



## The Linear and Nonlinear Electro-Mechanical Fin Actuator

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### Abstract

Electromechanical actuators are used in a wide variety of aerospace applications such as missiles, aircrafts and spy-fly etc. In this work a linear and nonlinear fin actuator mathematical model has been developed and its response is investigated by developing an algorithm for the system using MATLAB. The algorithm used to the linear model is the state space algorithm while the algorithm used to the nonlinear model is the discrete algorithm. The huge moment constant is varied from (-3000 to 3000) and the damping ratio is varied from (0.4 to 0.8).

The comparison between linear and nonlinear fin actuator response results shows that for linear model, the maximum overshoot is about 10%, rising time is 0.23 sec. and steady state occur at 0.51 sec., while For nonlinear model the maximum overshoot is about 5%, rising time is 0.26 sec. and steady state occurs at 2 sec.; i.e., the nonlinear fin actuator system gives faster and more accurate response than does the linear fin actuator system.

**Keywords:** *Electro-mechanical actuator, Fin, Nonlinear actuators, response.*

### 1. Introduction

The use of electromechanical actuation becomes increasingly popular in the aerospace industry as more importance is placed on maintainability. Electromechanical actuators (EMAS) are being used in the actuation of flight critical control surfaces and in thrust vector control [Milan R. Ristanovic, Dragan V. Lazic and Ivica Indin 2008].

Systems whose actuation mechanisms display both direct, i.e., mechanical work to electrical energy conversion, and converse effects between electrical charge and mechanical work employ electromechanical effects [Anusha Anisette 2007].

Electro-mechanical servo systems have been steadily used in fin position servo systems of guided missiles, because of their momentary overdrive capability, low quiescent power/ low maintenance characteristics and long-term storability. During a flight, fin position servo systems have many uncertainties due to

disturbances, parameter variations, and electrical noises and so on. Furthermore fin position servo systems are subjected to aerodynamic load disturbances, such as the deflection angle of the control fin, the angle of attack and Mach number [Chung-Hee Yoo, Young-Cheol Lee and Sang-Yeal Lee 2005].

In control system design, although linear control theory has wide range of applicability, very often some “nonlinearities” very often must be taken into account.

Although in the last few years the stability analysis of a single – input single – output (SISO) system with saturating actuator was studied using a circle or Popover’s criteria to analyze the stability of saturating system via PI control. Since these criteria can only apply (SISO) to stable plants, a complicated rearrangement of these systems is needed when applied to unstable plants. We should point out that such a technique of analysis is not easily extended to a multivariable case.

The objective of this paper is to formulate and solve the problem using the state – space model and discrete algorithm. After the fin’s position is assumed its velocity and acceleration is found by integrating the position equation to show the velocity and acceleration transient response. Finally a design algorithm is proposed and a comparison is done between the linear and nonlinear fin actuation results.

**2. Fin Actuator Models**

Two types of fin actuator models are discussed in this work : A linear second – order model and a nonlinear second – order model.

**2.1. The Linear Model**

A simple model that could describe an actuator, s dynamics is a linear second – order system with damping zeta ( $\zeta$ ) and natural frequency omega ( $\omega_n$ ). The transfer function of a second – order system is given below, where ( $\delta$ ) is the output and ( $\delta_c$ ) is the input. Figure (1) shows one of many possible methods of

implementing the transfer function as a block diagram.

$$G(s) = (\delta / \delta_c) = (\omega_n^2 / S^2 + 2 \zeta \omega_n S + \omega_n^2) \dots(1)$$

The differential equation describing system dynamics is [Milan R. Ristanovic, Dragan V. Lazic and Ivica Indin 2008, Scott J. Moody1989]:

$$\delta'' = \omega_n^2 (\delta_c - \delta - \delta' \frac{2\zeta}{\omega_n}) \dots (2)$$

The system state equations are :

$$\dot{X} = AX + B f(t) \dots(3)$$

$$\begin{bmatrix} \dot{X1} \\ \dot{X2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} \delta_c \dots(4)$$

$$\delta = [1 \quad 0] \begin{bmatrix} X1 \\ X2 \end{bmatrix} \dots(5)$$

Where  $\delta (X1)$  is the fin position ,  $\delta' (X2)$  is the fin velocity , and  $\delta'' (\dot{X}2)$  is the fin acceleration.

