Al-Khwarizmi Engineering Journal, Vol. 17, No. 3, Sptember, (2021)
P. P. 22- 28

Al-Khwarizmi Engineering Journal

Robust Computed Torque Control for Uncertain Robotic Manipulators

Maryam S. Ahmed* **Ali Hussien Mary****
Hisham H. Jasim***
*,**,****Department of Mechatronics/ Al-Khwarizmi College of Engineering/ University of Baghdad*
*Email: Maryamsadeq97@gmail.com
**Email: Alimary76@kecbu.uobaghdad.edu.iq
***Email: mschisham@gmail.com

(Received 27 July 2021; Revised 20 August 2021; Accepted 1 September 2021)
https://doi.org/10.22153/kej.2021.09.002

ABSTRACT

This paper presents a robust control method for the trajectory control of the robotic manipulator. The standard Computed Torque Control (CTC) is an important method in the robotic control systems but its not robust to system uncertainty and external disturbance. The proposed method overcome the system uncertainty and external disturbance problems. In this paper, a robustification term has been added to the standard CTC. The stability of the proposed control method is approved by the Lyapunov stability theorem. The performance of the presented controller is tested by MATLAB-Simulink environment and is compared with different control methods to illustrate its robustness and performance.

Keywords: *Computed Torque Control, Linear Matrix Inequality, Robotic Control, Robotic Manipulato.*

1. Introduction

Today, the robotics systems enter into many applications such as, industrial applications, manufacturing, medical fields, and space application [1] . Precise positioning is an important desired feature of the robot manipulator. The robot manipulator can be considered as a nonlinear system that suffering from high nonlinearity and external disturbance. The main problem is how to control the robot taking into account the system uncertainty and external disturbance. However, in recent years, significant and rapid progress had been made in the field of control, to solve the control problem. Different control strategies have been proposed. PID control proposed by many researchers because Due to its simplicity and ease of implementation. Abhishek and Dayal presented PID controller optimized by Particle Swarm Optimization (PSO) to stabilize the gait humanoid robot [2]. Ignacio proposed design an adaptive PID controller based reinforcement learning [3]. Structured and unstructured uncertainties and external interference, it is difficult to obtain an accurate dynamics model of the robotic arm. Therefore, many control schemes for unknown dynamic models of robot manipulators have been proposed. A robust control strategy is an important method that can handle the system uncertainties and disturbances. Sliding mode control (SMC) is an effect robust control method that applied successfully in control different robotic manipulators.in [4], SMC is combined with fuzzy logic technique to control robot manipulator in task space. Mary and Kara presented robust control method for control 2 links robotic arm by using Linear Matrix Inequality (LMI) to tune the control gains [5]. In (Wu and Huang , 2021), fraction fractional calculus utilized to design hybrid controller with the sliding mode control for trajectory tracking of mobile robot manipulator. In (Chen et al. , 2019), radial basis function neural network used with SMC for control n links robot arm taking into account the actuator dynamic. However, the Chattering is the important drawback in SMC [8,9]. CTC is an efficient control used in robotic control problem. CTC method require knowing the dynamic of robotic arm and in practice, determine the accurate dynamic of robotic arm may be difficult [10,11,12]. Thus, this paper proposed a new method that can improve the robustness of the standard CTC against system uncertainty and external disturbance.

2. Dynamic Model of the Robotic Manipulator

The dynamic model of the robotic arm system can be written as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = \tau \quad \ldots(1)$$

where $q, \dot{q}, \ddot{q} \in R^n$ refer respectively to the angular position, velocity, and acceleration vectors of robotic manipulator joints, $M(q) \in R^{nxn}$ refers to the inertia matrix, $C(q,\dot{q}) \in R^n$ denotes a centripetal and Coriolis vector force, $F(\dot{q}) \in R^n$ is the friction torque vector, $G(q) \in R^n$ represents loading of the gravity, $\tau_d \in R^n$ is the external disturbance, and $\tau \in R^n$ refers to the joints torque vector. In particular applications, getting the accurate dynamic of the robotic manipulator are not easy due to model uncertainties and external disturbance. Therefore, the model uncertainty can be included in the dynamic model as follows:

