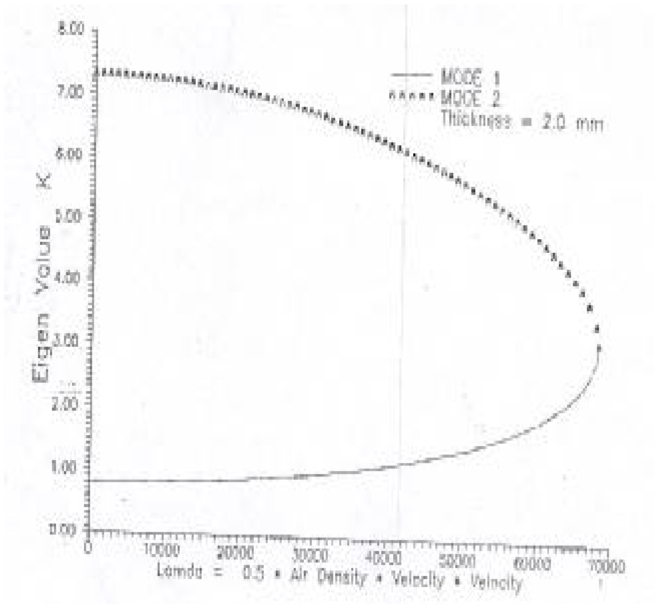

Fig.8 The bending and torsional modes for the cantilever tapered plate with thickness 2mm.

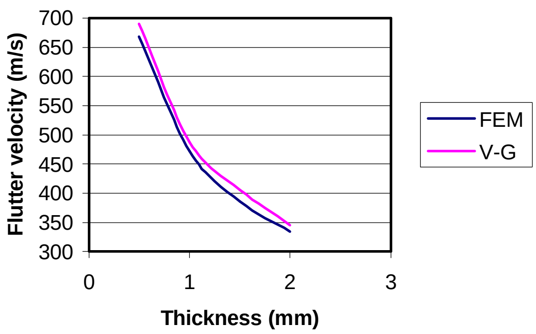

Fig. 9

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Nomenclature

Symbol	Meaning	Units
F	Total force	N
h	The vertical coordinate of the axis of rotation	M
	Mass of wing	
m	First moment of inertia	Kg
Sα	Kinetic energy	Kg.m
T	Strain energy	N.m
U	Up-wash velocity of the airfoil	N.m
Wα	Stiffness matrix,	m/s
[Kij]	Mass matrix and	
[Mij]	Aerodynamic matrix	
[Aij]	Second moment of inertia about shear center.	
Iα	Stiffness for translation spring	kg.m2
	Stiffness for tensional spring	
Kh	Total moment	N/m
Kα	The distance measured from the shear center	N/m
M	Downward displacement of the airfoil	N.m
x	Angular displacement for wing	m
	Air density	
Z		m
α		rad
ρ		kg/m3

حدود سرعة الرفرفة للصفائح المسلوبة و المستطيلة الناتئـة

قصي حاتم جبر

هيئة التعليم التقني / الكلية التقنية-بغداد

الخلاصة:

يمكن أن تسبب القوى الإيروديناميكية والمرنـة تذبذبا للهيكل حيث تولد تردد عالي لأسطح المطيار إضافة الى اللاإستقرارية التي تحدث في الطائرة أثناء الطيران والتي يمكن أن تسبب إنهياراً بسرعة تسمى بسرعة الرفرفة. في هذا البحث تم إيجاد حدود سرعـة الرفرفة للصفائح المستطيلة وتم مقارنتها مع نتائج منشورة. تم بحث عدة عناصر تصميمية مثل النسبة الباعية والسمك وتأثيراتها على سرعة الرفرفة. وجد بأن النسق الهيكلي يلعب دورا مهما في إيجاد سرعة الرفرفة والتقارن بين نسق الإنحناء والإلتواء السبب الرئيس في الرفرفة.