The system response with time in linear model show in fig .2, fig.3 and fig.4 .

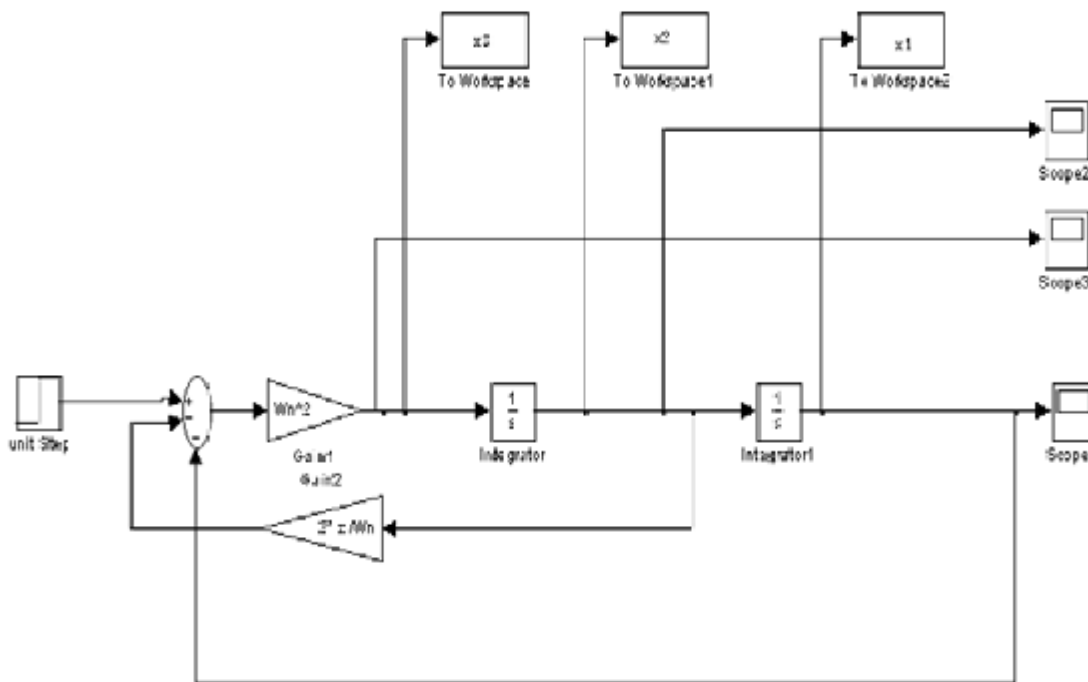


Fig. 1. Second-Order Linear Block Diagram .

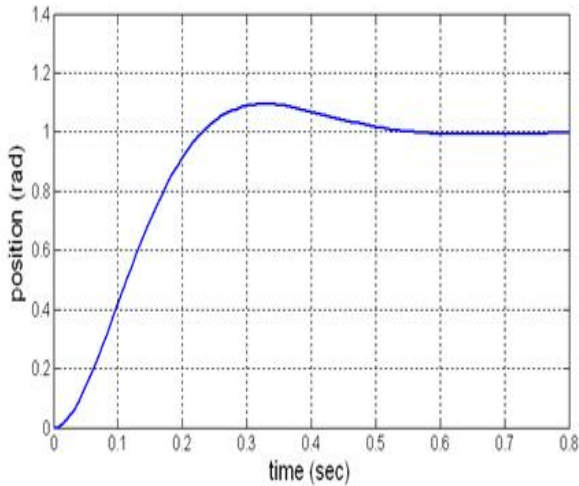


Fig. 2. Linear Position Unit Step Response.

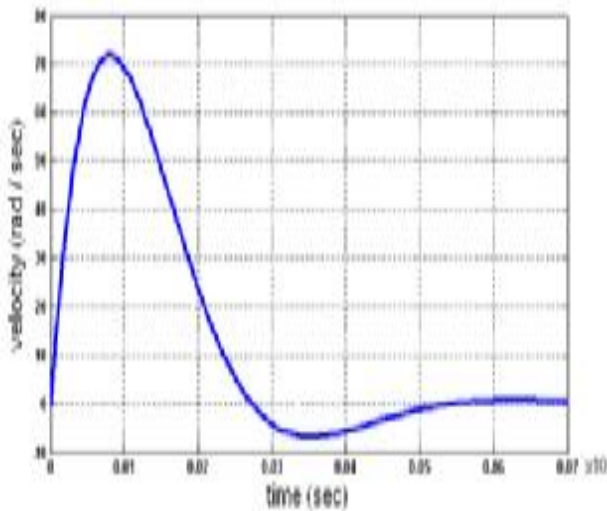


Fig. 3. Linear Velocity Unit Step Response.