$$M(q) = M_o(q) + \Delta M(q) \quad \ldots(2)$$

$$C(q,\dot{q}) = C_o(q,\dot{q}) + \Delta C(q,\dot{q}) \quad \ldots(3)$$

$$F(\dot{q}) = F_o(\dot{q}) + \Delta F(\dot{q}) \quad \ldots(4)$$

$$G(q) = G_o(q) + \Delta G(q) \quad \ldots(5)$$

where $M_o(q)$, $C_o(q,\dot{q})$, $F_o(\dot{q})$, and $G_o(q)$ represent the nominal model of the robotic system and it can be known, whereas $\Delta M(q), \Delta C(q,\dot{q})$, $\Delta F(\dot{q})$ and $\Delta G(q)$ denote the uncertainty part of the robotic manipulator dynamic model and it cannot be determined exactly. In this paper, we assume the following:

Assumption 1. The desired trajectories q_d with their first and second derivatives $\dot{q}_d$ and $\ddot{q}_d$ are continuous and bounded as follows:

$$|q_d(t)| \le M_{d1}, |\dot{q}_d(t)| \le M_{d2}, |\ddot{q}_d(t)| \le M_{d3}, \quad \ldots(6)$$

with M_{d1} , M_{d2} , and M_{d3} being positive constants[5].

3. Computed Torque Control

Computed torque control is an important control method that applied successfully in robotic system control when there is no model uncertainty Then, the dynamic model in (1) becomes

$$M_o(q)\ddot{q} + C_o(q,\dot{q})\dot{q} + F_o(\dot{q}) + G_o(q) = \tau \quad \ldots(7)$$

The standard CTC is

$$\tau = M_o(q)\big(\ddot{q}_d - k_p\, e - k_v \dot{e}\big) + C_o(q,\dot{q})\dot{q} + F_o(\dot{q}) + G_o(q) \quad \ldots(8)$$

$$e = q - q_d \quad \ldots(9)$$

where e is the tracking error, k_p and k_v are the proportional and derivative control gain matrices. By substituting (8) in (7),

$$\ddot{q}_d + k_p\, e + k_v \dot{e} = 0 \quad \ldots(10)$$

It is obvious that the roots of (10) will lie on the left half plane if the control gain matrices k_p and k_v are positive, which implies that the actual trajectory can track desired trajectory and error signal will converge to zero.

4. Proposed Robust Ctc Design

A robust CTC controller will be presented to improve the performance of the CTC. The proposed control law is

$$\tau = u^{CTC} + u^{ro} \quad \ldots(11)$$

$$u^{CTC} = M_o(q)\big(\ddot{q}_d - k_p\, e - k_v \dot{e}\big) + C_o(q,\dot{q})\dot{q} + F_o(\dot{q}) + G_o(q) \quad \ldots(12)$$

where

u^{CTC} is a standard computed torque that defined in (8).

u^{ro} is a robust control term that can handle the system uncertainties and external disturbance.

Design of robust compensator controller

To solve the problem robustness of the standard CTC, the following control is proposed:

$$u^{ro} = k\, sign(\gamma \mathrm{e(t)} + \dot{\mathrm{e}}\mathrm{(t)}) \quad \ldots(13)$$

Where

k is the gain of the robust term, γ is a positive constant.

Sign is the sign function.

Theorem 1. Consider the robotic manipulator dynamic model (1) with the proposed control law in (2), the closed loop controlled system will be stable and the tracking error and its derivative will converge to zero.

Proof. The Lyapunov function candidate selected as follows:

$S(\mathrm{t}) = \gamma \mathrm{e}(\mathrm{t}) + \dot{\mathrm{e}}(\mathrm{t})$ ...(14)

$\dot{q}_r = \dot{q}_d + \gamma(q_d - q)$ …(15)

By simple calculations it can be obtained the following

$\dot{S}(t) = \dot{q}_r - \dot{q}$ …(16)

$\ddot{S}(t) = \ddot{q}_r - \ddot{q}$ …(17)

$V(t) = \frac{1}{2} S^T M S$ …(18)

$\dot{V}(t) = S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S$ …(19)