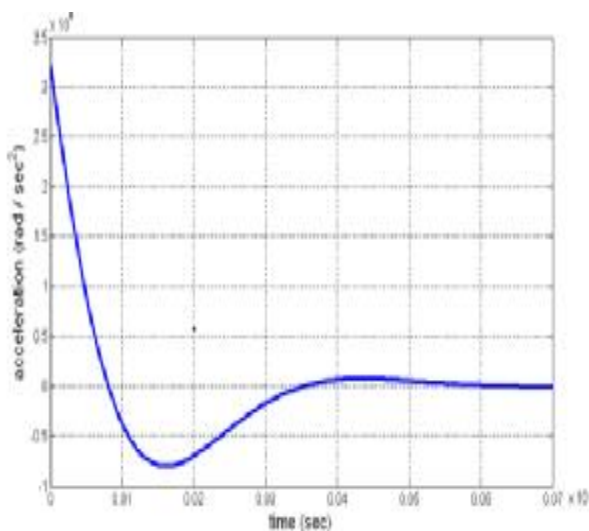


Fig. 4. Linear Acceleration Unit Step Response.

**Analytical Solution**

Since there is a difference between the solutions to the differential equations for actual and simulated systems, an analytical solution will be developed for comparison. The input ( $\delta_c$ ) will be a unit step [Scott J. Moody1989]:

$$\delta_c = 1 \quad \dots (6)$$

After Laplace transfer:

$$\delta_c = 1/S \quad \dots(7)$$

From the transfer function:

$$\delta (S) = G (S) . \delta_c (S) = ( \omega_n^2 / S ( S^2+2 \zeta \omega_n S+ \omega_n^2 ) ) \quad \dots(8)$$

$$\delta (t) = L^{-1} [ \delta (S) ] \quad \dots(9)$$

From a table of Laplace transform, find the solution to  $\delta (t)$ :

$$\delta (t) = [ ( 1/ \omega_n^2 ) - (1/ \omega_n^2 \sqrt{1 - \zeta^2}) e^{-\zeta \omega_n t} \sin ( \omega_n \sqrt{1 - \zeta^2} + \tan^{-1} ( \sqrt{1 - \zeta^2} / \zeta ) ) ] \omega_n^2 \quad \dots(10)$$

Where  $\tan^{-1} ( \sqrt{1 - \zeta^2} / \zeta ) = \cos^{-1} \zeta$

$$\delta (t) = [ 1 - (1/ \sqrt{1 - \zeta^2}) e^{-\zeta \omega_n t} \sin ( \omega_n \sqrt{1 - \zeta^2} t + \cos^{-1} \zeta ) ] \quad \dots(11)$$

For a solution to the first integral,  $\dot{\delta} ( t )$ :

$$\dot{\delta} ( t) = L^{-1} [ S \delta (S) - \delta ( 0^+ ) ] \quad \dots(12)$$

Assume  $\delta ( 0^+ ) = 0$

$$S \delta (S) = ( \omega_n^2 / (S^2+2 \zeta \omega_n S+ \omega_n^2 ) ) \quad \dots(13)$$

From the Laplace transform table:

$$\dot{\delta} ( t) = \omega_n^2 [ ( 1/ \omega_n \sqrt{1 - \zeta^2}) e^{-\zeta \omega_n t} \sin ( \omega_n \sqrt{1 - \zeta^2} t ) ] \quad \dots(14)$$

$$\dot{\delta} ( t) = [ \omega_n e^{-\zeta \omega_n t} \sin ( \omega_n \sqrt{1 - \zeta^2} t ) / \sqrt{1 - \zeta^2} ] \quad \dots(15)$$

For example using an  $\omega_n$  of 144 rad/sec, and a  $\zeta$  of 0.6, we get:

$$\delta (t) = 1 - 1.25 e^{-86.4t} \sin( 115.2t + 0.927 ) \text{ rad} \quad \dots(16)$$

$$\dot{\delta} (t) = 180 e^{-86.4t} \sin( 115.2t ) \text{ rad/sec} \quad \dots(17)$$

The system response with time in linear model with different values of zeta to find the best one of stability to position show in fig .5 , velocity show in fig.6 and acceleration slow in fig.7

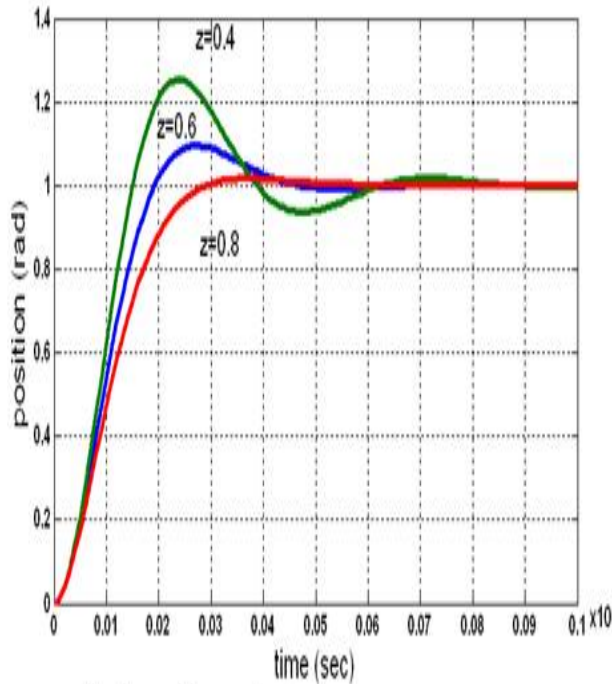


Fig. 5. Linear Position Unit Step Response .

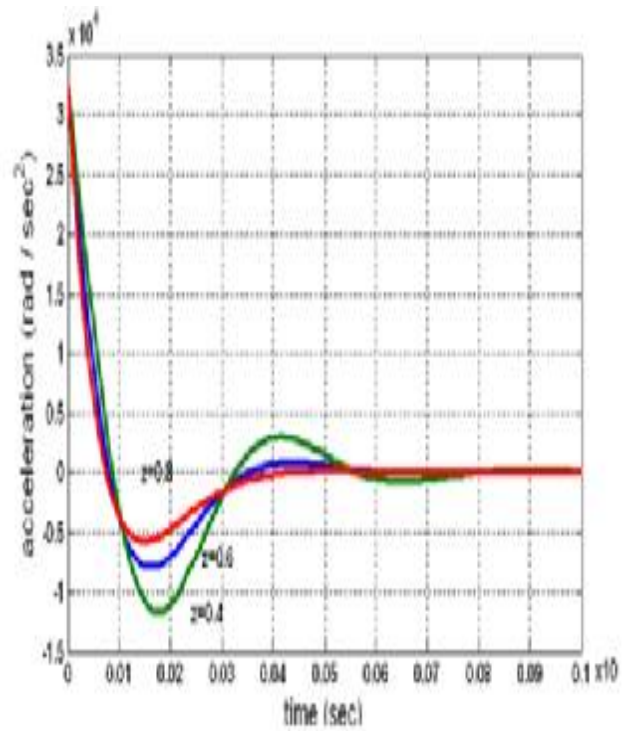


Fig. 7. Linear Acceleration Unit Step Response .