$= S^T M \dot{S} + S^T C S$ ...(20)

$= S^T [M(\ddot{q}_r - \ddot{q}) + C(\dot{q}_r - \dot{q})]$ …(21)

$= S^T [M\ddot{q}_r + C\dot{q}_r - M\ddot{q} - C\dot{q}]$ …(22)

$= S^T [M\ddot{q}_r + C\dot{q}_r + F + G - \tau]$ …(23)

Sub (11) in (23)

$\dot{V}(t) = S^T [M\ddot{q}_r + C\dot{q}_r + F + G - u^{CTC} + u^{ro}]$ …(24)

Sub (12) and (13) in (24)

$= S^T \big[M\ddot{q}_r + C\dot{q}_r + F + G - M_o(q)\big(\ddot{q}_d - k_p\, e - k_v \dot{e}\big) - C_o(q,\dot{q})\dot{q} - F_o(\dot{q}) - G_o(q) - k\, sign(\mathrm{S})\big]$ …(25)

Let

$\rho(t) = M\ddot{q}_r + C\dot{q}_r + F + G - M_o(q)\ddot{q}_d - C_o(q,\dot{q})\dot{q} - F_o(\dot{q}) - G_o(q)$ …(26)

$= S^T \big[\rho(t) - M_o(q)\big(k_p\, e + k_v \dot{e}\big)(q) - k\, sign(\mathrm{S})\big]$ ...(27)

$\dot{V}(t) = S^T \big[\rho(t) - k_p M_o(q) e - k_v M_o(q) \dot{e}(\mathrm{t}) - k\, sign(\mathrm{S})\big]$ …(28)

$\dot{V}(t) = S^T \big[\rho(t) - M_o(q) k_v (k_v{}^{-1} k_p e + \dot{e}(\mathrm{t})) - k\, sign(\mathrm{S})\big]$ …(29)

If parameters k_p and k_v selected as follows

$k_v{}^{-1} k_p = \gamma$ …(30)

Sub (30) in (29), yields:

$\dot{V}(t) = S^T [\rho(t) - k_v M_o S - k\, sign(\mathrm{S})]$ …(31)

$= -S^T k_v M_o S + S^T [\rho(t) - k]$ …(32)

$\leq -\|M_o(q)\| \|k_v\| \|S\|^2 - \|k\| \|S\| + \|\rho(t)\| \|S\|$ …(33)

If k selected large enough greater than $\rho(t)$, then

$\dot{V}(t) \leq -\|M_o(q)\| \|k_v\| \|S\|^2$ …(34)

which implies that:

$\dot{V}(t) \leq 0$ …(35)

As a result the stability of the controlled system in (1) with the proposed control law in (11) is guaranteed.

5. Simulation Results

In this section, the performance and robustness of the proposed control method is tested by applying it on the two link robotic arm. Moreover, the proposed method is compared with the standard CTC method. Additionally, integral absolute value error (IAE) performance index is used to examine the tracking error performances in this comparison that can be expressed as follows:

$IAE = \int_0^{tf} |e(t)| dt$ …(36)

The dynamic model of two link robotic manipulator is [21]:

$$\begin{bmatrix}\tau_1\\ \tau_{21}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{12} & A_{12}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix}-2b\dot{q}_2 & -z\dot{q}_2\\ b\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + \begin{bmatrix}v_1\dot{q}_1\\ v_2\dot{q}_2\end{bmatrix} + \begin{bmatrix}p_1 sgn(\dot{q}_1)\\ p_2 sgn(\dot{q}_2)\end{bmatrix} + \begin{bmatrix}G_1\\ G_2\end{bmatrix} \quad …(37)$$

with

$A_{11} = m_1 l_1^2 + m_2[l_1^2 + l_2^2 + 2 l_1 l_2 \cos(q_2)]$

$A_{12} = m_2[l_1 l_2 \cos(q_2) + l_2^2]$

$A_{22} = m_2 l_2^2$

$b = 2 m_2 l_1 l_2 \sin(q_2)$

$z = m_1 l_1 l_2 \sin(q_2)$

$G_1 = m_1 L_{c1} g \cos(q_1) + m_2 g[L_{c2} \cos(q_1 + q_2) + L_1 \cos(q_1)]$

$G_2 = m_2 l_{c2} g \cos(q_1 + q_2)$

where $q_1$and q_2 are angular positions, $\tau_1$and τ_2 are torques, L_1 and L_2 are lengths, m_1 and m_2 are masses, v_1 , and v_2 are coefficients of viscous friction, and p_1 and p_2 are coefficients of dynamic friction of Link1and Link2, respectively. The parameters of the robotic manipulator are selected as: $m_1 = 10\, kg$, $m_2 = 2\, kg$, $l_1 = 1.1$ m, $l_2 = 0.8$ m, $P_1 = P_2 = 30$,and $V_1 = V_2 = 10$. The desired trajectory is $q_d(t) = [q_{d1}\;\; q_{d2}]^T$, where

$\mathrm{q_{d1}} = \sin(2\pi t)$ …(38)

$q_{d2} = \sin(2\pi t)$ …(39)

The controller gains selected as follows: $k_v = 3, k_p = 5, k = 15$.The robustness of the proposed method and standard CTC are checked by varying the parameters of the robotic system by 15% of their nominal. Moreover, a $\sin(t)$ disturbance signal is applied. At second 3, Angular position and error in this position of robotic manipulators are shown in Figures 1 and 2 for the proposed and CTC methods. These figures indicated faster response of the proposed method. The proposed method needs approximately 0.06 seconds until tracking error converges to zero. Tables 1 and 2 list the IAE for proposed control scheme and CTC methods for Link1 and Link2 respectively.

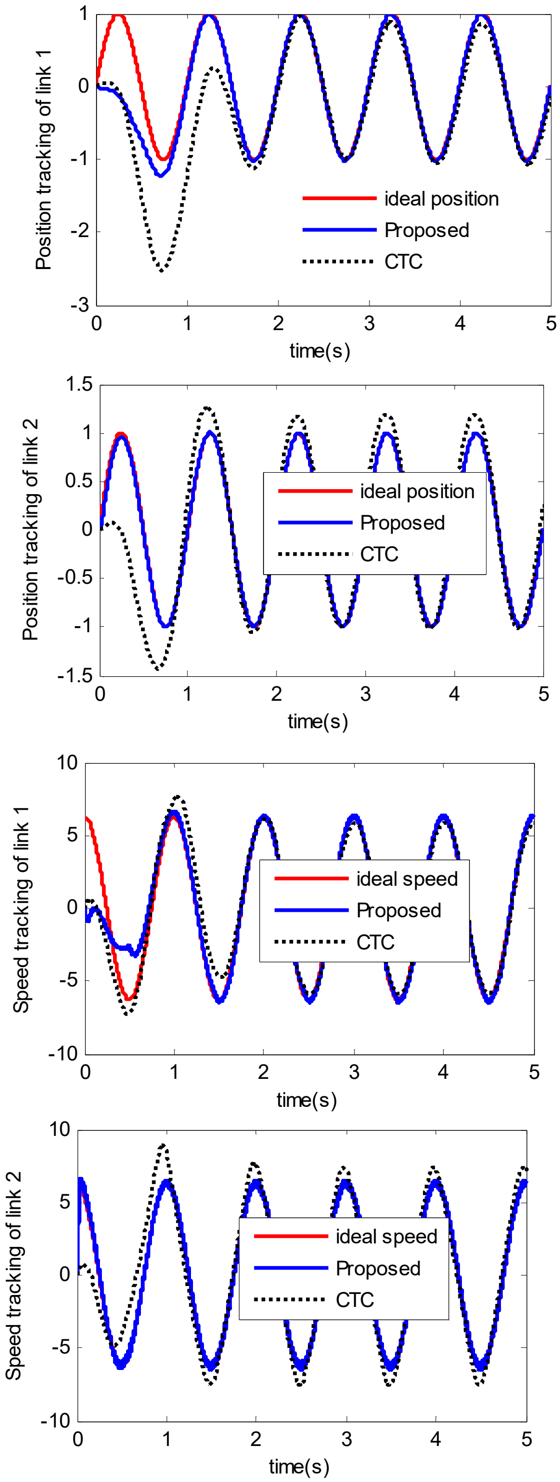

Fig. 1. Results for system uncertainty.