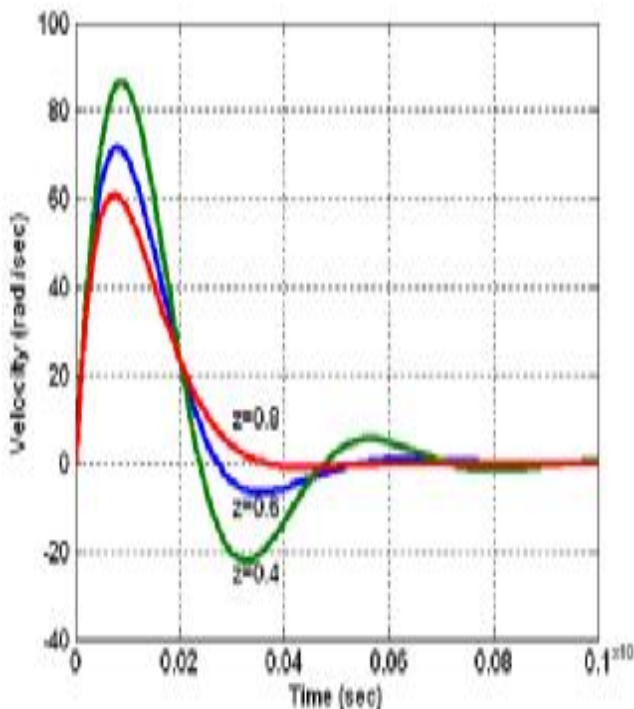


Fig. 6. Linear Velocity Unit Step Response .

## 2.2. The Nonlinear Model

A second type of model is one that contains physical limitations, which were added to the linear second-order model to yield a second-order nonlinear model.

The second-order linear model was modified to include characteristics typical of an actuator motor. This resultant nonlinear model more closely emulates the real thing. These characteristics are inherent limitations of the physical system and are nonlinear. They include; position limits ( fin stops ), velocity limits ( slew rate limits ), acceleration limits ( finite torque ), a dead band in the rate feedback , and aerodynamic hinge moments. Fig.(8) shows a block diagram of second-order nonlinear model

As it can be seen from the block diagram, there is no trivial analytical solution for the differential equation [Scott J. Moody1989]:

$$\ddot{\delta} = \omega_n^2 ( \delta_c - \delta_{LIM} - RATEFB - \dot{\delta} \frac{2\zeta}{\omega_n} ) - HM \dots (18)$$

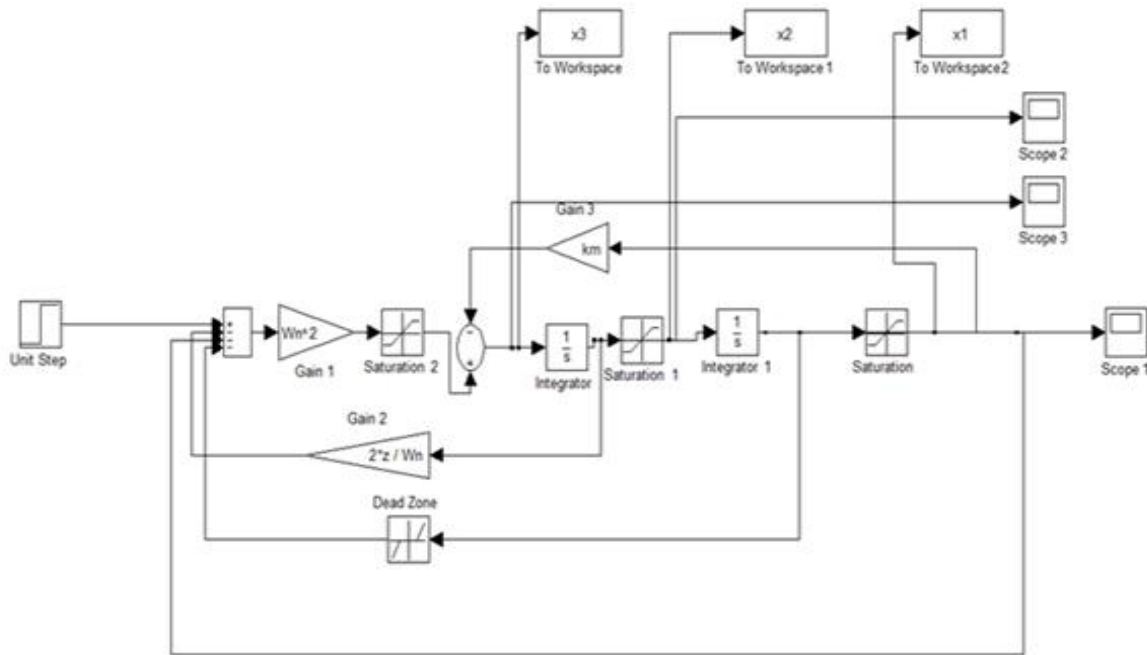


Fig. 8. Second Order Non-Linear Block Diagram .

### 2.3. The Nonlinearities in the Control System

Nonlinearities in control systems may appear due to one or more combination of the following [Choudhury 2008]:

- a. The process may be nonlinear in nature.
  - b. The control system may have a nonlinear characteristic.
  - c. The control system may develop nonlinear faults (the work in this paper focuses on this type of nonlinearities by studying each fault and its effect on the servo system).
  - d. A nonlinear disturbance may enter the system
- The main nonlinearities discussed in this paper are dead zone and saturation.

### 3. Simulation Results

The response is for unit-step input because it is the type of input used in the control systems, and the simulations are carried out using MATLAB.

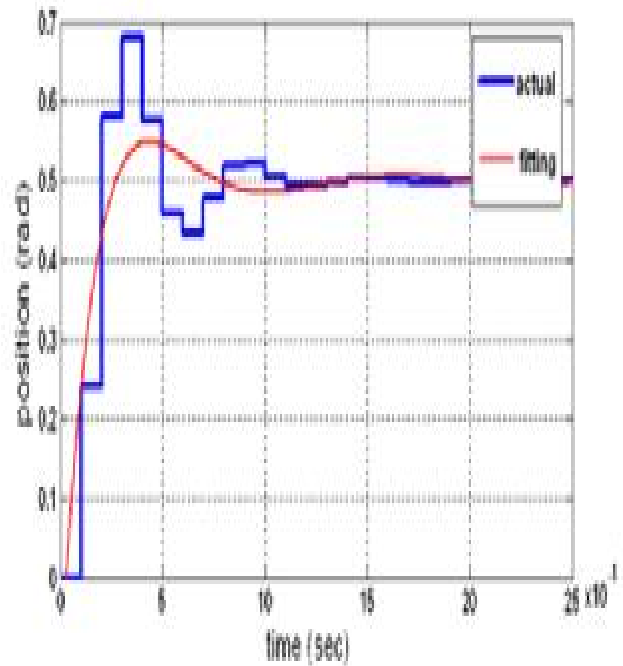


Fig. 9. Non-Linear Position Unit Step Response.

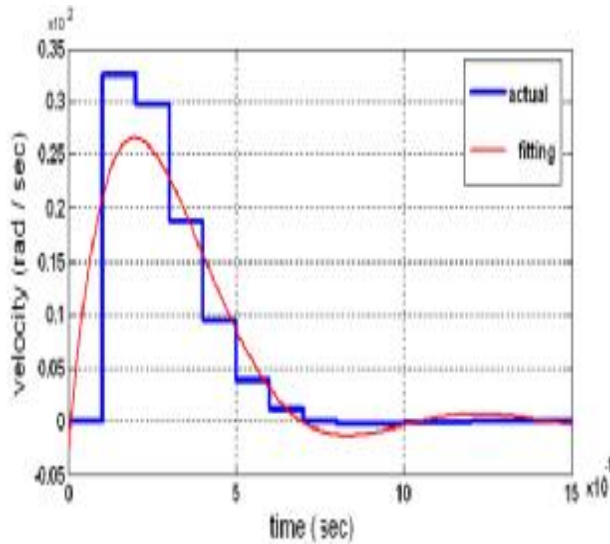


Fig. 10. Non-Linear Velocity Unit Step Response

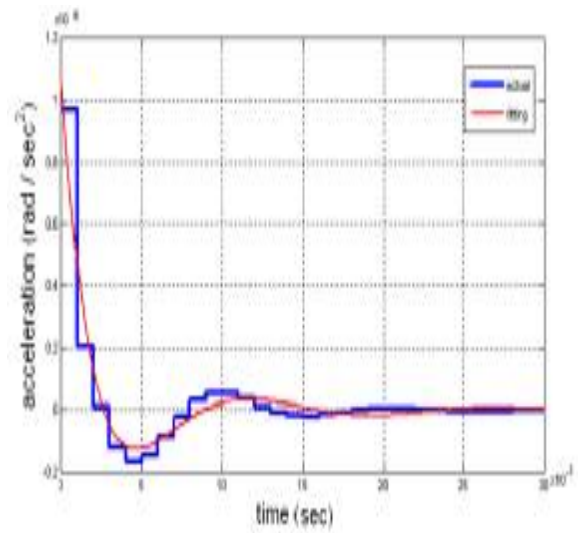


Fig. 11. Non-Linear Acceleration Unit Step Response.