These indices indicate that the performance of the prosed control method is better than standard CTC method in reduction tracking error. Finally, it should be noted that all simulation results illustrate good robustness of proposed method against system uncertainty and external disturbance.

Table 1,
Performance of controllers under system uncertainty

	Proposed	CTC
Link 1	0.3464	2.1859
Link 2	0.0419	1.7772

Table 2
Performance of controllers for disturbance rejection

	Proposed	CTC
Link 1	0.4679	2.7099
Link 2	0.0485	2.2542

6. Conclusion

In this paper, a robust control method proposed for control a robotic manipulator. Robotic manipulator is non-linear system, especially with the inability to represent the system perfectly due to the change in the parameters and the external disturbance. Although, CTC is good control method, but it's not robust to system uncertainties. This paper improves CTC method by adding a new robust term

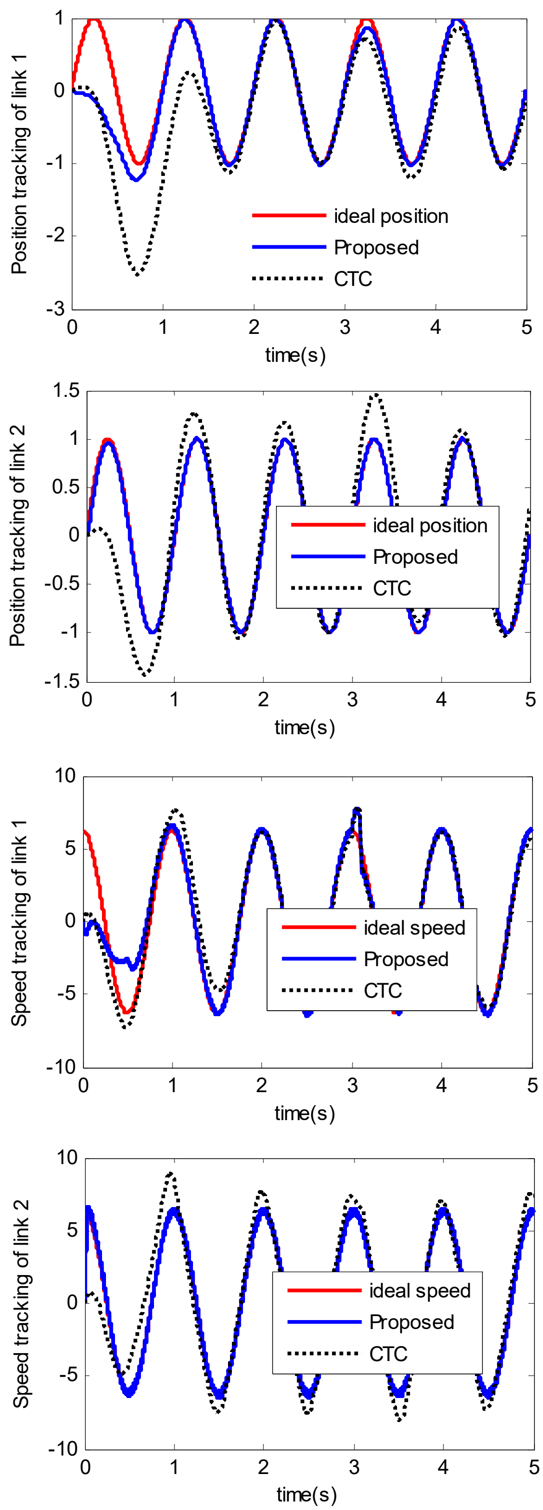

Fig. 2. Results for disturbance rejection.

Moreover, Lyapunov theorem stability has been used to approve stability of the proposed control method. It was concluded from the simulation results that there was a significant improvement in the performance of the proposed control in response to external disturbance and uncertainties in parameters of robotic manipulator. In future work, reinforcement learning can be used to select the optimal gains for the proposed controller parameters. Moreover, the proposed algorithm can be tested in the real robotic manipulator.