The position response with time in non-linear model with different value of Km show in fig .12

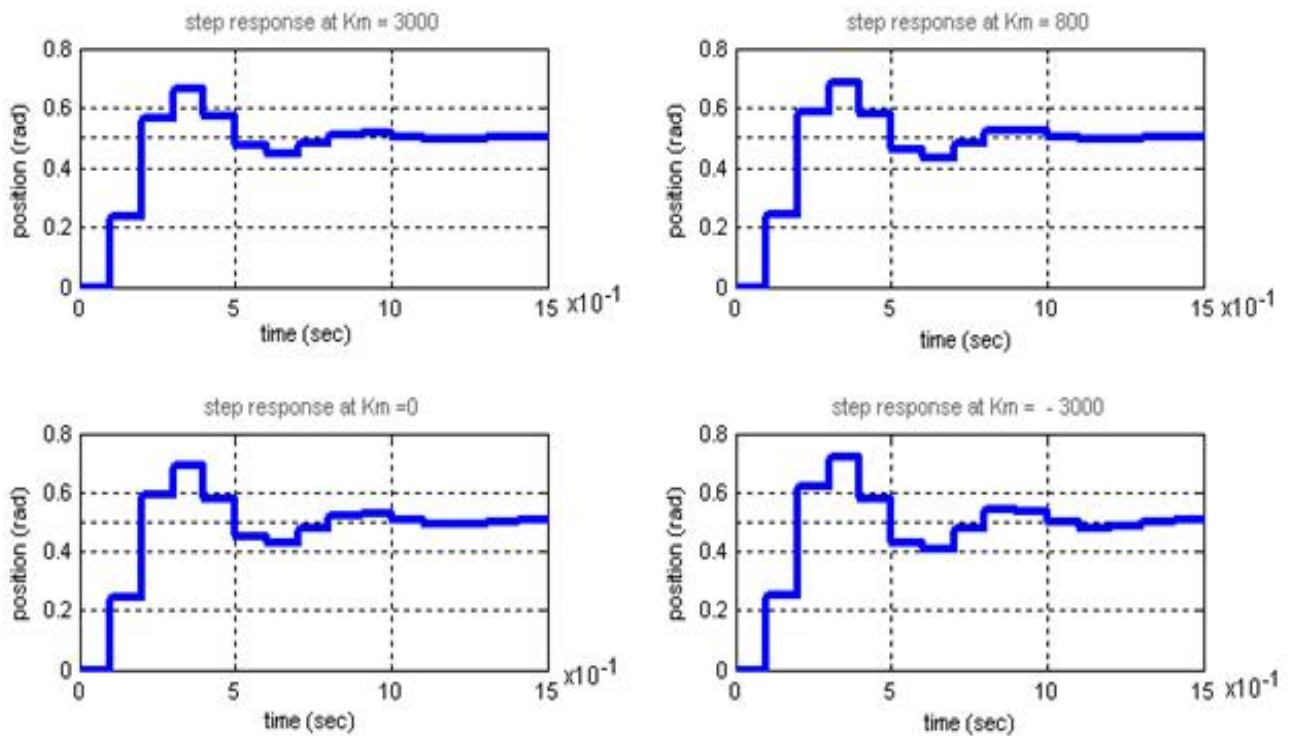


Fig. 12. Non-Linear Position Unit Step Response with Different Km.

#### 4. Results and Discussion

The results of the simulation as follow

Fig.(2) shows the position unit–step response with time for linear model, the maximum overshoot is about 10% , rising time is 0.23 sec. and steady state occur at 0.51 sec.

Fig.(3) shows the velocity unit–step response with time for linear model, the max value of velocity is 70 rad/sec. and the steady state occur at 0.51 sec.

Fig.(4) shows the acceleration unit–step response with time for linear model, the max value of acceleration is  $3.2 * 10^4$  rad/sec<sup>2</sup>. and the steady state occur at 0.51 sec.

Fig.(5) shows position unit–step response with time for linear model when  $\zeta$  is between( 0.4 - 0.8), the maximum overshoot is between (30% - 2%), while rising time between( 0.15 - 0.3) sec. and the steady state occur between (0.8 - 0.4) sec. i.e. if we decreased  $\zeta$ , the overshoot increased and rise time is faster.

Fig.(6) shows the velocity unit–step response with time for linear model model when  $\zeta$  is between( 0.4 - 0.8), the max value of velocity is between ( 90 – 60) rad/sec. and the steady state occur between( 0.8 – 0.4) sec.

Fig.(7) shows the acceleration unit–step response with time for linear model when  $\zeta$  is between( 0.4- 0.8), the max value of acceleration is between (  $3.3 * 10^4$  -  $3.1 * 10^4$ ) rad/sec<sup>2</sup>. and the steady state occur between( 0.8 – 0.4) sec.

Fig.(9) shows the position unit–step response with time for nonlinear model ,the maximum overshoot is about 5% , rising time is 0.26 sec. and steady state occur at 2 sec.

Fig.(10) shows the velocity unit–step response with time for nonlinear model, the max value of velocity is 27 rad/sec. and the steady state occur at 2 sec.

Fig.(11) shows the acceleration unit–step response with time for nonlinear model, the max value of acceleration is  $10^4$  rad/sec<sup>2</sup>. and the steady state occur at 2.2 sec.

Fig.(12) shows the nonlinear position unit–step response when various hinge moment constants were added between (- 3000 to 3000 ), the overshoot increased and the rise time is faster. The hinge moment HM is a function of the fin deflection and a hinge moment constant Km.

The results of the simulation show that the decreasing in maximum overshoot and the increasing in rising and steady state times for the nonlinear model because of nonlinearities effectiveness. However the nonlinear second

order system gives faster and more accurate response especially in the presence of system parameter variations and external disturbances than did the linear system.

#### 5. Conclusions

For linear model the transient response depended on the value of  $\zeta$ , so if we decrease  $\zeta$ , the overshoot increased and rise time is faster, while for nonlinear model the rise time is lengthened but the overshoot was less. In another hand when various hinge moment constants were added we found that the overshoot increased and the rise time was faster . So, the nonlinear second order system closely modeled the actuator's dynamics and physical characteristics than did the linear system and gave faster and more accurate response especially in the presence of system parameter variations and external disturbances.

#### Notation

t	time	sec
s	Laplace transform	
km	constant	
HM	Hinge Moment	m. N

#### Greek letters

$\xi$	Damping ratio
$\Delta$	Time change (t2-t1)
$\omega_n$	Natural Frequency rad /sec
$\delta_c$	Unit step function (input)
$\delta$	Fin position (output) rad
$\delta'$	Fin velocity rad /sec
$\delta''$	Fin acceleration rad /sec

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## مشغل الزعنفة الكهروميكانيكي الخطي واللاخطي

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### الخلاصة

تستعمل المشغلات الكهروميكانيكية في أنواع مختلفة من التطبيقات الفضائية مثل الصواريخ والطائرات وحشرات التجسس وغيرها. في هذا البحث تم بناء نموذج رياضي لمشغل الزعنفة الخطي واللاخطي وقمنا بالتحقيق حول استجابته وذلك ببناء خوارزمية للمنظومة وتطبيقها برمجيا باستخدام MATLAB. الخوارزمية المستخدمة للأنظمة الخطية هي خوارزمية state space بينما الخوارزمية المستخدمة للأنظمة اللاخطية هي الخوارزمية المنفصلة، وقد تم حساب قيم متغيرة لثابت العزم المضخم ما بين (-3000 إلى 3000) وتغير قيم نسبة التخميد ما بين (0.4 إلى 0.8). من المقارنة بين نتائج الاستجابة لمشغل الزعنفة الخطي واللاخطي وجد ان maximum overshoot 10% و rising time 0.23 ثانية ويحدث الاستقرار عند 0.51 ثانية لمشغل الزعنفة الخطي بينما لمشغل الزعنفة اللاخطي يكون maximum overshoot 5% و rising time 0.26 ثانية ويحدث الاستقرار عند 2 ثانية، أي انه مشغل الزعنفة اللاخطي يعطي استجابة اسرع وادق من مشغل الزعنفة الخطي.