7. References

[1] A. Mary, T. Kara and A. Miry, "Inverse kinematics solution for robotic manipulators based on fuzzy logic and PD control", Al-Sadiq International Conference on Multidisciplinary in IT and Communication Techniques Science and Applications(AIC-MITCSA), vol.pp.160-165, 2016.

[2] A. Kashyap and D. Parhi, "Particle Swarm Optimization aided PID gait controller design for a humanoid robot", ISA Transaction, vol. 114, pp.306-330, 2020.

[3] I. Carlucho and G. Acosta, "An adaptive deep reinforcement learning approach for MIMO PID control", ISA Transactions, vol.102, pp.280-249, 2020.

[4] [4] X. Yin, L. Pan and S. Cai, "Robust adaptive fuzzy sliding mode trajectory tracking control for serial robotic manipulators", Robotics and Computer-Integrated Manufacturing, vol. 72, pp. 101884, 2021.

[5] A. Mary and T. Kara," Robust Proportional Control for Trajectory Tracking of a Nonlinear Robotic Manipulator: LMI Optimization Approach", Arabian Journal for Science and Engineering, vol. 41, pp. 5027-5036, 2016.

[6] X. Wu and Y. Huang, "Adaptive fractional-order non-singular terminal sliding mode control based on fuzzy wavelet neural networks for omnidirectional mobile robot manipulator", ISA Transactions, vol. 115, 2021.

[7] Z. Chen, X.Yang and X. Liu, "RBFNN-based non-singular fast terminal sliding mode control for robotic manipulators including actuator dynamics", Neurocomputing, vol. 362, pp. 72-92, 2019.

[8] T. Kara and A. Mary," Robust trajectory tracking control of robotic manipulators based on model-free PID-SMC approach" Journal of Engineering Research, vol. 6, pp. 170-188, 2018.

[9] T. Kara and A. Mary," Adaptive PD-SMC for Nonlinear Robotic Manipulator Tracking

Control" Studies in Informatics and Control, vol. 26, pp. 49-58, 2017.

[10] Th. Costa, F. Lara-Molina, A. Junior and E. Taketa,"Robust H∞ Computed torque Control for Manipulators", IEEE Latin America Transactions, vol. 16, pp. 398-407, 2018.

[11] M. Alwan, Hassan and R. Zaid Hikmat," Motion Control of Three Links Robot Manipulator (Open Chain) with Spherical Wrist", Al-Khwarizmi Engineering Journal 15(2) ,13-23. 2019.

[12] C. Chen, C. Zhang, T. Hu, H. Ni1, and W. Luo,"Model-assisted extended state observer-based computed torque control for trajectory tracking of uncertain robotic manipulator systems", International Journal of Advanced Robotic Systems, vol. 15, pp. 1-12, 2018.

التحكم المتين في متحكم عزم دوران ذراع الانسان الالي

مريم صادق احمد* علي حسين مري هشام حسن جاسم*****
،،***قسم هندسة الميكاترونكس/ كلية الهندسة الخوارزمي/ جامعة بغداد*
*البريد الالكتروني: Maryamsadeq97@gmail.com
**البريد الالكتروني: Alimary76@kecbu.uobaghdad.edu.iq
***البريد الالكتروني: mschisham@gmail.com

الخلاصة

يدرس هذا البحث السيطرة على ذراع روبوت حيث تم إقتراح بناء منظومة سيطرة بالاعتماد على طرق السيطرة الذكية. إذ إن ذراع الروبوت تمثل نظام لا خطيا وخاصة مع عدم القدرة على تمثيل النظام بشكل مثالي بسبب ازعاجات الحمل والإخطاء التي تحصل عند نمذجة النظام وهو مايسمى uncertainty system وهذا من خلال تحسين تحكم عزم الدورانCTC,استخدمت نظرية Lyapunov للحصول على استقرارية النظام. و تم اختبار أداء المتحكم المقترح بواسطه matlab - simulink ومقارنتها مع متحكم عزم الدوران المحسوب التقليدي, حيث توضح نتيجة المحاكاة قوة الطريقة المقترحة ومتانتها والحصول على تتبع مسار جيد وفق الاداء الطلوب.